Fano Factor of Scintillation Detectors from Photoelectron Correlations

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Abstract—In this work we propose a new experimental approach to studying the statistics of the scintillation light from scintillation detectors. Two photodetectors are used for detecting the light output from the two faces of a thin scintillator crystal. From the correlation of the signals from the two photodetectors, the Fano factor \( F_{\text{opt}} \) of the scintillation light is determined. Data on several inorganic crystalline scintillators (CsI:Na, LaBr\(_3\):Ce, and CdWO\(_4\)) are presented, and in LaBr\(_3\):Ce it is observed that the correlations are negative and \( F_{\text{opt}} << 1 \) (sub-Poisson statistics).

Index Terms—Fano factor, Photoelectron anticorrelation, sub-Poisson statistics.

I. INTRODUCTION

The Fano factor of an integer-valued random variable is defined as the ratio of its variance to its mean[1]; a Fano factor of one corresponds to Poisson statistics. In scintillation detectors, the Fano factor can be defined for the number of optical photons or for the resulting photoelectrons in a photodetector.

Inorganic crystalline scintillators are an excellent medium for radiation detection, with high stopping power, good scintillation yield, fast decay time and ease of handling[2]. Semiconductor detectors such as silicon (Si), or germanium (Ge), however, show much better energy resolution, less than that predicted by Poisson statistics and as expected from a small Fano factor. For scintillation detectors, the best experimental energy-resolution measurement is worse than that predicted by Poisson statistics. It is always observed that the Fano factor for the photoelectrons in inorganic crystalline scintillators is greater than one, as illustrated in the widely published figure below[3], [4].

These observations are often expressed by the conventional wisdom: "Scintillation detectors are always limited by Poisson statistics." Motivated by the current trends of scintillation detectors (high conversion efficiency, high quantum efficiency of the photodetector and good electronics), we believe it is time to reexamine the conventional wisdom.

In this paper we present an alternative experimental method of measuring the Fano factor of scintillation detectors, not based on the width of spectral lines. Instead, the method is based on detecting the scintillation light with two photomultipliers (PMTs), each of which collects approximately half of the light, and observing the correlations of the output signals.

II. THEORY

The theory used in this paper is presented in more detail in a pending paper[5], but the main results are summarized here. Background material on Poisson and sub-Poisson statistics is reviewed in [6] and [7].

The interaction of an incident gamma ray in a scintillation detector produces a random number \( N \) of optical photons, with a mean \( \bar{N} \) and a variance \( \sigma_N^2 \). The Fano factor for the optical photons is defined as

\[
F_N = \frac{\sigma_N^2}{\bar{N}}. \tag{1}
\]

A fraction \( \eta \) of the of the \( N \) optical photons will reach the photodetector and produce \( n \) photoelectrons, where \( n \) is also a random number, with a mean \( \bar{n} = \eta \bar{N} \) and a variance \( \sigma_n^2 \). The Fano factor for photoelectrons is

\[
F_n = \frac{\sigma_n^2}{\eta \bar{N}}, \tag{2}
\]

where \( \eta \) is the product of the geometric collection efficiency and the quantum efficiency (QE) of the photodetector. It can
be shown \[6\] that the two Fano factors are related by the following equation:

$$F_n = 1 + \eta (F_N - 1).$$  \hspace{1cm} (3)

This equation shows that the number of photoelectrons \(n\) is a Poisson random variable if the number of optical photons is Poisson \((F_N = 1)\), and it approaches a Poisson if the overall efficiency is poor \((\eta << 1)\), regardless of \(F_N\).

A fundamental equation, known for decades\[6\], shows that the photoelectrons produced in two different detectors by the same scintillation event are correlated according to:

$$\langle (n_1 - \bar{n}_1)(n_2 - \bar{n}_2) \rangle = \bar{n}_1 \bar{n}_2 (F_N - 1),$$ \hspace{1cm} (4)

where the angle brackets denote statistical averaging. This result shows that Poisson sources \((F_N = 1)\) produce uncorrelated counts in different photodetectors, and that sources with \(F_N \neq 1\) produce approximately uncorrelated photoelectrons if the efficiencies \(\eta_1\) and \(\eta_2\) are small. The key to observing the photoelectron correlations is therefore to have efficient light collection and high QE on the photodetectors.

We do not, however, directly observe the number of photoelectrons. Typically we use a photomultiplier (PMT) and some chain of electronics to produce a signal \(s_j\) on detector \(j\) \((j = 1, 2)\). The signal is given by \(s_j = \eta_j G_j \bar{n}\), where \(G_j\) is the gain of the PMT and associated electronics.

A full treatment of the statistics of the signals, given in \[5\], includes the randomness of each optical photon and their collection and conversion to photoelectrons. If we assume that the two PMTs and electronic chains are identical and omit the subscripts on \(G_j\) and \(\eta_j\), the covariance matrix for the two signals is found to be

$$K^{(s)} = \frac{\bar{G}^2 \bar{n}}{\eta(1 + \alpha) + \eta^2 (F_N - 1)} \begin{bmatrix} \eta(1 + \alpha) + \eta^2 (F_N - 1) & \eta^2 (F_N - 1) \\ \eta^2 (F_N - 1) & \eta(1 + \alpha) + \eta^2 (F_N - 1) \end{bmatrix}$$  \hspace{1cm} (4)

where \(\bar{G}\) is the mean overall gain (PMT plus electronics) and \(\alpha\), defined as \(\text{Var}(G)/\bar{G}^2\), accounts for the PMT gain noise.

From the off-diagonal elements of \(K^{(s)}\), we can see that if \(F_N < 1\) implies negative correlations, but the factor of \(\eta^2\) shows that the effect disappears rapidly for inefficient light collection or low QE. The variances (diagonal elements) have one term that scales as \(\eta\) and another that scales as \(\eta^2\). Thus, for small \(\eta\) we get the Poisson result (variance = mean and no correlation).

We note also that the PMT gain noise affects not only the variances but not the covariances of the PMT signals. Thus, sub-Poisson statistics are easier to see by correlation measurements than by energy-resolution measurements.

To be able to estimate the Fano factors from the measured signals, we define a correlation coefficient, denoted \(r_{12}\), by

$$r_{12} = \frac{K^{(s)}_{12}}{\sqrt{K^{(s)}_{11} K^{(s)}_{22}}} = \frac{\eta (F_N - 1)}{1 + \alpha + \eta (F_N - 1)}. \hspace{1cm} (5)$$

If this quantity is estimated from the photopeak data, we can obtain an estimate of the Fano factor of the photoelectrons by\[5\]

$$\hat{F}_n = \frac{1 + \alpha \hat{r}_{12}}{1 - \hat{r}_{12}},$$  \hspace{1cm} (6)

where the caret denotes an estimate.

If \(\eta\) is known or estimated independently, we can also use \(\hat{r}_{12}\) to estimate the underlying Fano factor for the optical photons as

$$\hat{F}_N = 1 + \frac{\hat{r}_{12}}{1 - \hat{r}_{12}} \cdot \frac{1 + \alpha}{\eta}.$$  \hspace{1cm} (7)

### III. Experimental Setup

The experimental setup (Fig. 2) consists of a small thin crystal (between 0.5 and 1 mm thick and 3 by 3 mm to 10 by 10 mm laterally), and two identical PMTs, each coupled to one side using optical grease (V-788 optical coupling compound from Visilox Systems). We used two PMT pairs, Hamamatsu R6233-100 PMTs with super-bialkali (SBA) photocathodes, which had a peak QE of 35% as reported by the manufacturer, and Hamamatsu R7600-200 PMTs with ultra-bialkali (UBA) photocathodes (QE of 45%).

In the case of a slow scintillator, the anode signals were fed to a preamplifier and a shaping amplifier. For a fast scintillator (LaBr₃), however, the anode signals were sometimes fed directly to the oscilloscope by a 50 Ω characteristic impedance coaxial cable terminated in 50 Ω resistance inside the oscilloscope. An AND gate in the oscilloscope was used for trigger. Both pulses were stored on the disk for off-line analysis. In each run we saved between 30,000 and 50,000 pulses from each channel. From each pulse we extracted the energy either from the pulse height or the integral under the pulse waveform.

Because CsI(Tl) and LaBr₃ are hygroscopic, they were received from the manufacturer embedded in mineral oil. Each time we used a sample, we removed it from the mineral oil, cleaned it using a fine emery paper, and coupled it to the two PMTs using the optical grease. The scintillator was irradiated with different gamma ray sources, but the most used is Cs-137 (gamma-ray energy at 662 keV). Both flood and collimated beams were used in different experiments.

![Fig. 2. System for collecting light from opposite sides of a thin scintillator.](image)
IV. DATA PRESENTATION AND DISCUSSION

To investigate the possible relationship between the two PMTs signals $s_1$ and $s_2$, data are displayed as scatter plots, where each event $(s_1, s_2)$ serves as coordinates of one point. We can also sum the two channels and construct the energy spectrum.

The data obtained in a typical experimental run are shown in Fig. 3. In this case the sample was 10mm×10mm×1mm slab of CsI(Tl) and the SBA PMTs were used. The photopeak is the elliptical grouping of points in the upper right portion of the scatter plot (Fig. 3 left), corresponding to the higher-energy peak in the spectrum (Fig. 3 right). Also evident on both the scatter plot and the spectrum is a portion of the Compton continuum; lower-energy events are absent because of the threshold imposed during data collection.

The most important observation from this figure, however, is that the photopeak signals for the two PMTs are positively correlated, because the photopeak events form a band running from lower left to upper right (if $s_1$ increases, $s_2$ increases). The photopeak energy resolution was about 6% full width at half maximum (FWHM), which is typical for CsI(Tl) at 662 keV.

A different behavior is shown in the data in Fig. 4, which were obtained on a 7mm×6mm×1mm slab of LaBr$_3$(Ce) with a collimated 662 keV gamma ray from Cs-137 isotope and the SBA PMTs. In the scatter plot of this figure, the photopeak shows a negative correlation, when $s_1$ increases, $s_2$ decreases (major axis of the elliptical cluster of points is perpendicular to the diagonal). The photopeak in the spectrum plot shows an excellent energy resolution of 2.1% (compared to 2.7-3.2% reported in the literature[8], [9]). Below the photopeak, the subsidiary peak is the escape peak at 624.6 keV which is from the strongest x-ray line, La Kα (37.4 keV); this peak is also seen in the scatter plot as a small cluster of points below the photopeak.

V. RESULTS

To investigate the correlations quantitatively we isolated the photopeak by truncating the scatter plots and binning them into pixel arrays as shown on the upper left in Fig. 6.

To estimate the covariance matrix $K(s)$ for the photopeak signals, we performed a least-squares fit of the photopeak array to a correlated 2D Gaussian function truncated to the same region as the isolated photopeak.

<table>
<thead>
<tr>
<th>Photocathode</th>
<th>$r_{12}$</th>
<th>$\hat{F}_\alpha$</th>
<th>$\hat{F}_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBA</td>
<td>-0.32 ± 0.17</td>
<td>0.72 ± 0.06</td>
<td>0.10 ± 0.16</td>
</tr>
<tr>
<td>UBA</td>
<td>-0.36 ± 0.17</td>
<td>0.68 ± 0.07</td>
<td>0.09 ± 0.20</td>
</tr>
</tbody>
</table>

Using (5), (6) and (7), we obtained the results summarized in Table 1. To estimate $\eta$, we corrected the manufacturer's
quoted QE, presumably measured at normal incidence in air, for the reflectivity of the photocathode[10], [11] and for an expected increase in QE away from normal incidence[12], [13], and we also assumed that the geometrical collection efficiency of each PMT was 0.5. The resulting estimates of $\eta$ were 0.28 for SBA and 0.35 for UBA.

The most extensive data were obtained from LaBr$_3$ samples. We used many samples, different experimental procedures, with and without the shaping amplifier, and different ways of isolating the photopeak. Error bars indicate standard deviation over multiple samples, multiple surface finishes, multiple acquisition parameters and multiple methods of data analysis.

The table shows that the observed Fano factor for photoelectrons $F_n$ is significantly less than one and that the inferred optical Fano factor for optical photons $F_N$ is very small. The error bars do not rule out $F_N=0$. The latter conclusion holds no matter what we assume for eta.

VI. CONCLUSION

In summary, we have presented an new experimental method for estimating the Fano factor of the optical photons and resulting photoelectrons produced by gamma rays in scintillation detectors. We used more than 10 LaBr$_3$ samples, and repeated the measurement many times for each sample. All samples showed negative photopeak correlations for all experimental procedures and methods of data analysis. Three samples of CsI have been investigated, and all showed positive photopeak correlations.

The conversion of correlation coefficients to electron Fano factors is hampered by inaccurate estimates of PMT gain variance, and the conversion of electron Fano to photon Fano is hampered by inaccurate estimates of effective PMT QE. It would take a more careful study of the reflectivity of the photocathodes to refine the optical Fano factors.

Nevertheless, the optical Fano factor of LaBr$_3$ is very low, comparable to that of good semiconductor detectors; the error bars do not rule out $F_N=0$.

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REFERENCES