RCS CHARACTERIZATION USING THE ALPHA-STABLE DISTRIBUTION*

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ABSTRACT

The radar backscatter from complex sources, such as ships and ocean waves, can vary rapidly with target aspect or time. The radar cross section (RCS) of such targets is usually described in statistical terms using one of the many statistical models that are available. These models, however, tend to fit less well when the amplitude fluctuations begin to vary over wider extremes and become impulsive in nature. To better handle this condition, the alpha-stable distribution is shown to model RCS over a wide range of amplitudes. The alpha-stable distribution is derived from the generalized central limit theorem and contains the Gaussian (or Rayleigh) distribution as a subset. The alpha-stable distribution is shown to fit examples of Ship RCS as well as sea clutter examples. The performance of various envelope detectors including the maximum likelihood detector for the alpha-stable distribution is shown for a low signal-to-noise (SNR) case.

1.0 INTRODUCTION

The envelope of the backscatter from complex radar targets can vary rapidly with changing aspect angle (Nathanson, 1969). Because of the amplitude fluctuations, the RCS is best characterized using statistical terms such as averages, medians and distributions. Commonly used distribution functions (single pulse; envelope or envelope squared) for target modeling analysis include: Rayleigh (Swerling cases 1 and 2), chi-square (Swerling cases 3 and 4 for four degrees-of-freedom), Rician, log-normal, and Weibull. The finite RCS, beta distribution (Maffett, 1989) has also been advocated. The density functions and their parameters form RCS distribution models or statistical RCS models. Because of large mean-to-median ratios, the RCS distribution of ships, for example, have been represented by the log-normal distribution (Nathanson, 1969). The models are used to predict target detectability.

Next, when modeling the radar backscatter from ocean waves, sea clutter returns change rapidly with time. The radar returns are generally more spiky for horizontally polarized measurements and for high spatial resolution measurements. The K-distribution is currently a widely accepted model for representing the amplitude or envelope statistics of sea clutter (Armstrong, 1991 and, Nohara 1991). This distribution is based on the assumption that the radar return from a region consists of the sum of independent returns (speckle with “short” decorrelation time) that vary in intensity (chi-distributed) with time (modulation with “long” decorrelation time). Knowledge of the distribution can result in detector designs that are tailored to the clutter statistics.

Applying conventional distributions to targets or sea clutter can become less accurate when the returns are more impulsive or spiky in nature (large mean-to-median ratio). Estimates of statistics such as the mean RCS (arithmetic average of the envelope squared, or mean-square of the envelope) can give inconsistent results. To avoid giving too much weight to the large spikes, the conventional approaches are to ignore the spikes (treat as outliers), to use geometric averages (mean of the log in dB), or to use the median RCS. This becomes the basis for modeling using empirical distributions such as log-normal or Weibull. The quality of the fit of these models to the distribution function determines their range of applicability.

2.0 ALPHA-STABLE DISTRIBUTION

The alpha-stable distribution is proposed as a statistical model that can closely describe RCS over a wide range of amplitudes. This distribution appears to be particularly applicable for radar returns from targets and sea clutter that are impulsive or spiky in nature. The alpha-stable distribution is described in recent literature (Nikias, 1995). Alpha is


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the characteristic exponent that varies from zero to two. For an alpha of two, the distribution is Gaussian and the envelope is Rayleigh distributed. As alpha becomes smaller than two, the distribution becomes more impulsive (the tail of the envelope distribution becomes thicker) with infinite variance (including all higher-order moments). For alpha less than or equal to one, the mean is infinite. From the generalized central limit theorem, the alpha-stable distribution is the only limiting distribution for sums of independent and identically distributed (IID) random variables. The Gaussian distribution (alpha equal to two) results when the IID random variables have finite mean and variance. Alpha less than two occurs when the IID random variables have infinite mean or variance. An example is the amplitude distribution of static from an AM radio with an antenna located in the middle of a lightning storm.

When the RCS is closely modeled by the alpha-stable distribution, estimates of the mean RCS, are expected to be inconsistent and appear to be non-robust or non-stationary. The mean RCS is the second moment of the envelope. For the alpha-stable distribution, however, lower-order moments, the median and alpha can be consistently estimated from measurements of the envelope of the radar returns. For sea clutter, these parameters can be estimated from the time series where the radar illuminates a fixed area (down-range resolution and beam-width), sea state, grazing angle, wave direction, etc. For target RCS, the distribution parameters can be estimated over different ranges of target aspect, radar frequency, etc. RCS is fully described by the median RCS and alpha.

The I and Q samples from a radar system are taken to have a bivariate, isotropic, alpha-stable distribution (Nikias, 1995) where the distribution of the envelope, \( \alpha \), is

\[
f(a) = a \int_0^\infty s e^{-\gamma s^\alpha} J_0(as) ds \quad \text{where} \quad a = \sqrt{x_1^2 + x_2^2}
\]

Alpha, \( \alpha \), is the characteristic exponent that varies from zero to two, and \( \gamma \) is the scale parameter or dispersion. Graphs of the envelope density function for various alpha are given in figure 1. Closed form solutions only exist for alpha of 1.0 and 2.0, so numerical integration of equation (1) is required. The density function is normalized by the negative first-order moment, \( m_{-1} \), of the envelope. The moment is given by

\[
m_{-1} = \frac{\Gamma(1/\alpha)}{\alpha \gamma^{1/\alpha}} \quad \text{and estimated by} \quad \hat{m}_{-1} = \frac{1}{K} \sum_{k=1}^K \frac{1}{a_k}
\]

For a range of alpha from about 1.3 to 2, the median envelope is approximately 1.5 times the reciprocal of \( m_{-1} \) (Pierce, 1996). Alpha is estimated by first calculating \( \hat{m}_{-1} \) and then counting the number of times the envelope exceeds various thresholds in the tail of the distribution. These probability estimates are compared to probability calculations using the density function at various values for alpha; the alpha with the closest fit is chosen. If sufficient data are available, then the histogram is compared to the density function. Constant false alarm thresholds could be based on \( \hat{m}_{-1} \) or the estimate of the median envelope.

### 3.0 SEA CLUTTER RCS

Examples of fitting sea clutter histograms to the alpha-stable envelope distribution are given in figure 2 and 3 for an alpha of 1.3. Horizontally and vertically polarized (H Pol and V Pol) data were collected with an X-band radar. The clutter data were taken in a low sea state 2 at a 0.9° grazing angle, 0.5 nautical mile offshore looking into the wind. The down-range spatial resolution is 2 meters and the cross-range resolution is 35.6 meters. The amplitude is normalized by the negative first-order moment. From figure 2, the alpha-stable fit to the H Pol histogram is nearly perfect. From figure 3, the histogram tail curves downward and the alpha-stable fit overestimates the tail. For V Pol, the overall fit is good, but the tail more closely fits the K-distribution with a shape parameter of 0.25. For H Pol the alpha-stable fit is excellent.

### 4.0 SHIP RCS (COMPLEX TARGET)

An example of fitting the RCS histogram of a range boat to the alpha-stable envelope distribution is given in figure 4. The amplitude is normalized by the negative first-order moment. The histogram includes all aspects from 0° to 360°;
all aspects are equally likely for detection. The total ship RCS closely fits the alpha-stable distribution for an alpha of 1.8. Also included on this figure are the Rayleigh (alpha of two) and the log-normal (5 dB stdv, dash line). The log-normal is not a good fit to the histogram. The total ship RCS can be specified by two parameters: an alpha of 1.8 and the median RCS of 117.3 dB (the ship data are uncalibrated). Figure 5a shows the median RCS in dB as a function of aspect angle over 2° windows. Figure 5b shows alpha estimated over the same windows. Between flashes in the RCS (fuzzball region), the alpha is shown to be greater than two (2.05 nominal) which in theory is not possible. This result suggests that the threshold is exceeded less often than it should for a Rayleigh distribution. It is believed that the RF gain was set too high between flashes and the radar electronics went into soft saturation. Other data suggest that alpha in the fuzzball region can be closer to 1.9. Specifying median RCS is common practice; however, specifying alpha completes the description.

5.0 DETECTION IN LOW SNR

The envelope detection of a target in two extreme cases is examined: a small Rayleigh distributed target in spiky noise (to simulate a small target plus sea clutter), and a complex, spiky target in Rayleigh distributed noise (to simulate a ship plus radar system noise). The detection statistics are obtained using Monte-Carlo techniques with synthetic, statistically independent data samples and a fixed threshold. The size of each block of samples was set to $N = 512$. The SNR is defined by the ratio of the negative first-order moments for the signal and noise (approximately equivalent to a ratio of medians for the range of alpha from 2 to 1.3).

The maximum likelihood (ML) detector is derived using an approach similar to the univariate case (Tsihrintzis, 1995). The signal and signal-plus-noise distributions are needed. For the bivariate isotropic alpha-stable distribution, the characteristic function (Nikias, 1995) is

$$
\phi(t_1, t_2) = e^{-\gamma |t|^{\alpha}} \text{ where } |t| = \sqrt{t_1^2 + t_2^2}
$$

for which the envelope is given in equation (1). For the sum of two bivariate random variables (signal plus noise),

$$
\phi(t_1, t_2) = \exp\left(-\gamma_1 |t|^{\alpha_1} - \gamma_2 |t|^{\alpha_2}\right) \text{ with envelope } f(a) = \int_0^{\infty} \exp\left(-\gamma_1 |s|^{\alpha_1} - \gamma_2 |s|^{\alpha_2}\right) J_0(sa) \, ds
$$

To set the SNR, the dispersion is related to $m_{-1}$ in equation (2). The optimum or ML detector test statistic is

$$
\Lambda_{ML} = \frac{1}{N} \sum_{n=1}^{N} \log \left( \frac{f_{\alpha_1, \alpha_2, \gamma_1, \gamma_2}(a_n)}{f_{\alpha_1, \gamma_1}(a_n)} \right) - K
$$

where $K$ is an arbitrary constant. This expression is evaluated using numerical integration. As expected, for a Rayleigh target in Rayleigh noise, the optimum detector is the quadratic (envelope squared or power) detector.

For a progressively more spiky target in Rayleigh noise, evaluation of equation (5) shows that the optimum detector varies from a quadratic toward a cubic detector as the target alpha is varied. Receiver operating characteristics (ROC) for several detectors are given in figure 6. The curves give the SNR at which the detector has a probability of detection of 0.5 for a false-alarm rate of 0.001. The optimum detector results are not plotted since they overlay the quadratic and cubic results. Figure 6 shows how the detectability of a spiky target increases by more than 20 dB as the spikeness increases; the alpha-stable model allows detector performance to be quantified. Log weighting, as seen in the Log detector, gives poor performance.

For a Rayleigh target in spiky noise, the ML detector was designed from equation (5) for a noise alpha of 1.5, target alpha of 2.0 and SNR of -12 dB. The shape of this detector is given in figure 7a where the amplitude is normalized by the $m_{-1}$ for the noise-only, and $K$ is adjusted to make the detector zero for zero input. As an example, this ML detector was used as a mismatched detector over a range of noise alpha in the ROC given in figure 7b along with the linear and log detectors. The ML detector's performance is nearly constant with alpha. The linear detector shows performance loss...
of up to 10 dB as noise spikeness increases; the quadratic detector (not shown) is even worse. The log detector shows improvement, but it is not as good as the mismatched ML detector. To achieve peak performance, alpha and $m_{-1}$ would be estimated and the ML detector designed accordingly.

6.0 ADMINISTRATIVE INFORMATION

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7.0 REFERENCES


8.0 FIGURES

Figure 1. Envelope Distribution for Bivariate, Isotropic, Alpha-Stable Distributed Noise

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Figure 2. Envelope Distribution for H Pol Sea Clutter

Figure 3. Envelope Distribution for V Pol Sea Clutter

Figure 4. Total Ship RCS
Figure 5a. Ship RCS over 2° Windows

Figure 5b. Alpha over 2° Windows

Figure 6. Detection ROC for Spiky Target (0.5 PD, 0.001 PFA)

\[ \Lambda_{\nu} = \frac{1}{N} \sum_{n=1}^{N} a_n^{\nu} \]

- \( \nu = 1 \) Linear
- \( \nu = 2 \) Quadratic
- \( \nu = 3 \) Cubic

\[ \Lambda_{\text{Log}} = \frac{1}{N} \sum_{n=1}^{N} \log(a_n) \]

- Log

\( N = 512 \)

Figure 7a. ML Detector for Spiky Noise

Figure 7b. Detection ROC for Spiky Noise (0.5 PD, 0.001 PFA)