Enhance Neher-McGrath Formulation to Model Current Uncertainty

Chang-Jen Lan, Ph.D., P.E.
Town of Jupiter, Jupiter, FL 33458
cjl01@gmail.com

Abstract— Electric current fluctuates constantly in the circuit due to uncertainty of demand and load on the grid, as well as other factors. The current variation imposes significant effects on ampacity along the circuit, but is largely ignored. Users are advised to select “next size up” wire to address such an issue empirically. Whether this empirical rule ensures safe installation of wire and equipment is questionable. This study starts with the Neher and McGrath (NM) formulation and derives uncertainty of the operating temperature. Using the reliability constraint on the rated temperature, the uncertainty expression is substituted back to the NM equation to solve for the maximum current. The result is a stochastic version of the NM equation, which can be used to derive the derated ampacity for overcurrent protection. A comparison of the existing NEC tabulated ampacity values indicates that, depending on the degree of current variation, the “next size up” rule may not be sufficient.

I. INTRODUCTION

Electric current in a transmission line fluctuates constantly in normal operation, due to the following reasons:

- Changing demand and load on the grid especially during peak times
- Changing voltages
- Changing resistance due to heat-up and cool-down in transmission lines
- Geo-magnetically induced current (GIC) due to solar flares

Demand uncertainty from consumers could be significant even during normal days of operation. Depending on its type, the generator may not be spontaneous to generate sufficient power or react quickly enough to accommodate the fluctuation in electricity demand. All these contribute to the load variations on the grid. In practice, voltage is allowed to change continuously within 5% of variation and 10% of variation during short period of time. In addition, the changing resistance in the wires and the GIC will also cause the currents to vary. See Figure 1 for the magnitude of GIC changes over time that will potentially affect the high-voltage transmission system [1].

Should unusual current spikes occur, fuses or other surge protection devices will act to prevent failure or large-scale damage from happening to the system. However, during normal circumstances, the system as designed is expected to operate robustly to accommodate variation in currents or voltages to minimize repairs, maintenances and the inconvenience of service interruption to consumers. For economic consideration, the system should not engage surge protectors frequently for system protection due to regular current variation that can be expected, because the cost of repairs and service interruption might be prohibitive. To that end, current variation should be factored into wire selection and other design processes of the power delivery system.

The current National Electrical Code [2] adopted and mandated in many States of the US codifies the maximum current that can be carried in the conductor, before sustaining immediate or progressive deterioration primarily to insulation. In addition to various sets of tabulated ampacity values, two derating schemes are also provided in NEC to adjust ampacity for different ambient temperature assumption and number of conductors in a raceway or cable. Current variation, however, is not specifically factored into ampacity derating. As a rule of thumb, users are empirically advised to use “next size up” conductor, but whether this empirical rule ensures safe and efficient operation remains questionable. Further investigation on this subject is desirable.

This study starts with a review of the Neher and McGrath (NM) equation, which is recommended by NEC for
determining ampacity if the tabulated design values were not used. The operating temperature can then be expressed as a function of current, conductor resistance property and thermal resistances. Based on this functional relationship, the coefficient of variation of the operating temperature can be derived as a function of the coefficient of variation of current and the current itself. Using the reliability constraint on the rated temperature, the operating temperature is substituted back to the NM equation to solve for the maximum current. The result is a stochastic version of the NM equation that specifically takes into account the current variation. The writer compares the NEC rated ampacity with the modified ampacity rating under the current variation.

II. REVIEW OF THE NEHER-MCGRATH FORMULATION

In the following, the writer reviews the Neher and McGrath (NM) Equation [3], which is quoted in the NEC manual for determining the ampacity if the tabulated values were not used. To facilitate the computation in this paper, tabulated values used to determine the AC resistance due to skin and proximity effects have been fitted using a regression function as described hereafter in Equation (3) so the following section is not solely a review but also contains additional information provided by the writer to facilitate the discussion for later methodological development. The original NM equation can be written as:

\[ I = \sqrt{\frac{T_c - T_a}{R_{dc}(1 + Y_c)R_{ca}}} \]  

(1)

Where

- \( T_c \) = operating temperature (°C);
- \( T_a \) = ambient temperature (°C);
- \( R_{ca} \) = thermal resistance between conductor and ambience (°C-ft/watt);
- \( R_{dc} \) = DC conductor resistance (Ohms/ft); and
- \( Y_c \) = component of AC resistance resulting from the skin effect (Y_e) and the proximity effect (Y_{cp}); = \( Y_{es} + Y_{cp} \) since resistances act in series.

The AC resistance, \( R_{dc} \), is converted from the DC resistance through \( R_{dc}(1 + Y_c) \). \( R_{dc} \) is a function of operating temperature and can be calculated as:

\[ R_{dc} = \frac{K}{CM} = \frac{1.02 \varphi \tau + T_e}{CM} \tau + 26 \]  

(2a)

where \( K \) is in CM-Ohms/ft;
- \( \tau \) = absolute inferred temperature of zero resistance (= 234.5°C for copper);
- \( \varphi \) = resistivity (CM-ohms per ft) of conductor at 20°C = 10.371 for 100% IACS (International Annealed Copper Standard) copper; and
- \( CM \) = circular mil.

For convenience, also write \( K = K'(\tau + T_e) \), where \( K' = \frac{1.02 \varphi}{\tau + 26} = 0.04157 \) for copper. The AC resistance can then be expressed as:

\[ R_{ac} = \frac{K'}{CM}(\tau + T_e)(1 + Y_c) \]  

(2b)

The Neher and McGrath (NM) equation is adopted by the NEC manual to estimate the ampacity under engineering supervision. However, no detailed guidelines are given to engineers how the formula is implemented. In particular, the calculation of \( Y_s, Y_p \) and \( R_{ca} \) are nontrivial and rather specific to the type of wire and installation. This review section attempts to fill the gap.

Determination of \( Y_s, Y_p \) can be referred to the original NM paper. The skin effect function values were originally tabulated, but the functional form was not provided for the computerized application. To avoid the inconvenience of table or chart look-up, these tabulated values (in Table III of the NM paper) are fitted into the following two-piece regression function in this study:

\[ Y_{es} = F(x_s) = 0.00522x_s^{0.837}, \quad x_s \leq 2.78 \]

\[ = 0.3645x_s - 0.779, \quad x_s > 2.78 \]

(3)

where \( x_s = 0.875\sqrt{\frac{k_s}{R_{dc}}}, f = \text{frequency of AC circuit in Mhz} \) and \( k_s = \text{skin effect correction factor}, = 1.0 \) for conventional concentric conductor. On the other hand, for proximity effect, first calculate \( x_p = 0.875\sqrt{\frac{k_p}{R_{dc}}} \) and \( F(x_p) \) by substituting \( x_p \) into Equation (3). Then the AC resistance due to proximity effect can be estimated as:

\[ Y_{cp} = \frac{1.18F(x_p)}{f(x_p) + 0.27}(\frac{\varphi}{S})^2 + 0.312F(x_p)(\frac{\varphi}{S})^4 \]  

(4)

where

- \( D_c = \text{conductor diameter (in)} \);
- \( S = \text{axial spacing between adjacent cables (in)} \);

Determination of the thermal resistance is rather empirical and its value varies with type of wire, insulation and conduit, number of wires in the conduit, with or without jacket, etc. According to Neher and McGrath (1957), the overall thermal resistance is the sum of the following thermal resistances occurred in the order from conductor, through insulation, jacket and conduit to the ambience (earth or air).

(1) Thermal resistance from insulation

\[ R_1 = 1.2k_i\log_{10}\left(\frac{D_i}{D_c}\right) \]  

(5a)

where

- \( k_i = \text{thermal resistivity of insulation material (°C-m/watt)} \) = 5 for rubber;
- \( D_c = \text{conductor diameter (in)} \); and
- \( D_i = \text{diameter over insulation (=}D_c+2\text{-thickness of insulation). Refer Table 310.104(A) for the thickness of insulation.} \)

(2) Thermal resistance from jacket

\[ R_2 = \frac{1.04n_c}{k_j} \frac{\tau}{D_j} \]  

(5b)
where

\[ k_j = \text{thermal resistivity of jacket material (°C-m/watt)} = 5 \]

for rubber;

\[ t = \text{thickness of jacket; and} \]

\[ D_j = \text{diameter over jacket}. \]

(3) Thermal resistance from power cables in conduit (duct)

\[ R_a = \frac{nd}{mD_i + B} \]

(5c)

where

\[ n = \text{number of current carrying conductors}; \]

\[ m = \text{multiplier for } D_i \text{ to obtain effective diameter of cables}; = 1 \text{ when } n = 1; = 1.65 \text{ when } n = 2; = 2.15 \text{ when } n = 3; \text{ and } = 2.5 \text{ when } n = 4; \text{ and} \]

\[ A, B = \text{constant associated with type of conduit (duct); } A = 3.2 \text{ and } B = 0.19 \text{ for metallic conduit}. \]

(4) Thermal resistance between power cables and conduit

Suspended in air

\[ R_a = \frac{9.5n}{1 + 1.7D_i(e+0.41)} \]

(5d)

where

\[ D_s = \text{inside diameter of conduit (duct) (in); and} \]

\[ e = \text{emissivity (dimensionless) on the surface of the conduit (duct); } e = 0.1 \text{ for polished surface}; e = 0.46 \text{ for galvanized pipe}; e = 0.77 \text{ anodized aluminum pipe}; e = 0.92 \text{ for PVC pipe}; e = 0.95 \text{ for all others or refer to the emissivity table published in many commercial websites}. \]

Buried in earth

\[ R_a = 1.2nk_e\left(\log_{10}\frac{D_e}{D_s} + LF \cdot \log_{10}\frac{4df_b}{D_s}\right) \]

(5e)

where

\[ k_e = \text{dry earth thermal resistivity (°C-cm/watt)} = 67; \]

\[ D_e = \text{diameter at the beginning of the earth portion of the thermal circuit (in); defaulted at 1.6 } D_i; \]

\[ LF = \text{lost factor (defaulted as 0.75)}; \]

\[ d = \text{depth of burial (in); and} \]

\[ f_h = \text{mutual heating factor (assumed 1.0 if no information is available)}. \]

Upon determining these components, the overall thermal resistance \( R_{ca} = R_1 + R_2 + R_3 + R_4 \) if all the form of thermal resistance exist. After determining \( T_c \), one could calculate the \( K \) value and \( R_{ca} \) to determine the conductor resistance instead of using the resistance value at the rated (default) temperature.

**III. Stochastic Version of the NM Formulation**

Ampacity is tabulated in NEC manual and two schemes are provided to adjust ampacity for ambient temperature and number of conductors bundled in raceway or cable different from the default settings. In this section, the writer attempts to factor in current variation into the NM equation to determine safe ampacity rating under the current variation. This implies that the operating temperature of conductor should be subjected to a reliability constraint. To derive the variation in the operating temperature, one needs to express the operating temperature in terms of current first. Substitution of \( K \) into the Eq. (1) and rearranging terms yield the operating temperature of conductor as follows:

\[ T_c = \frac{T_{n} + \frac{K'}{C_M(1 + Y_c)}R_{ca}}{1 - \frac{K'}{C_M(1 + Y_c)}R_{ca}} \]

(6)

To evaluate the fluctuation in operating temperature \( T_c \) due to the fluctuation in current \( I \), the first-order approximation (aka the delta method) is applied to derive the standard deviation of the \( T_c \) as

\[ \sigma_{T_c} \approx \frac{dT_c}{dI} \sigma_I = \frac{2w(t+T_n)R_{ca}}{(T_{n} + twf^2)(1-wf^2)} \sigma_I \]

(7)

where \( w = \frac{K'}{C_M(1 + Y_c)}R_{ca} \). Expressing it in term of coefficient of variation (CV) yields

\[ CV_{T_c} = \frac{2w(t+T_n)R_{ca}}{(T_{n} + twf^2)(1-wf^2)} \]

(8)

The coefficient of variation is simply the ratio of the standard deviation of a random variable to its mean. It normalizes the dispersion of a random variable in term of mean such that one could conveniently realize the relative dispersion among a set of random variables. It is a useful dimensionless statistic for comparing the relative variation from one random variable to another, even if the means are significantly different from each other.

To verify the quality of approximation, the author conducts the Monte Carlo simulation with a large amount of realizations. It is found that the quality of the first-order approximation is fairly acceptable (0.3%-2%). The Equation (9) or (10) can therefore be used with confidence. As shown in Figures 2, it is suggested that higher amperage could cause the operating temperature to fluctuate more than the degree of variation in amperage, especially in the smaller conductors. Note that the data for #6 wire at 70 and 80 amps are eliminated because the corresponding operating temperature exceeds the highest rated temperature in NEC.
verify that Equ. (10) reduces to Equ. (1) when accommodate variation in actual amperage. One could easily but the “rated” amperage should be decreased to immediate or progressive deterioration remains unchanged, amperage that can be carried in conductor without sustaining variation characterized by

\[ T_e + z_a CV_T C_e \leq T_R \]

Substitution of Equation (8) into the above reliability constraint leads to

\[ T_e \leq \left( 1 + z_a \frac{2w(\tau+T_a)^2}{(\tau w+I)^2(1-w^2)} CV_I \right)^{-1} T_R \]

Replacing Equation (9) into (1) and solving for \( I \) yields

\[ I \leq \left( \frac{\frac{1}{R_c}}{\frac{1}{R_c} + 2\frac{1}{R_e}} \right)^{1/2} \]

where \( T = 2T_R + \tau - T_a + 2z_a CV_I (\tau + T_a) \).

The resulted current is the downgraded ampacity rating for a specific rated temperature under the effects of current variation characterized by \( CV_I \). The actual maximum ampereage that can be carried in conductor without sustaining immediate or progressive deterioration remains unchanged, but the “rated” amperage should be decreased to accommodate variation in actual ampereage. One could easily verify that Equ. (10) reduces to Equ. (1) when \( CV_I \) is set to zero. Equ. (10) can therefore be viewed as the stochastic version of the NW equation.

IV. DERATED AMPACITY FOR OVERCURRENT PROTECTION

A simulation study is conducted here to illustrate the effect of varying currents on the ampacity rating and the operating temperature. In particular, how much ampacity derating will be resulted to account for overcurrent. In the NEC manual, the ampacity of the “600” conductor is rated 420 amps at 75°C, which is very close to the ampacity calculated in this study (= 424 amps). The coefficient of variation for currents is assumed as 0.08, which means the standard deviation of the current is 8% of the average amperage. With \( CV_I = 0.08 \) and \( z_a = 1.96 \) (95% confidence interval), the calculated maximum amperage through Equation (10) is 371 amps, which suggests that the rated ampacity in the Table should be downgraded 50 amps from 420 amps (approximately 12%) to allow the conductor to be operated safely under varying currents. As shown in Figures 3, among a total of 100 realizations, one could find there are two time instances that the current exceeds 420 amps, which is close to 2.5% of probability occurred outside the upper side of the confidence bounds.

In the mean time, the operating temperatures resulted from these simulated currents are calculated via Equ. (6). The average of these 100 realizations of temperatures is 63.8°C, as compared to the deterministic mean of 63.2 calculated by substituting 371 amps in Equ. (6). This is because the stochastic mean is greater than the deterministic mean in this case where the temperature is a convex function of currents. Similarly, one could find two operating temperatures exceed the rated temperature of 75°C, which coincides well with 2.5% of probability occurred outside the upper side of the confidence bounds.

Lastly, ampacities are also calculated for several large size conductors (500-750 kcmil) at 60°C, 75°C and 90°C rated temperatures, and compared with the rated ampacity tabulated in NEC. The results indicate that, depending on the degree of current variation, the “next size up” rule may not be sufficient to meet the ampacity requirement under the effect of current variation. As the current variation increases, two or three size bigger conductors might be necessary. For example, with 5% coefficient of variation in current, the next size up rule might be applicable in general. If the coefficient of variation increases to 8%, two-size bigger wire will be called for. In addition, when the size jump is not uniform, e.g., transition from regular 100 kcmil interval to 50 kcmil such as 700 to 750, it may call for three-size bigger wire.

<p>| TABLE I. CONDUCTOR AMPACITY AT 60°C RATED TEMPERATURE |</p>
<table>
<thead>
<tr>
<th>Size of wire (kcmil)</th>
<th>NEC</th>
<th>Stochastic NM</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>320</td>
<td>285</td>
</tr>
<tr>
<td>600</td>
<td>355</td>
<td>325</td>
</tr>
<tr>
<td>700</td>
<td>385</td>
<td>355</td>
</tr>
<tr>
<td>750</td>
<td>400</td>
<td>370</td>
</tr>
</tbody>
</table>

<p>| TABLE II. CONDUCTOR AMPACITY AT 75°C RATED TEMPERATURE |</p>
<table>
<thead>
<tr>
<th>Size of wire (kcmil)</th>
<th>NEC</th>
<th>Stochastic NM</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>380</td>
<td>340</td>
</tr>
<tr>
<td>600</td>
<td>420</td>
<td>385</td>
</tr>
<tr>
<td>700</td>
<td>460</td>
<td>425</td>
</tr>
<tr>
<td>750</td>
<td>475</td>
<td>440</td>
</tr>
</tbody>
</table>
TABLE III. CONDUCTOR AMPACITY AT 90°C RATED TEMPERATURE

<table>
<thead>
<tr>
<th>Size of wire (kcmil)</th>
<th>NEC</th>
<th>Stochastic NM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CV=0.05</td>
<td>CV=0.08</td>
</tr>
<tr>
<td>500</td>
<td>430</td>
<td>385</td>
</tr>
<tr>
<td>600</td>
<td>475</td>
<td>435</td>
</tr>
<tr>
<td>700</td>
<td>520</td>
<td>480</td>
</tr>
<tr>
<td>750</td>
<td>535</td>
<td>500</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In circuit design, two important considerations in wire size selection are ampacity and allowable voltage drop, which can be affected by the current variation. Depending on its degree of variation, such effects could be significant but are largely ignored in the literature and national standards. A stochastic version of the Neher-Grath formulation is derived to specifically account for the effect of current variation on ampacity. It provides a mechanism for wire size selection or evaluation of safe ampacity rating for overcurrent protection without involving an ad hoc rule such as next size up. Although the conductor resistance property and thermal resistance of insulation carry uncertainty as well, their effects on ampacity are relatively insignificant compared to current variation and can therefore be neglected. The focus on the current variation well serves the purpose for assessing the overall effect on the rated ampacity.

From the comparison with the tabulated ampacity in NEC, the ampacity under the effect of current variation is derated to accommodate the variation in actual amperage and provide overcurrent protection. As a result, users should select bigger wires but there is no provision of specific guidelines in nationwide standards except the “next size up” rule of thumb in practice. It is found in this study that, depending on the degree of variation, the next size up rule may not be adequate and it might call for two or three size bigger wires. The issue is more prominent when the size jump is not uniform. By modifying rated ampacity using the proposed stochastic NM formulation, users can be provided with appropriate ampacity rating information for selecting the wire size, either through table look-up or computer program. The reliability of the proposed rated ampacity is strongly contingent upon the quality of the current uncertainty information. A current sensor can be installed on the circuit to provide continuous monitoring and more reliable uncertainty information.

ACKNOWLEDGMENTS

The writer is grateful for the valuable comments from Mr. Thomas Driscoll and Mr. Thomas Lepore. The content of this paper is sole opinion of the writer.

REFERENCES