Directional Wide Band Time Reversal Digital Beam forming FIR Filter Design using Bore-sight Calibration Data

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Abstract

The time reversal wide band digital beam forming and calibration technique is briefly described. New techniques using the bore-sight probing RF signal alone to derive the time reversal FIR (Finite Impulse Response) filter set are discussed. In the digital beam forming simulation, a set of 50-tap FIR filters measured from a 16 channel of 1 to 8GHz tunable 500Mhz bandwidth front end digitized system is used as RF front end. The simulation results of the previous time reversal approach and new approaches are compared.

Introduction

There are two approaches to perform the wide band beam forming for a wide band array antenna. The first one is the hardware approach by using a set of programmable true time delays. Since the true time delay is a RF device, it has loss, impedance miss match, and noise figure problems. Also the time delay resolution requirement is very rigid particularly at the high RF frequency. The second one is the firmware approach. In this approach, the time delay is digitally implemented in FPGA (Field Programmable Gate Array) hardware. Since the signal being dealt with is at IF frequency, the resolution requirement is relaxed and easily implemented digitally. Time reversal technique is very useful for wide band digital beam forming and array channel calibration [1]. It creates a set of FIR (Finite Impulse Response) filters for each interested aspect angle. Each FIR filter set includes the system calibration and time delay functions. However, this approach has some drawbacks for real-time applications. First, the set of FIR filters for a specific angle has to be generated with a prior probing signal transmitted at that direction. Second, this approach lacks flexibility for adaptive beam forming and beam nulling.

Figure 1 is a typical 16 element wide band phase array antenna with RF front end. The symbol ⊗ is the operator of convolution. The system in the box, which includes the time delays to antenna elements, can be described as a linear system. Hence it can be represented by a transfer function (frequency response) matrix H(w) in the frequency domain and an impulse response matrix h(t) in the time domain. H(w)* is the conjugate of H(w). In Figure 1, x(t) is an input RF wide band signal and X(w) is the frequency domain expression. Assume that h(t), the system impulse response, can be predetermined by some system identification technique. The time reversal beam forming concept can be
represented by the following equations and block diagrams shown in Figure 2.

The input \( x(t) \) is a plane wave before it reach the antenna. Thus, for each channel of the antenna system, it can be treated as in phase signal

\[
x(t) = \cos\left( w_c t + \frac{1}{2} a t^2 \right)
\]

before it reaches any element of the array. The calculated time reversal signal at the output of FPGA for each channel should be in phase. The digital beam forming is achieved by adding up the calculated \( x(t) \) from each channel.

![Block diagram of 16 element wide band phase array antenna with RF front end](image)

**Figure 1.** Block diagram of 16 element wide band phase array antenna with RF front end

\[
X(w)H(w) = Y(w)
\]

\[
\frac{Y(w)H^*(w)}{|H(w)|^2} = \frac{X(w)H(w)H^*(w)}{|H(w)|^2} = X(w)
\]

\[
x(t) * h(t) = y(t)
\]

**Figure 2.** Functional diagram of Figure 1 in frequency domain and time domain

The input \( x(t) \) is a plane wave before it reach the antenna. Thus, for each channel of the antenna system, it can be treated as in phase signal before it reaches any element of the array. The calculated time reversal signal at the output of FPGA for each channel should be in phase. The digital beam forming is achieved by adding up the calculated \( x(t) \) from each channel.

\[
x(t) = \cos(w_c t + \frac{1}{2} a t^2)
\]

In practice, \( H(w) \) of the hardware system can be measured by sending a probing RF chirp signal

\[
x(t) = \cos(w_c t + \frac{1}{2} a t^2)
\]

Where \( w_c \) is a carrier frequency and \( a \) is the chirp rate. Since only the relative phases
between channels are important in beam forming, we assume the initial phase of input RF signal is 0 for all discussion in this paper. The chirp rate is chosen so that the total chirp range is equal to the Nyquist frequency. The output of the system, y(t), is digitized and stored in the hard disk. The y(t) is an IF signal with the same chirp rate because x(t) has been down converted in the system. A software generated IF de-chirp reference x(-t) will be used to convolve with the collected data. The impulse response of the system can be obtained because the chirp signal convolve with its matched de-chirp signal will be an impulse signal δ(t).

\[ x(t) \ast x(-t) = \delta(t) \quad \text{and} \quad y(t) \ast h(t) = y(t) \]

\[ y(t) \ast x(-t) = h(t) \]

The time reversal FIR filter can be obtained by taking the inverse FFT of \( H^*(w)/|H(w)|^2 \).

\[ h'(t) = \text{IFFT} \left( \frac{H^*(w)}{|H(w)|^2} \right) \]  

Original Time Reversal digital beam forming FIR filter design [1]

The description above is an overall time reversal wideband digital beam concept. The detailed internal operation can still be discussed by using Figure 1.

For the \( m^{th} \) antenna, the received RF chirp signal is

\[ x_m(t) = \cos(w_c(t-m\tau) + \frac{1}{2}a(t-m\tau)^2) \]  

(3)

After going through amplifiers, a mixer, and a low pass anti-aliasing filter, the signal at the \( m^{th} \) channel output of ADC can be expressed as

\[ y_m(t) = \cos(w_{IF}(t-m\tau) - m\theta_0 + \frac{1}{2}a(t-m\tau)^2) \ast \delta(t) \]

(4)

where \( w_{IF} = w_c - w_0, \theta_0 = w_0\tau \), and \( w_0 \) is a local oscillator frequency.

\[ h_m(t) = h_m(t) \ast h_m(t) \ast h_m(t) \ast h_m(t) \ast h_m(t) \]

Let \( c_{IF}^{'}(n) \) and \( c_{IF}^{'*}(n) \) be a chirp and de-chirp at IF frequency in the digital format

\[ c_{IF}^{'*}(n) = \cos(w_{IF}(N-n) - \frac{1}{2}a(N-n)^2) \]  

(5)

\[ c_{IF}^{'}(n) = \cos(w_{IF}(n-m\tau) - \frac{1}{2}a(n-m\tau)^2 - m\theta_0) \]  

(6)

The impulse response of the \( m^{th} \) channel system in digital form is

\[ g_m(n, m\tau, m\theta_0) = y_m(n) \ast c_{IF}^{'*}(n) = c_{IF}^{'}(n-m\tau, m\theta_0) \ast h_m(n) \ast c_{IF}^{'*}(n) \]  

(7)

After taking the FFT

\[ G_m(k) = \text{FFT}(g_m(n, m\tau, m\theta_0)) \]  

(8)

the time reversal FIR filter for the \( m^{th} \) channel, \( h'(n, m\tau, m\theta_0) \), can be obtained by the inverse FFT operation

\[ h'(n, m\tau, m\theta_0) = \text{IFFT} \left( \frac{G'_m(k)}{[G_m(k)]^2} \right) \]  

(9)

Time Reversal digital beam forming FIR filter design using the bore-sight probing chirp signal

In equation (7), \( c_{IF}^{'}(n-m\tau, m\theta_0) \) is down converted IF chirp signal for the \( m^{th} \) antenna element and \( c_{IF}^{'*}(n) \) is the de-chirp reference generated by computer. It follows from (7) that the time reversal FIR filter must be designed.
(calibrated) according to the aspect angle, because the down converted IF chirp signal $c_{IF}(n - m \tau, m\theta_0)$ is dependent on the time delay. To lift this constraint, two new methods will be developed along this approach.

The following lemma is used to derive the new time reversal FIR filter.

**Lemma:** The convolution of the down converted IF chirp signal at the $m$th antenna and the de-chirp reference has the following relation, i.e.,

$$c_{IF}(n - m \tau, m\theta_0) \otimes c_{IF}^\prime(n)$$

= $c_{IF}(n) \otimes c_{IF}^\prime(n - m \tau, m\theta_0)$  \hspace{1cm} (10)

Proof of this lemma is given in the Appendix.

**Method I**

Applying this commutative property (10) to (7) yields

$$g_m(n, m\tau, m\theta_0)$$

= $c_{IF}(n) * h_m(n) \otimes c_{IF}^\prime(n - m \tau, m\theta_0)$  \hspace{1cm} (11)

Now the down converted IF chirp signal for the $m$th antenna element becomes $c_{IF}(n)$ and the de-chirp reference is $c_{IF}^\prime(n - m \tau, m\theta_0)$. Since $c_{IF}(n) \otimes h_m(n)$ represents the bore-sight collected data due to $\tau = 0$, we can generate the impulse responses for all aspect angles by generating the proper de-chirp reference $c_{IF}^\prime(n - m \tau, m\theta_0)$ based on the required delays.

The time reversal FIR filter $h'(n, m\tau, m\theta_0)$ can be obtained by the same FFT and inverse FFT operations as in the original design of (8) and (9). This method may require a relatively long de-chirp reference.

**Method II**

Using the trigonometric identity, we have

$$\cos(w_p(N-(n-m\tau))) + \frac{1}{2} a(N-(n-m\tau))^2 - m\theta_0)$$

= $\cos(w_p(N-(n-m\tau))) + \frac{1}{2} a(N-(n-m\tau))^2 \times \cos(m\theta_0)$

$+ \sin(w_p(N-(n-m\tau))) + \frac{1}{2} a(N-(n-m\tau))^2 \times \sin(m\theta_0))$

= $FD(n-m\tau)*(\cos(w_p(N-(n-m\tau)))\times \cos(m\theta_0) + \frac{1}{2} a(N-(n-m\tau))^2 \times \sin(m\theta_0))$

Here $FD(n-m\tau)$ a fractional delay FIR filter and

$$s_{IF}' = \sin(w_{IF}(N-m) + \frac{1}{2}(N-m)^2)$$

Then the de-chirp reference has the following expression

$$c_{IF}^\prime(n-m\tau, m\theta_0) = FD(n-m\tau) \otimes (c_{IF}(n) \times \cos(m\theta_0) + s_{IF}'(n) \times \sin(m\theta_0))$$

(12)

We can rewrite (11) as

$$g_m(n, m\tau, m\theta_0) = c_{IF}(n) \otimes h_m(n) \otimes c_{IF}^\prime(n - m \tau, m\theta_0)$$

= $(c_{IF}(n) \otimes h_m(n) \otimes c_{IF}^\prime(n) \times \cos(m\theta_0) + c_{IF}(n) \otimes h_m(n) \otimes s_{IF}'(n) \times \sin(m\theta_0)) \otimes FD(n-m\tau)$

Let

$$h_{cm}(n) = c_{IF}(n) \otimes h_m(n) \otimes c_{IF}^\prime(n)$$

and call it the in-phase system FIR filter and

$$h_{sm}(n) = c_{IF}(n) \ast h_m(n) \ast s_{IF}'(n)$$

and call it the quadrature–phase system FIR filter. Both $h_{cm}(n)$ and $h_{sm}(n)$ are independent of the aspect angle and can be obtained by the bore-sight probing RF Chirp signal. They are stored in the computer disk. For different aspect angles, the corresponding system characteristic impulse responses are
\[ g_m(n,m\tau, m\theta_0) \]
\[ = (h_{cn}(n) \times \cos(m\theta_0) + h_{sm}(n) \times \sin(m\theta_0)) \]
\[ \otimes FD(n - m\tau) \]

Then the time reversal FIR filter \( h'(n, m\tau, m\theta_0) \) can be obtained by the same FFT and inverse FFT as in the original design of (8) and (9).

**In house RF front end characterization**

The 16-channel MCWESS (Multi-Channel Wideband Electronic Support System) front end system for data collection includes Chirp signal generator, power splitter, 16 Mid-Atlantic LCR-100 tuners, and 16 Acqiris/Agilent DC282 digitizer. Figure 3 is the functional block diagram of the system. Using the approach described above, the system impulse response \( h_m(n) \) of 16 channel s can be obtained.

**Simulation Result**

For the fidelity, 16 50-tag FIR filters, \( h_m(n) \), is used to represent the MCWESS RF front end in the modeling and simulation. Also, the element of the antenna array is simulated as an ideal antenna with isotropic antenna pattern. Figure 4 is the block diagram of simulation model

![Figure 3 Functional diagram of the 16-channel in house front end system](image)

Figure 3 is the functional diagram of the 16-channel in house front end system.

![Figure 4 Simulation Block Diagram](image)

Figure 4 is the block diagram of the simulation model.

First, a 2 µs linear chirp from 3.00 to 3.50 GHz and 7.00 to 7.50 GHz were simulated as probing signals illuminating at \(-10^\circ\) and the ADC data were collected. The FIR filters \( h'(n, m\tau, m\theta_0) \) of all channels at that angle can be obtained by using the original time reversal concept. Next we move the simulated probing signal to bore-sight and collect the ADC data. Then \( h'(n, m\tau, m\theta_0) \) at \(-10^\circ\) also can be obtained by methods I and II. The 200ns pulses of linear chirp from 3.10 to 3.40 GHz and 7.00 to 7.50 GHz are simulated as probing signals. They are illuminated from \(-60^\circ\) to \(60^\circ\) with respect to bore-sight to generate wide band array antenna pattern (bore-sight at \(0^\circ\)). Figures 5, 6, and 7 are the results of applying the time reversal FIR filters constructed by the conventional approach, method I, and method II, respectively. It can be seen that these methods produce almost the same broad band patterns.
Conclusion and Recommendation

This paper has presented two designs of the time reversal beam forming FIR filters for all aspect angles using only the bore-sight chirp illumination. The time reversal beam forming filters are functions of the delay. Method I eliminates the need to calibrate the filters for every aspect angle. In method II, the delay for each channel can be independently programmed. This makes the wide band adaptive beam forming and beam nulling become achievable. A wide band array antenna RF front end system has been built. And experiments using this system will be scheduled in near future to validate the effectiveness of the new approaches.

Appendix
Proof of the Lemma on the commutative property of chirp functions

Without losing the generality, let’s assume

\[ c_{IF}(n, \phi) = \cos \left( 2\pi k(n) + \frac{1}{2} a(n)^2 + \phi \right) \]

\[ c'_{IF}(n, \phi) = \cos \left( 2\pi f(N-n) + \frac{1}{2} a(N-n)^2 + \phi \right) \]

\[ FFT(c_{IF}(n, \phi)) = \sum_{n=0}^{N-1} \cos \left( 2\pi k(n) + \frac{1}{2} a(n)^2 + \phi \right) e^{-\frac{j2\pi nk}{N}} \]

\[ FFT(c'_{IF}(n, \phi)) = \sum_{n=0}^{N-1} \cos \left( 2\pi k(N-n) + \frac{1}{2} a(N-n)^2 + \phi \right) e^{-\frac{j2\pi nk}{N}} \]

Let \( m = N-n \)

\[ FFT(c'_{IF}(n, \phi)) = \frac{1}{m} \sum_{m=N}^{N-1} \cos \left( 2\pi km + \frac{1}{2} am^2 + \phi \right) e^{-\frac{j2\pi nk}{N}} \]

\[ \therefore FFT(c'_{IF}(n, \phi)) = \text{conjugate of } FFT(c_{IF}(n, \phi)) \]

\[ G_1(k) = \frac{1}{2} \sum_{n=0}^{N-1} e^{j\left(\frac{1}{2} a(n-\tau)^2\right)} e^{-\frac{j2\pi nk}{N}} \]

\[ G_2(k) = \frac{1}{2} \sum_{n=0}^{N-1} e^{-j\left(\frac{1}{2} a(n-\tau)^2\right)} e^{-\frac{j2\pi nk}{N}} \]

\[ K_1(k) = \frac{1}{2} \sum_{n=0}^{N-1} e^{j\left(\frac{1}{2} m^2\right)} e^{-\frac{j2\pi nk}{N}} \]

\[ K_2(k) = \frac{1}{2} \sum_{n=0}^{N-1} e^{-j\left(\frac{1}{2} m^2\right)} e^{-\frac{j2\pi nk}{N}} \]

\[ G_1(k) = K_1(k)e^{-j2\pi k\tau} \]

\[ G_2(k) = K_2(k)e^{-j2\pi k\tau} \]

\[ F_1(k) = \sum_{n=0}^{N-1} \cos \left( \frac{1}{2} a(n-\tau)^2 + \phi \right) e^{-\frac{j2\pi nk}{N}} \]

\[ = \frac{1}{2} \sum_{n=0}^{N-1} \left( e^{j\left[\frac{1}{2}(n+\tau)^2 + \phi\right]} + e^{j\left[\frac{1}{2}(n-\tau)^2 + \phi\right]} \right) e^{-\frac{j2\pi nk}{N}} \]

\[ = G_1(k)e^{j\phi} + G_2(k)e^{-j\phi} \]

\[ F_2(k) = \sum_{n=0}^{N-1} \cos \left( \frac{1}{2} a(n)^2 \right) e^{-\frac{j2\pi nk}{N}} \]

\[ = \frac{1}{2} \sum_{n=0}^{N-1} \left( e^{j\left[\frac{1}{2}(n+\tau)^2\right]} + e^{j\left[\frac{1}{2}(n-\tau)^2\right]} \right) e^{-\frac{j2\pi nk}{N}} \]

\[ = K_1(k) + K_2(k) \]

\[ FFT(c_{IF}(n-\tau, \phi) \ast c'_{IF}(n)) \]

\[ = \left( G_1(k)e^{j\phi} + G_2(k)e^{-j\phi} \right) \left( K_1(k) + K_2(k) \right)^* \]

\[ = G_1(k)e^{j\phi}K_1(k)^* + G_2(k)e^{-j\phi}K_2(k)^* \]

\[ F_2(k) \ast F_2(k) \]

\[ = K_1(k)e^{-j2\pi k\tau}e^{j\phi}K_1(k)^* + K_2(k)e^{-j2\pi k\tau}e^{-j\phi}K_2(k)^* \]

\[ F_1(k) \ast F_2(k) \]

\[ = \left( K_1(k) + K_2(k) \right) \left( K_1(k)^*e^{j\phi}e^{-2\pi k\tau} + K_2(k)^*e^{-j\phi}e^{-2\pi k\tau} \right) \]

\[ = K_1(k)K_1(k)^*e^{-j2\pi k\tau}e^{j\phi} + K_1(k)K_2(k)^*e^{-j2\pi k\tau}e^{-j\phi} \]

\[ + K_2(k)K_1(k)^*e^{-j2\pi k\tau}e^{j\phi} + K_2(k)K_2(k)^*e^{-j2\pi k\tau}e^{-j\phi} \]

Since \( K_1(k) \) only has non zero values from 0 to \( (N/2)-1 \) and \( K_2(k) \) only non zero values from \( N/2 \) to \( N-1 \). Their product is zero

\[ FFT(c_{IF}(n-\tau, \phi) \ast c'_{IF}(n)) \]

\[ = K_1(k)K_1(k)^*e^{-j2\pi k\tau}e^{j\phi} + K_2(k)K_2(k)^*e^{-j2\pi k\tau}e^{-j\phi} \]

\[ = FFT(c_{IF}(n) \ast c'_{IF}(n-\tau, \phi)) \]
Therefore, we prove that

\[ c_{IF} (n - \tau, \phi) \ast c'_{IF} (n) = c_{IF} (n) \ast c'_{IF} (n - \tau, \phi) \]

**Reference**

“Digital Wideband Phased Array Calibration and Beam forming Using Time Reversal Technique”
