WiMAX Ambiguity Function for PCL Systems

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Abstract—Passive coherent location (PCL) radar systems use signals of opportunity as the off-board source of electromagnetic illumination. The analysis of the structure, spectral, and spatial properties of emerging commercial waveforms becomes a critical component of future PCL radar systems design. This paper presents an ambiguity function analysis of the emerging IEEE 802.16 OFDM signal known as “WiMAX” for passive radar applications. The simulated WiMAX wave structure shows desirable range and Doppler resolution properties for passive radar applications.

I. INTRODUCTION

The introduction of passive radar technology presented a novel way to create radar products by using off-board emissions in bistatic or multistatic environments. An ideal passive radar would be capable of using any commercial or military electromagnetic spectrum available to detect and track targets of interest [1]. Imaging applications using passive radars are also possible and have been a popular research topic in recent years [2], [3]. Waveform properties such as transmitting power, area coverage, and ambiguity function are of special interest to any passive coherent location (PCL) radar designer. However, the success of PCL systems depends largely on the availability and properties of signals not intended for radar use. The analysis of structure, spectral, and spatial properties of emerging commercial waveforms becomes a critical and necessary component of future developments of PCL radar systems.

This paper presents the ambiguity function properties of the emerging IEEE 802.16 OFDM signal known as “WiMAX”. For the analysis, the bistatic ambiguity function presented in [4] is implemented in MATLAB. Section II of the paper briefly covers basic properties of the WiMAX 802.16 standard found in the commercial literature. The similarity to other radar waveforms is presented and a radar “pulse train” defined using signal cycles. In Section III the bistatic ambiguity function algorithm is presented followed by a discussion of results in Section IV. Final conclusions and future work are offered in Section V.

II. IEEE 802.16 (WiMAX)

The Worldwide Interoperability for Microwave Access or WiMAX, is a wireless networking standard which aims to address interoperability across IEEE 802.16 standard based products. It is a wireless networking connection for broadband access directly competing against widely-used cable, DSL, and T1 systems. Currently, there are more than 500 fixed and mobile WiMAX trials and commercial deployments happening in 147 countries [5]. Worldwide deployment of WiMAX systems, along with its well-structured signal modulation scheme, provides unique opportunities for emerging PCL systems applications. Operating in the frequency band between 2 and 11 GHz, fixed WiMAX systems can provide 5 to 10 km of service area with a maximum data rate of 70 Mbps in a scalable 20 MHz channel [6]. This capability is provided to users without the need of a direct line-of-sight (LOS) to the base station [6].

The IEEE 802.16 standard [7] defines three physical layers for WiMAX networks. One consists of a single carrier used for LOS transmissions. The other two layers are used for non-LOS transmissions. The physical layer used for this analysis follows the 802.16e standard and consists of a 5MHz signal using a 512-point FFT OFDM multiplexing scheme per the simulation code by [8]. The 512 subcarriers are used to carry data, pilot symbols, training symbols for channel estimation, and null or guard symbols. The length of the signal is selected based on computational limitations and the signal’s periodic properties which will be subsequently
discussed. Given that the WiMAX transmission network is designed to operate in the C or X band, an arbitrary carrier frequency of $f_c = 2.4$ GHz is assumed.

The real part of the complex simulated WiMAX time-domain data burst used for this analysis is shown in Figure 2. The signal frequency spectrum is shown in Figure 2. Although the WiMAX signal simulation code is not formally validated, the signal shows typical characteristic of an OFDM signal structure in time and frequency domain; as such, it is considered adequate for this initial evaluation.

The information in the data subcarriers is assumed to be a random process modeled as white gaussian noise. Figure 3 shows a signal time average over 50 realizations. The averaging removes the random data revealing an exploitable periodic and predictable wave structure. The periodicity of the pilot and null subcarriers of the WiMAX signal provide a unique opportunity not seen in other signals of opportunity. For example, in using analog FM or TV signals, the frequency content varies with the radio or TV program content [9].

For the purpose of this paper, a cycle will be defined as one full period of the WiMAX signal based on its autocorrelation function shown in Figure 4. A peak occurs approximately every 5040 samples or 0.2 msec. The simulated WiMAX communication burst consists of 1.6 msecs of the downlink transmission, sampled at $f_s = 24.5$ MHz. The bistatic ambiguity function is evaluated using a one-cycle burst and an eight-cycle burst. The eight-cycle burst results in ambiguity functions similar to those obtained by the use of pulse trains in typical radar systems, with relatively good resolution in both delay and Doppler.

### III. Ambiguity Function Algorithm

The ambiguity function for this research incorporates bistatic geometry as defined in [4]. Using the north-referenced coordinate system shown in Figure 5 and a target located North of the bistatic baseline, the total transmission time delay proportional to the total range $R = R_R + R_T$ is defined as:

$$\tau(R_R, \theta_R, L) = \frac{R_R + \sqrt{R_R^2 + L^2 + 2R_RL\sin(\theta_R)}}{c}$$  \hspace{1cm} (1)$$

where $R_R$ is the range from the target to the receiver, $L$ is the bistatic baseline or range between receiver and transmitter, and $\theta_R$ is the angle from the North axis to the target as shown in Figure 5. Assuming the transmitter and receiver are stationary, the Doppler shift is defined by [4]:

$$w_D(V, \cos(\beta/2), \phi) = \frac{2w_D}{c} V \cos(\phi) \cos(\beta/2)$$  \hspace{1cm} (2)$$
where $V$ is the target velocity vector, $w_c$ is the carrier angular frequency, $\phi$ is the angle between the velocity vector and the bistatic bisector, and $\beta$ is the bistatic angle. The cosine term in (2) is defined as:

$$
cos(\beta/2) = \sqrt{\frac{1}{2} + \frac{R_R + L \sin(\theta_R)}{2\sqrt{R_R^2 + L^2 + 2R_RL \sin(\theta_R)}}}
$$

Further, it is shown in [4] that the target-receiver range in terms of total range $R$ and target-receiver angle $\theta_R$ is:

$$
R_R = \frac{L^2 - R^2}{2R - 2L \sin(\theta_R)}
$$

Note that for the bistatic geometry, target-receiver range $R_R$ and in turn the associate range rate, will vary differently than for a monostatic radar system. Consequently, a target that moves in the bistatic geometry with velocity $V$ will also produce non-linear Doppler frequencies with respect to the total range $R$. It is clear that as the transmitter-receiver baseline distance $L$ approaches zero, $R_R$ approaches the monostatic range $R/2$. For these reasons, the bistatic ambiguity function only makes sense in terms of target-receiver or target-transmitter range [4].

Given the bistatic geometry described above, the bistatic ambiguity function can be defined as [4]:

$$
|\Psi(\cdot)| = \left| \int_{-\infty}^{\infty} s(t - \tau_a(\cdot))s^*(t - \tau_h(\cdot))e^{j(w_{Da}(\cdot)-w_{Da}(\cdot))t} dt \right|^2
$$

where the subscripts $h$ and $a$ represent the hypothesized and actual signal parameters. The delay $\tau$, Doppler $w_D$, and the ambiguity function $|\Psi(\cdot)|$ are all functions of the bistatic geometry parameters $R_R$, $L$, and $\theta_R$ defined in (1)-(4).

The simulation of (5) requires the incorporation of bistatic geometry effects. The algorithm input is a complex signal $s(t)$ with sampling frequency $f_s$ and duration $T$. A uniformly distributed set of hypothesized range and target velocity values are chosen to define the ambiguity function extent. Delay and Doppler values are computed using (1) and (2). Lastly, (5) is used to compute the ambiguity function using the complete set of delay-Doppler pairs. Note that Doppler effects on only the carrier frequency are assumed; that is, effects on other frequency components of the wideband signal are considered negligible.

To validate the bistatic ambiguity function, the algorithm is tested using a single gaussian pulse (SGP) model [10] with identical wave and geometric parameters as those found in [4]. Figure 6 shows the SGP ambiguity function contours for $\theta_R = 60$ degrees, $L = 100$ km, $R_R = 60$ km, and $V = 600$
TABLE I: Signal Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WiMAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency $f_c$</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>“Pulse” Duration (one cycle) $T$</td>
<td>0.2 m sec</td>
</tr>
<tr>
<td>Sample Frequency $f_s$</td>
<td>24.5 MHz</td>
</tr>
</tbody>
</table>

m/s. The simulation results match the ambiguity function contours shown in [4].

IV. RESULTS

The core of the evaluation lies on the ambiguity function evaluated for probable field geometries based on the advertised WiMAX signal and network properties. The “monostatic” evaluation is based on the bistatic geometry construct with $L = 0$ and $\theta_R = 0$; representing the best-case scenario. Figure 7 shows the monostatic geometry. Figures 8 and 9 show the bistatic geometry with a target-receiver range of 5km, a bistatic baseline of 20km, and $\theta_R$ values of 60 and 80 degrees respectively. To explore the ambiguity function characteristics associated with longer ranges, a 100km bistatic baseline with a 60km target-receiver range is also considered.

Two time-limited WiMAX waveforms are evaluated. The first is a single cycle of the WiMAX signal analogous to a single modulated radar pulse. The second waveform consists of a “pulse train” of 8 WiMAX cycles as shown in Figure 1. The evaluation of larger number of cycles is avoided mostly to maintain reasonable computational load. Table I summarizes the WiMAX signal parameters used.

Figure 10 shows the monostatic ambiguity function for a single cycle. Note the narrow response in the range domain, typical of a pseudorandom radar waveform with desirable range resolution. Although the range resolution is a direct result of the correlation of the “noisy” signal (large bandwidth), recall the data stream average over 50 Monte Carlo runs in Figure 3 resulted in a “good” correlation waveform. On the other hand, the Doppler domain shows a Doppler tolerant waveform. The Doppler resolution is directly related to the waveform length, hence it is expected that for longer integration times, Doppler resolution will improve. This is evident by the eight cycle ambiguity function results shown in Figure 15 and Figure 16.

Figures 11-14 show the bistatic ambiguity functions of a single WiMAX cycle for $\theta_R$ values of 10, 30, 60, and 80 degrees. The figure sequence illustrates the degradation of the ambiguity function as the target approaches the bistatic baseline. Note that range resolution begins to degrade only very close to the baseline ($\theta_R = 80$ degrees). Doppler resolution degrades much faster to the point where it is practically lost close to the baseline. These results are typical
Interesting results are obtained by integrating eight WiMAX cycles in the coherent processing interval. Figure 15 shows the monostatic ambiguity function for the 8-cycle waveform. A thumbtack response is obtained. The longer integration time mimics a typical radar phase-coded pulse train, as evidenced by the repeated secondary peaks at symmetric intervals. These secondary peaks are 7dB lower relative to the main peak. This ambiguity function is similar to those seen using phase coding and frequency shifting techniques [11].

Figure 16 and Figure 17 show the 8-cycle bistatic ambiguity function for $\theta_R$ of 60 and 80 degrees respectively. Different from the single cycle case in large bistatic angles, the 8-cycle ambiguity function maintains better resolution in both Doppler and range. When the target approaches the baseline, the peak spreads in both directions; but maintains a relatively good form conducive to target state estimation.

To expand the evaluation of the range and Doppler ambiguity properties of the 8-cycle signal burst, a long range monostatic case is presented to explore the range ambiguities associated with the periodic nature of the signal. Figure 18 shows the WiMAX ambiguity function for the
monostatic case with $R_R = 60$ km. The periodic nature of the WiMAX signal structure is apparent by noting the large range ambiguities at approximately every 30 km. Large sidelobes can also be observed within 10 km of the central peak. Note that as simulated and defined in this paper, a full WiMAX cycle consists of 0.2 msec, corresponding to a 60 km ambiguity. The ambiguities shown in the figure correspond to other periodic attributes of the simulated signal (i.e., cyclic prefixes, training symbols, and pilot symbols). Nevertheless, these ambiguities appear at larger than the practical operating ranges of 5-10 km for a typical WiMAX network and for now, can be assumed to have minimal impact.

V. CONCLUSION

The ambiguity function for a simulated WiMAX burst is evaluated for several bistatic radar geometries using one-cycle and 8-cycle bursts. Results show that the simulated waveform has bistatic properties useful to PCL systems. Using longer integration times improved Doppler resolution and reduced ambiguity function degradation in large bistatic angles.

Based on the simulation results, the WiMAX signal is considered a practical signal of opportunity for PCL applications. The creation of WiMAX wide-area networks and its world-wide deployment in both urban and rural areas, provides a unique opportunity for the design and implementation of a PCL system with global reach. The challenge lies on the exploitation of the real signal in a true bistatic or multistatic environment. Further exploration of WiMAX network properties and other waveform characteristics is necessary to build a solid knowledge foundation for future WiMAX-based PCL systems research and design.

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REFERENCES