Low Frequency Antenna Analysis

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Abstract—An arbitrary VHF antenna array was considered for the comparison of two methods of analysis. The first method was based on a Fourier transform decomposition of the far-field radiation integral written in Matlab. In this approach the current on the antenna array is taken to be comprised of simplified functions. The current has three rectilinear components with each component being a product of three one dimensional rectilinear functions. The far-field is then obtained by a one dimensional Fourier transform of each of rectilinear function for each of the vector components. The array nature of the antenna is analyzed via a traditional summation approach and represented in closed form. The second method used was a traditional computational electromagnetic simulation using the Numerical Electromagnetic Code (NEC). The Fourier transform method ignores mutual coupling but is very computationally fast. The NEC method fully considers mutually coupling and is computationally much slower.

I. INTRODUCTION

Conventional television broadcasts around the world continue to rely on the Very High Frequency (VHF) radiation band for signal transmission. Given the current and potential growth of VHF broadcasts, VHF antenna analysis and design is of interest. The ability to analyze and design arbitrary, unconventional VHF antennas, including arrays, is an inherently valuable and critical asset. Our work demonstrates the accurate, mathematically and physically based, analysis of VHF antenna arrays capable of independently radiating in multiple directions. Both numerically based and physically based simulations are our approaches. This approach, however, is simply an introduction of the in-depth analysis potential given more time and resources.

II. THEORETICAL PHYSICALLY BASED SOLUTION

A. The Array Factor

The approach implemented allows us to develop the far field solutions for any arbitrary antenna structure. To begin the analysis, we first focus on characterizing an Array Factor. The original antenna element, and its cloned antennas can be incorporated into a single multiplying Array Factor (AF). In a 3D antenna with identical elements and equal current amplitude $I_o$, the AF is defined as:

$$AF = I_o f_M(\Psi_x) f_N(\Psi_y) f_L(\Psi_z)$$

where $f_M, f_N, f_L$ are the Sub-Array Factors in the $x, y, z$ vector directions respectively, and the underscores signifies a complex quantity. In the case of our 2D antenna, the Sub-Array Factor normal to the antenna array plane will have a value of one. The remaining Sub-Array Factors can be treated as independent 1D Array Factors with an equal phase taper and equal element spacing. The Sub-Array factors are given by:

$$f_M(\Psi_x) = 1$$

$$f_N(\Psi_y) = \frac{\sin(N\Psi_y)}{N\Psi_y} e^{j\Psi_y(N-1)/2}$$

$$f_L(\Psi_z) = \frac{\sin(L\Psi_z)}{L\Psi_z} e^{j\Psi_z(L-1)/2}$$

where the generating functions, $\Psi_x, \Psi_y, \Psi_z$ are defined by

$$\Psi_x = 0$$

$$\Psi_y = \beta d_y \sin \phi \sin \theta - \delta_y$$

$$\Psi_z = \beta d_z \cos \theta - \delta_z$$
with \( \beta \) related to the wavelength \( \lambda \) by \( \beta = 2\pi / \lambda \), and where \( \delta_x \), \( \delta_y \) are arbitrary linear phase tapers. Therefore, a 2D antenna with identical elements, equal spacing and phase taper in each vector coordinate is:

\[
AF = \int \int_{\mathbb{R}^2} \Psi \, d\psi .
\]

With the Array Factor developed, the next step is to identify the current distribution for a single element in the antenna. With the individual elements having rectilinear construction, the focus will remain on the rectangular coordinate system for this formation. With the assumption that current components in each vector direction are separable and independent, the current can be represented as:

\[
\hat{J}(\vec{r}_R, \omega) = \hat{x}J_{xx}(x', \omega)J_{xz}(z', \omega) + \hat{y}J_{yy}(y', \omega)J_{yz}(z', \omega) + \hat{z}J_{zz}(z', \omega). \tag{7}
\]

where the over-scores (over-bars) indicate a vector.

Conveniently, a Fourier Transform can be applied to each current component from real space into phase space such that:

\[
\tilde{\mathcal{J}}(\vec{r}_R, \omega) = \mathcal{F}\{J(\vec{r}_R, \omega)\} = \int \mathcal{J}(\vec{r}_R, \omega)e^{-j\vec{r}_R \cdot \vec{r}'_R} \, d\vec{r}'_R \tag{8}
\]

where

\[
\tilde{\mathcal{J}}(\theta, \phi, \omega) = \hat{x}J_x(\theta, \phi, \omega) + \hat{y}J_y(\theta, \phi, \omega) + \hat{z}J_z(\theta, \phi, \omega) \tag{9}
\]

\[
\vec{r}'_R = \hat{x}'x' + \hat{y}'y' + \hat{z}'z'. \tag{10}
\]

However, this process assumes a lossless media in the region of the antenna to allow for a closed form solution to be found. Thus, the separation of variables approach results in each individual Fourier Transform being multiplied to account for the entire Fourier Transform. This allows for a maximum of nine single Fourier Transforms to represent any current in phase space by:

\[
\tilde{\mathcal{J}}(\vec{r}_R, \omega) = \hat{x}J_{xx}(x', \omega)J_{xz}(z', \omega) + \hat{y}J_{yy}(y', \omega)J_{yz}(z', \omega) + \hat{z}J_{zz}(z', \omega). \tag{11}
\]

With the Array Factor and current distribution transform identified, the far field solutions can be easily found. First, the magnetic vector potential \( \vec{A}_R \) is given as:

\[
\vec{A}_R(\vec{r}, \omega) = \mu \Psi \tilde{\mathcal{J}}(\vec{r}_R, \omega) \tag{12}
\]

where \( \mu \) is the relative permeability media between antenna and far field, and \( \Psi \) is the free space Green's Function defined as

\[
\Psi = \frac{e^{-j\chi r}}{4\pi r} \tag{13}
\]

where \( \chi \) is the complex propagation constant for the media between the antenna and the far field.

Since the far field solutions are ideally represented in spherical coordinates, we transform the magnetic vector potential from rectangular to spherical unit vectors. This is accomplished by:

\[
\vec{A}_S(\vec{r}, \omega) = \mathcal{T}_S \cdot \vec{A}_R \tag{14}
\]

where \( \mathcal{T}_S \) is a dyadic transform from rectangular to the spherical vector components. The resulting far electric and magnetic fields are found by

\[
\vec{E}_S(\vec{r}, \omega) = \frac{z}{\mu} \vec{A}_S(\vec{r}, \omega) \tag{15}
\]

where \( z = \sigma \omega + j\omega \mu \)
array in the instance, consider two sub-arrays in the vertical plane. The inter-element spacing varies slightly in the center of the antenna, but we have assumed equal spacing in each direction. This assumption will not allow for an exact beam width to be identified, but rather a close approximation.

Let's first consider a full 84 element array. The number of elements in the y direction N=14, and the number of elements in the z direction L = 6, are applied to produce:

$$f_N (\Psi_y) = \left[ \frac{\sin \left( \frac{7 \Psi_y}{2} \right)}{\Psi_y} \right] e^{j \frac{\Psi_y}{2} (14-1)} \quad \text{(17)}$$

$$f_L (\Psi_z) = \left[ \frac{\sin \left( \frac{6 \Psi_z}{2} \right)}{\Psi_z} \right] e^{j \frac{\Psi_z}{2} (6-1)} \quad \text{(18)}$$

One can choose a phase taper to steer the beam in the desired direction. We have also assumed no phase taper in the azimuthal y direction. This is a safe assumption considering the antenna under investigation rotates continuously.

Another possible option would be two separate sub-arrays within the antenna. This would reduce the power radiated, but would allow another possibility for multiple beams. For instance, consider two sub-arrays in the y direction and a full array in the z direction. In this case, the y array factor would be broken into two component. The sub-array factor in y would then become:

$$f_{N1} (\Psi_{y1}) = \left[ \frac{\sin \left( \frac{7 \Psi_{y1}}{2} \right)}{\Psi_{y1}} \right] e^{j \frac{\Psi_{y1}}{2} (14-1)}$$

and

$$f_{N2} (\Psi_{y2}) = \left[ \frac{\sin \left( \frac{7 \Psi_{y2}}{2} \right)}{\Psi_{y2}} \right] e^{j \frac{\Psi_{y2}}{2} (14-1)}$$

where $\Psi_{y1}$ and $\Psi_{y2}$ would be dependent on the selected phase taper. The total Array Factor for a two 6 by 7 element arrays is given by:

$$AF = I_0 (f_{N1} + f_{N2}) f_L \quad \text{(20)}$$

C. The Element

In order to define particular current distributions, a critical assumption must be made about which elements are active and passive. We are considering each antenna element to consist of three folded over dipoles and a regular dipole, where the central folded over dipole acts as an active element. For our closed form mathematical development, we have simplified each antenna element as a single active dipole. Doing so will simplify the current distributions, allowing for the previous method to be applied. Since the development includes only simple dipoles, a safe assumption is only z directed applied currents.

With these assumptions defined, we can consider a constant z directed current. The full Fourier Transform in phase space reduces to:

$$\tilde{\mathbf{J}}_{\mathbf{R}} (\mathbf{R}, \omega) = I_0 \left( \frac{\beta L}{2} \right) \sin \left( \frac{\beta L}{2} \right) \frac{\sin \beta L \cos \theta}{\beta L \cos \theta} \quad \text{(21)}$$
where $L$ is the length of the dipole. This transform in phase space is simply a sinc function. Another case would be a cosine current distribution in the $z$ direction on the dipole. In this case, the Fourier transform would reduce to:

$$
\hat{J}_R(\beta_z, \omega) = \frac{2L \cos(\beta_z L / 2)}{\pi \left[ 1 - (2 / \pi \beta_z L / 2)^2 \right]}.
$$

(22)

Each current distribution leads to a different radiation pattern in the far field. With a particular Array Factor and current distribution identified, the previous procedures allow for far field pattern calculations. Figure 1 illustrates the far field pattern of a full antenna array maintaining a constant $z$ directed current with a phase taper of -15 degrees. This pattern is indicative of the antenna's ability to produce a single main beam at the horizon.

As previously discussed, it is possible to divide the full antenna into sub-arrays. By applying different phase tapers, one may produce multiple beams in separate locations. Figure 2 demonstrates the concept with the array broken into two arrays in the $y$ direction. Each sub-array then contains a phase separation of approximately 45 degrees. Each beam may then be steered by altering the phase taper.

After extensive testing with various configurations, we concluded that the VHF antenna array can not optimally produce more than a single main beam simultaneously. The distance between each element in the antenna affects the beamwidth for produced beams. In order to produce a beam without distortion from another lobe, the phase taper between subarrays must be large. Initial results suggested at least 35 degrees is required for smooth beam patterns. Though it is possible to produce multiple beams at once, doing so requires a relatively large phase taper (nearly 35 deg) with a much reduced power. Thus, in order to focus the main beam in various angles, multiple iterations of beams are required.

III. COMPUTATIONAL SOLUTION

To compare our mathematical approach to numerical simulations, we employed the use of the Numerical Electromagnetic Code (NEC). Utilizing the GNEC software package we modeled the VHF antenna array and simulated various antenna patterns which are shown in the Figures 3 and 4. Each element consists of 4 sub-elements of a single dipole followed by three folded dipoles with the central folded dipole as the active element.

IV. CONCLUSION

We have demonstrated the comparison between a simple rapid theoretically based analysis of an array antenna and a computational electromagnetic approach. The later accounts for complete mutual coupling and all other effects of the antenna structure and is hence more accurate. The former is computationally rapid and is adequate for initial evaluation purposes in the event that computational software is not available.

Based on the given results, it seems inefficient for the VHF antenna array considered herein to radiate more than a single main beam as depicted in Figure 5. This is confirmed both by both simulations.

ACKNOWLEDGMENT

The views expressed in this paper are those of the authors and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the United States Government.
Figure 3. 84 element VHF array

Figure 4. Zero phase taper antenna pattern.

Figure 5. Simultaneous multiple lobe radiation pattern.