Angle of Arrival Measurement Using Wideband Linear Phased Array

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Abstract - A radio frequency (RF) signal detection algorithm is developed based on two-dimensional Fast Fourier Transform (FFT) on phased array time series data. The sensitivity is calculated based on two-frame signal detection criterion. A collection system consisting of a linear phased array and multiple wideband digital receivers is utilized. Simulation results are presented based on the system parameters of the channelization frequency plan including the frequency range, LO, mixer, and sampling frequency. Experimental angle of arrival data are processed, and the results are shown to be in good agreement with the simulation results.

I. INTRODUCTION

As modern radar signal becomes more sophisticated in terms of its short pulse and wideband nature, the electronic warfare (EW) receiver needs to have wideband surveillance and direction finding (DF) capabilities. The wideband capability enables characterization and identification of the threat signals, while the direction finding capability measures the angle of arrival (AOA) and enables location of the threat signal. Both capabilities are essential for improving survivability in the electronic warfare environment [1].

Current wideband digital receiver systems are typically based on a channelization scheme [1]. A signal is received by an antenna, and amplified by a low noise amplifier (LNA). The signal is then fanned out into sub-band channels. A band-pass filter in each channel allows a certain frequency band to pass. A local oscillator (LO) down converts the signal into an intermediate frequency (IF) in each channel using a mixer and a low-pass filter. An analog to digital converter (ADC) digitizes the IF signal into a baseband signal. Each channel requires a tuner subsystem to provide a range of LOs to cover the wideband signals. This channelization scheme can be implemented to have instantaneous wideband coverage. In this implementation, the system requires a mixer for each channel. For a phased array system, this implementation requires same set of components following each antenna element, which may be impractical. The channelization scheme can also be simplified with one shared mixer with a tuner capability. This system needs a control mechanism to scan through channels to function as a wideband receiver. Its instantaneous bandwidth is that in the individual channel covering only the sub-band’s bandwidth.

Phased array receiver system is the modern day choice for DF. The multiple antenna elements provide not only the DF capability, but also an extra gain due to spatial coherence compared to single antenna receiver system. As a consequence, the sensitivity improves. The AOA can be measured by processing the spatial series data, similar to the processing the time series data using the Fast Fourier Transform (FFT) for time-frequency conversion. The peak FFT component in the frequency bin contains the frequency and power information of the input signal. Likewise, the peak FFT component in the spatial FFT bin contains the direction and power information of the input signal. Applications involving direction surveillance, the phased array is usually scanned through a range of angles. For these applications, microwave lens [2], Butler Matrix [3], Blass Matrix, and time delay line [4] are among the analog beamforming methods that can be used. Digital time delay line and FIR filter are among the digital beamforming implementation methods [5]. The phase relation among the multiple spatial channels is modified by these implemented devices, and the superposed signal from these channels has a maximum coherent gain toward a certain direction. By scanning this
maximum-gain direction, the receiver system can determine the AOA of the emitting source. This system needs only one ADC to digitize the combined signals from the multiple channels. It simplifies the ADC component count, but due to the scanning nature, the receiver may miss signals, especially those with short pulse characteristics. Ideally, the AOA surveillance covers a wide instantaneous angle so as not to miss any signals. The system needs to have an ADC for each antenna channel, and the signal processing involves two-dimensional (2D) FFT operations, which has a high computational cost.

With the advance of RF and digital technology including FPGA, the instantaneous wide angles and wideband is a natural path to take. In this presentation, we discuss the signal detection algorithm in a phased array antenna receiver system. First we describe the wideband digital receiver system, followed by a brief description of a signal detection protocol. Simulation using floating point discrete time series data is developed based on a signal detection protocol. The sensitivity and the AOA measurement accuracy is evaluated using this simulation. We also present preliminary AOA experimental data and their analysis results. The results are compared with the simulation results.

**II. SIMULATION AND EXPERIMENT**

**A. Phased array wideband digital receiver system**

The phased array system in this study is an in-house testbed system [4]. The antenna is a two-dimensional 16 X16 phased array covering 1-8GHz. The array is divided into sixteen 1x16 sub-arrays. Each sub-array has a 16 elements covering vertical direction. In this study, the elements in each sub-array are connected together. Each sub-array has a digital receiver consisting of an RF tuner and ADC with a 500 MHz instantaneous bandwidth. The signal from each sub-band is down converted into an input band between 750 MHz and 1250 MHz before an ADC. This ADC samples the signal with a sampling frequency of 1.333 GHz. The frequency plan in the front-end is shown in Table I and Fig. 1.

The 16 sub-arrays result in this system that can be considered as a one-dimensional phased array of 16 elements covering azimuthal angle from -60° to 60°. The spacing between elements is 0.45". This spacing is smaller than half a wavelength at 8 GHz. There is no grading lobe in the covered frequency range. However, for the low frequency input signal, the electrical length of the array is not as long, and the AOA measurement is less accurate compared to the high frequency input signal.

![Fig. 1. The frequency out vs. frequency in. The bandwidth is 500 MHz, and input band is from 750 to 1250 MHz with center frequency of 1000 MHz.](image)

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![Fig. 2. (A): The simulation flow chart. The S/N is referred to the simulated analog signal. After LO, bandpass filter, and ADC, the S/N is equivalently increased by 10 dB. (B): SNR after ADC vs. SNR in the simulated analog signal.](image)

**Fig. 2.** (A): The simulation flow chart. The S/N is referred to the simulated analog signal. After LO, bandpass filter, and ADC, the S/N is equivalently increased by 10 dB. (B): SNR after ADC vs. SNR in the simulated analog signal.
B. Simulation Flow Chart

Fig. 2(A) shows the simulation flow chart. The analog input signal is simulated using a discrete time series with a very high sampling frequency. In this case we use a sampling rate of 26.66 GHz which is 20 times of the ADC's sampling rate (1.333 GHz) in the testbed system. The input signal is mixed with an LO, and the resulting IF is passed through a 500 MHz bandpass filter. The signal-to-noise ratio (SNR) referred in this study is the input value for the simulated analog signal. The relation between the S/N at the input and output ports in the flow chart is shown in Fig. 2(B). It shows that the SNR at the output of the ADC is different from those in the simulated analog input signal. The difference is about 10dB in the linear region. It can be explained as follows. The input signal is of real number (in contrast to complex data using I Q channels), and in each data frame, there are two signal components in the full FFT frequency domain. These two components have equal amplitude (since the components are conjugate to each other). After mixing with the LO, these two components split into four equal-amplitude components in frequency domain. Each component is a half of the original signal components, ie., it is reduced by 3dB. The sampling frequency of 26.66 GHz at the input ADC has a bandwidth of 13.33 GHz, which is 20 times wider than 0.666 GHz which is the baseband width of the output ADC. This means the noise power is reduced by 13dB after the filter. As a consequence, the output S/N has a net gain of 10 dB compared to the input S/N. The deviation from this linear relation in the large input S/N is due to the FIR filter. The filter used in the simulation is the type-I fourth-order Chebyshev bandpass filter [6]. The power loss through this filter increases as the input S/N increases.

C. Signal detection algorithm

The important function of the receiver is to detect pulse signal, and to report its signal characteristics in pulse description word (PDW) [1]. For single antenna element receiver, the PDW contains the information of frequency, pulse width, pulse amplitude, and time of arrival. The continuous discrete time series data from the ADC is processed by frame. Each frame has 128 data points, and the frame length is about 96 nano-seconds. The FFT is conducted on the data in each frame. The frequency resolution is therefore, 10.42 MHz. Two-frame signal detection protocol is used. If FFT components exceeding a pre-set threshold are detected in two successive data frames, and these components in the successive frames occur at the same or the nearest neighboring frequency bins, one considers that a signal is detected. The allowing for the nearest neighboring bins is to include the possibility that the noise-embedded off-frequency-bin signal may deviate from the integer bin location by one bin.

Once the signal is detected, the beginning of the second data frame is the time of arrival (TOA) of this pulse signal, and the tracking of this signal follows. During the tracking, the signal’s frequency and amplitude are recorded. At the end of the pulse, there is no detected signal in the data frame, and the time of departure (TOD) is recorded.

When a phased array receiver system is used, the PDW adds the AOA information. In this case, the two-frame signal detection protocol is still applied. The time-frequency conversion needs to expand to include the spatial-angle conversion. The 2D FFT operation is conducted on the frame data of all the spatial channels. Both time and spatial series data is preconditioned using Hanning window [1]. The time frequency FFT results in 128 components. Half of them are retained since the input time series data is of the real type. The first 5 and the last six frequency bins are neglected due to no interesting and potentially undesired alias information content. The final tracked number of frequency bins is 53 ( = 64-11). The FFT operation is conducted on the 16 spatial series of each frequency bin. Since the input for the spatial FFT is of complex data type, the resulting 16 spatial FFT bins need to be all tracked. The 2D size is, therefore, 53 X 16. A typical 2D FFT result is shown in Fig. 3(A), where the location of the peak indicates its frequency and AOA information. Fig. 3(B) shows an amplitude’s color plot. If the maximum amplitude’s position in the second frame occurs at the same or within the nearest neighbor of that in the first frame, the signal is considered detected.

Once the strategy is set, we need to determine the signal detection threshold. A signal containing only noise was launched, and the 2D FFT amplitude results were collected. These amplitudes are collected from a 1000 runs. The relative histogram of the ensemble is shown in Fig. 4. It resembles the probability density function which is a Rayleigh distribution [7]. If the false alarm rate for this signal detection scheme is set to be $10^{-7}$, the false detection rate of a single 2D FFT
component is calculated to be $p = 3.62 \times 10^{-6}$, which is the solution to the following equation:

$$10^{-7} = (16)(53)(9p^2)$$  

(1)

where 16 is the number of spatial bins, 53 is the number of the frequency bins, and 9 is the number of the possible location of the maximum 2D FFT component in the second frame as indicated in Fig. 3(B). Based on the value of $p$, a threshold is calculated, and it is also indicated in Fig. 4.

D. Sensitivity

The sensitivity of a receiver is a performance parameter indicating its weak signal detection measurement ability. It is usually represented by a SNR in dB where the detection probability is 90%. Fig. 5 shows the results of the detection probability vs. the SNR in the simulated analog signal. Three cases are shown. The default case of $M = 16$ element has a relative SNR of -22.3 dB, compared to -19.3 dB and -25.3 dB, respectively for $M = 8$ and 32. Sensitivity gains 3 dB when element number doubles. This is due to 6 dB increase in signal power; 3 dB increase in noise power, and the net gain of S/N is 3 dB when element number doubles. If there is no window function imposed on the spatial series, the overall gain of the 16-element system is about 12 dB compared to single element system. With the window function, the gain generally reduces by one dB [8].

Fig. 3, 2D FFT results. (A) The location of the maximum peak indicates the signal’s frequency and spatial FFT bin, from which the signal’s frequency and AOA can be calculated. (B) If the peak location in 2nd data frame is one of the “x”, the signal is considered detected.

Fig. 4. Relative occurrence of the FFT component amplitude. The probability density function is noted to be a Rayleigh distribution function. The straight line indicates the threshold.

Fig. 5. Detection probability vs. SNR. The relative SNR is referred to the one in simulated analog signal using a sampling rate of 26.66 GHz.

E. Experimental results

The uniformity of the 16 channel hardware is characterized. An RF emitter is placed broadside about
10 meter away from the phased array. The input power to the emitter is 10 dBm. Fig. 6(A) and (B) show the S/N and the relative phase angle of the measured data for each antenna channel responding to the input frequencies of 3, 5, and 7 GHz. The system shows significant non-uniformity in both S/N and phase. These phase imbalance is taken into account when spatial FFT is conducted to find the AOA.

\[ \theta_m = \sin^{-1}\left( \frac{m\lambda}{Md} \right) \]  

where \( m = -M/2 \text{ to } M/2, \lambda \) is the wavelength, and \( M \) is the total number of the array (i.e., 16 in this case). There are two signal processing methods using FFT to conduct AOA measurement. One is the direct FFT operation on 16 spatial points. The other is zero padding 16 complex frequency component inputs into a one-dimension vector size of 128.

![Fig. 6. Calibration results showing the non-uniformity in (A) S/N and (B) phase among the 16 channels.](image)

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The AOA is measured from the spatial FFT results. Assuming the peak component is at \( m \)-th spatial FFT bin, the AOA is calculated to be:

![Fig. 7. Comparison between the signal processing results using experiment and simulation data.](image)

Fig. 7. Comparison between the signal processing results using experiment and simulation data. The AOA is 30°, and the frequency is (A) 7 GHz, (B) 5 GHz, and (C) 3 GHz.

The direct method generates only 16 spatial components. Fig. 7 shows the results using the experimental data and the simulation data for 7 GHz (A), 5 GHz (B) and 3 GHz (C). It is seen that both FFT results are in good agreement. One also notes that as
frequency decreases, the physical AOA bin decreases. For the case of 3 GHz input signal, there are only three physical AOA bins. It appears that the AOA resolution suffers. However, one finds that the amplitude comparison method [2] is very useful in determining the fraction FFT bin correction from which the AOA can be accurately determined.

The zero padding method generates 128 spatial FFT bin. Evidently, the AOA can be determined with a good angle resolution, since the data length is substantially increases. Fig. 8 shows the FFT results using the zero padding method. The accuracy of AOA measurement using this method is paid by increasing the computation cost.

**F. AOA measurement error simulation**

Fig. 9 shows the AOA measurement simulation results. The system parameters of the instantaneous bandwidth, frequency range, element spacing, ADC’s sampling rate are used in the simulation. The phase imbalance and gain uniformity among channels are not considered. The results of the standard deviation (STD) and the mean of the measurement error vs. SNR are shown for several input frequencies. For this calculation, the ensemble is obtained by 1000 simulation run for each S/N. For each run, an input frequency is given, the initial phase is randomly chosen between 0 and 2π, and the emitter’s incident angle is randomly chosen between ± 60 degrees. The direct FFT operation on windowed 16 spatial points, combined with amplitude comparison is conducted to calculate AOA. It shows that either S/N or input frequency increases, the STD decreases. The mean of the measurement error seems to be close to zero indicating un-bias error.

**III. SUMMARY**

An equivalent one-dimensional (16 X 1) phased array wideband digital receiver system is described. A signal detection simulation code is developed based on this system. The 2D FFT conducted on the collected
signals, enabling the frequency and AOA to be determined simultaneously. The signal processing using FFT for the AOA measurement is discussed. Both the signal processing results using the experimental data and simulation data are found in good agreement. The simulation results of the system’s sensitivity, and the AOA measurement performance are also presented and discussed.

ACKNOWLEDGEMENT

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REFERENCES


Table I, Frequency plan of a channelization receiver covering 2-8 GHz using a conventional approach. The sampling frequency is 2560 MHz for all the sub-bands

<table>
<thead>
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<th>Input band (GHz)</th>
<th>Input range (MHz)</th>
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<th>Baseband Output (MHz)</th>
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