Augmentation of a Navigation Reference System with Differential Global Positioning System Pseudorange Measurements

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Abstract

To quantify the performance of existing or proposed navigation systems, the U.S. Air Force has, for the last several years, compared the performance of systems under test to the performance of a baseline navigation system known as the Completely Integrated Reference Instrumentation System (CIRIS). CIRIS obtains an accurate navigation solution by combining information from three major subsystems: inertial navigation system (INS) information, barometric altitude information, and range and range-rate information from ground transponders which are precisely surveyed. Although the navigation solution produced by CIRIS is accurate, it will soon be inadequate as the standard against which future navigation systems can be tested. This paper proposes an alternative to CIRIS - a hybrid Enhanced Navigation Reference System (ENRS) which takes advantage of a newer ring laser gyro strapdown INS (the LN-93), certain features of the current CIRIS, and certain features of differential Global Positioning System (DGPS) measurements. Analysis conducted using a Kalman filter development package known as the Multimode Simulation for Optimal Filter Evaluation (MSOFE) is presented. Both a large order baseline truth model for the ENRS (in which a full 24 satellite constellation is modeled) and full- and reduced-order Kalman filters are developed. Results suggest that the proposed ENRS (with DGPS aiding) provides a navigation solution at least one order of magnitude better than CIRIS.

Background

CIRIS is a transponder aided INS test reference currently used by the Air Force for the development and testing of new aircraft navigation systems [10, 1-1]. The Office of Primary Responsibility (OPR) for the CIRIS system is the Central Inertial Guidance Test Facility (CIGTF), 6555th Test Group, Air Force Systems Command (AFSC), Holloman AFB, NM. The operation of CIRIS involves flying the INS to be tested, referred to as the test article, and CIRIS through an aircraft trajectory of interest across a CIRIS transponder range. The data from each system is recorded during the flight and compared to analyze how well the test article performed. Up to this point in time, CIRIS has been considered more accurate than the test articles and has formed the baseline for determining the performance of aircraft INSs.

CIRIS determines the aircraft’s latitude and longitude with a 1σ accuracy of 13 ft horizontal and 40 ft vertical; the north and west velocity to 0.1 ft/sec (fps) 1σ; and the vertical velocity to 0.4 fps 1σ [10, 1-1]. This accuracy is due primarily to the Range/Range-Rate System (RRS) transponder aiding [3, 5]. Recently, there have been a few state-of-the-art aircraft INSs developed (and many more in the design stage) approaching the accuracy of CIRIS. Interestingly, many of these new INSs use the Global Positioning System (GPS) to increase their accuracy. In order to use CIRIS as a baseline against these new INSs, CIRIS must be enhanced to provide an order of magnitude more accurate navigation solution.

By using Differential Global Positioning System (DGPS) measurements to augment the navigation solution of CIRIS, it is possible to increase the accuracy of CIRIS to produce an order of magnitude better estimate of the navigation solution. This paper overviews the development of a 48 state post-processing Extended Kalman Filter (EKF) to augment the CIRIS navigation solution with DGPS pseudorange measurements. An EKF is developed instead of a smoother due to the limited computer storage capacity available. The EKF is limited to 48 states to ensure 24 hour turn-around time for post-processing real measurements during INS testing at CIGTF. An in depth description of the design of this EKF can be found [9].

The Truth Models section describes the truth models used in this research. The error states for the test reference (INS and RRS) are described and then a short explanation of the DGPS implementation and error states is given. Then the Measurement Equations section presents the DGPS measurement equation. Full- and reduced-order filter models are described in the Filter Models section. Results of 25-run Monte-Carlo simulation analysis are presented in the Results section. Finally, conclusions based on the performance of the ENRS are discussed.
Truth Models

The 89-state truth model used in this research is divided into 3 submodels based on the 3 subsystems which form the ENRS. The first submodel contains a 41-state INS model consisting of 10 Litton LX-93 INS error-states and a single baro-altimeter error-state. The second 26-state submodel defines the error-states associated with 6 transponders modeled in the RRS. The last submodel contains the 22 error-states associated with DGPS measurements from 4 space vehicles (SVs).

INS Truth Model

The 41 error-state INS truth model is derived from the complete 93-state Litton LX-93 INS truth model [7]. This error-model is composed of 13 general error-states as well as 28 dominant gyro, accelerometer, and baro-altimeter error-states. The 32 states not present in this truth model have been combined with other states or eliminated according to the recommendations of Lewontowicz and Keen [6].

The first 13 states in the 41-state INS truth model represent the general error-states, states which are combinations of several other states in the error model. Position, velocity, platform tilt, and vertical channel errors are in this group of error-states. Following the first 13 general error-states are 4 error-states modeled as first-order Markov processes. These states are the X, Y, and Z accelerometer noise states and the baro-altimeter state. The final 24 error-states are gyro and accelerometer error-states modeled as random biases. The gyro error-states include gravity, drift, and scale factor errors. The accelerometer error-states include bias, scale factor, and misalignment errors. The 41-state truth model performance has been compared to the 93-state truth model performance with excellent correlation. A discussion of this comparison and a complete description of the 41-state INS truth model is found in [9].

RRS Truth Model

The 26 error-state RRS truth model contains two types of error-states, two states common to all six transponders and four states which are transponder dependent. The first two states in the truth model are random bias states modeling the effects of user hardware (RRS interrogator) on range and range-rate calibration errors. These two states are modeled with initial variance values of $1 \text{ft}^2$ and $10^{-4} \text{ft}^2$, respectively. The final 24 states, 4 states for each of six transponders, model transponder x, y, and z position errors and an atmospheric propagation error. The position errors are modeled as random biases with initial variance values of $25 \text{ft}^2$ while the atmospheric error is modeled as a first-order Markov process with a time constant of 300 seconds and a white dynamics driving noise of $6.66 \times 10^{-12} \text{ft}^2$. Again a complete description of the RRS states can be found in [9].

DGPS Truth Model

GPS is designed to be an accurate, stand-alone navigation system. However, for this research, GPS is used as a subsystem to improve the navigation solution of the ENRS. GPS navigation information is obtained from EM signal propagation through the media (space and atmosphere) between the user (ENRS) and each of 4 SVs which the user "locks" into a reception channel of the GPS receiver. In a stand-alone GPS receiver, navigation information is obtained by receiving GPS SV ephemeris data which are broadcast continuously from each active ("locked-on") SV, correlating the phase of the signal with a matching signal in the GPS receiver, and correcting for known error sources to produce an accurate range estimate between the user and each SV which is monitored. In this research, uncorrected range measurements (known as pseudorange measurements) are channeled to a Kalman filter which provides estimates of the error sources. Common GPS error sources which are considered dominant in this research include receiver clock bias and drift, ionospheric and tropospheric (atmospheric) propagation errors, SV clock, and SV position errors. As in the RRS, GPS range measurements make refinements to the ENRS navigation solution possible.

Interometrics, Inc. is the government sponsored contractor responsible for the DGPS reference station at CIGTF [1, 3]. Personal interviews were conducted with Mr Darwin Abbey and Mr Scott Dance of Interometrics to determine how differential corrections to GPS are implemented at CIGTF. The following discussion of DGPS comes directly from these interviews and the DGPS error model described is a combination of Interometrics's description and a course given by Navtech Seminars on DGPS error models [1, 3, 8].

In order to apply differential corrections to GPS measurements, a ground based reference receiver (GBR) is needed as well as an airborne GPS receiver (ABR). Figure 1 shows the basic DGPS system as it is being implemented at CIGTF. The ground based receiver must be capable of tracking all SVs in view, possibly as many as eleven when the full GPS constellation is placed in orbit. The GBR's antenna position is permanently fixed and surveyed to within centimeter accuracy. A high accuracy rubidium clock is used in place of the GBR's normal clock, greatly decreasing the large clock errors common to GPS receivers. Using the transmitted SV ephemeris data, the GBR computes its position (different from the surveyed position) known as the ground truth. The data (pseudorange measurements, SV transmitted corrections, GBR applied corrections, ground truth, clock errors, etc.) is fed to a 30386 personal computer which then estimates the SV position errors, SV clock errors, and atmospheric delays with great precision. Because the GBR's true position is accurately known and its clock errors are much smaller than the ABR's, the SV position and clock errors and atmospheric delays are highly observable. This
is unlike a normal GPS model where the large ABR's clock errors and vehicle dynamics cause these states to be largely unobservable. These errors, now called differential corrections, are time tagged and stored on magnetic tape or disc. Note that the differential corrections could be immediately transmitted via a data link for real-time differential corrections if the need ever arises [1, 3].

Remembering that the EXRS is a post-processing filter, the raw airborne pseudorange measurements (which are also stored magnetically and time tagged) now have the differential corrections applied before they are analyzed in the post-processing filter. Of course, this assumes the GBR is tracking the same four SVs the ABR was tracking (a good assumption if the ABR is within the CIRIS test range). Using differential corrections in this manner, the SV clock error is eliminated from each pseudorange measurement and the SV position errors are nearly eliminated. Depending on the distance between the ABR and the GBR during the flight profile, the atmospheric propagation errors (ionospheric and tropospheric) can be almost totally eliminated for close trajectories or greatly reduced for flights within 200 miles of the GBR. Even with long-range DGPS (flights which extend more than 200 miles from the GBR), the post-processing in the 30386 personal computer eliminates some of the atmospheric propagation errors. The largest remaining errors in the pseudorange measurements are the ABR clock errors. With this basic knowledge of DGPS, a DGPS error model is now developed assuming that differential corrections have previously been applied to the raw pseudorange measurements from the ABR [1, 3, 8].

The DGPS error-model is composed of 22 states. The DGPS error-states are similar to any GPS error-state model except for elimination of the SV clock error and reduction of the atmospheric and SV position errors due to differential corrections. The first two error-states in the DGPS error model are the ABR clock error-states. These two error-states are modeled by [11]:

\[
\begin{align*}
\{ \dot{x}_{U clk_e}, \dot{x}_{U clk_r} \} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \{ x_{U clk_e}, x_{U clk_r} \} \\
\end{align*}
\]

where

\[
\begin{align*}
x_{U clk_e} &= \text{range equivalent of ABR clock bias} \\
x_{U clk_r} &= \text{velocity equivalent of ABR clock drift}
\end{align*}
\]

The initial state estimates and covariances for these states are [9]:

\[
\begin{align*}
\{ \dot{x}_{Uclk_e}(t_0), \dot{x}_{Uclk_r}(t_0) \} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
\end{align*}
\]

and

\[
P(t_0) = \begin{bmatrix} 9.0 \times 10^{14} ft^2 & 0 \\ 0 & 9.0 \times 10^{13} ft^2/sec^2 \end{bmatrix}
\]

Until the ABR clock error is determined, it is the single largest source of error in DGPS range measurements. While the two states discussed above apply to all DGPS measurements, the tropospheric, ionospheric and SV position errors are unique to each SV. Note that these errors have a much smaller contribution after differential corrections than they would in a GPS model [8]:

\[
\begin{align*}
\begin{bmatrix}
\delta R_{ref} \\
\delta R_{ion} \\
\delta z_{u_1} \\
\delta y_{u_1} \\
\delta z_{u_2} \\
\delta w_{ref} \\
\delta w_{ion} \\
\end{bmatrix} &= \\
\begin{bmatrix}
- \frac{1}{c^2} & 0 & 0 & 0 & 0 \\
0 & - \frac{1}{c^2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\delta R_{ref} \\
\delta R_{ion} \\
\delta z_{u_1} \\
\delta y_{u_1} \\
\delta z_{u_2} \\
\delta w_{ref} \\
\delta w_{ion} \\
\end{bmatrix} \\
+ \\
\begin{bmatrix}
w_{ref} \\
w_{ion} \\
0 \\
0 \\
0 \\
\end{bmatrix}
\end{align*}
\]

\[
P(t_0) = \begin{bmatrix}
1.0ft^2 & 0 & 0 & 0 & 0 \\
0 & 1.0ft^2 & 0 & 0 & 0 \\
0 & 0 & 0.35ft^2 & 0 & 0 \\
0 & 0 & 0 & 0.35ft^2 & 0 \\
0 & 0 & 0 & 0 & 0.35ft^2 \\
\end{bmatrix}
\]

and

\[
E\{w(t)w(t^*+\tau)\} = \\
\begin{bmatrix}
0.001 & 0 & 0 & 0 & 0 \\
0 & 0.0004 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0. \left( \text{ft}^2/\text{sec} \right) \cdot \delta(t) \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The set of equations above apply to a single SV. The initial covariance values in Equation (5) and atmospheric
error dynamics noise variances in Equation (7) were obtained from [8] and are modeled slightly larger than Abbey
and Dance [1, 3] recommend since a conservative model is developed. There are four such sets of matrix equa-
tions for DGPS SV errors modeled in this research. The error-state vector is completely specified in [9].

The submodels described above for the INS, RRS, and DGPS subsystems compose the state dynamics equations
(with initial conditions). These are entered into MSOFIE [2] so that true error-state values can be computed and
analyzed against the extended Kalman filter’s estimated error-states. Now that all the error-states used in the
models have been described, the 3 types of measurement equations can be overviewed.

Measurement Equations
In order for any Kalman filter to properly estimate state variables, external measurements encompassing some of
the state variables must be provided to it. In this research, 3 types of measurements are provided to the post-
processing EKF. The first type of measurement is the baro-altimeter measurement used to stabilize the vertical
channel in the INS. This measurement occurs at 1 second intervals. The second measurement type is the RRS
range measurement from each of six transponders, with measurements from all six occurring every six seconds.
These measurements are well documented in [10] and [9], respectively, if the reader wishes to further research these
areas. The final type of measurement, and the measurement this research focuses on, is the DGPS pseudorange
measurement.

The DGPS pseudorange measurement is best described as a GPS pseudorange measurement with differential cor-
corrections applied to it, as mentioned in the previous section. These measurements are obtained from 4 geometri-
cally optimal SVs (out of 24 total and 11 possible, when the full GPS constellation is in orbit) at 10 second inter-
vals. The full satellite constellation has been programmed into this simulation and geometric dilution of precision
(GDOP) calculations are performed to obtain the 4 SVs used for measurement purposes, with satellite changes oc-
curring when necessary for optimal GDOP considerations [11]. After applying differential corrections, the measure-
ment equation is modeled as:

\[ R_{DGPS} = R_t + \delta R_{trop} + \delta R_{ion} + \delta R_{Urb} + \nu \]  \hspace{1cm} (8)

where

- \( R_{DGPS} \) = DGPS pseudorange measurement
- \( R_t \) = true range
- \( \delta R_{trop} \) = range error due to tropospheric delay
- \( \delta R_{ion} \) = range error due to ionospheric delay
- \( \delta R_{Urb} \) = range error due to ABR clock error
- \( \nu \) = zero-mean white Gaussian measurement noise

The DGPS pseudorange equation above includes the true range (which can never be known exactly) along with
terms which reflect sources of error and uncertainty inherent to DGPS range measurements. With this equation in
hand, it is desirable to formulate a difference measurement in the DGPS model. However, to accomplish this,
two sources of range information must be obtained. The first source is the range measurement coming from the
DGPS reference station and modeled by Equation (8). The second range estimate is constructed by differencing
INS-indicated position and SV (broadcast) positions to calculate the range. This derivation is explained in detail
in [9], and its result yields:

\[ \delta z = R_{INS} - R_{DGPS} \]

\[ = - \left[ \frac{x_s - x_u}{|R_{INS}|} \right] \cdot \delta x_u - \left[ \frac{y_s - y_u}{|R_{INS}|} \right] \cdot \delta y_u + \left[ \frac{z_s - z_u}{|R_{INS}|} \right] \cdot \delta z_u \]

\[ + \left[ \frac{x_s - x_u}{|R_{INS}|} \right] \cdot \delta x_s + \left[ \frac{y_s - y_u}{|R_{INS}|} \right] \cdot \delta y_s + \left[ \frac{z_s - z_u}{|R_{INS}|} \right] \cdot \delta z_s \]

\[ - [1] \cdot \delta R_{trop} - [1] \cdot \delta R_{ion} - [1] \cdot \delta R_{Urb} + \nu \]  \hspace{1cm} (9)

The terms \( \delta x_u, \delta y_u, \) and \( \delta z_u \) directly relate to the INS position error terms (latitude, longitude, and altitude) while \( \delta x_s, \delta y_s, \) and \( \delta z_s \) are the SV position errors. The pseudorange measurement noise variance is \( 9 \text{ ft}^2 \) when DGPS pseudorange measurements occur every 10 seconds. The true whole-valued range \( (R_t) \) formerly present is cancelled in the differencing operation. The bracketed coefficients in the equation above appear in the EKF update equations. The full derivation of this equation along with the EKF update and propagation equations is found in [9]. The DGPS error-state truth model and measurement equations have now been shown, so now the two filter models used in this research are described.

The Filter Models
Two filters are developed to implement DGPS pseudorange measurements into the ENRS, a full-order filter of
89 error-states and a 48 error-state reduced order filter. The first filter is described as full-order since the 89 filter
states are modeled exactly like the 89 truth model states. For the reduced-order filter, 41 states are eliminated af-
after analysis determined their magnitudes are small when compared to other error-states being modeled.

The Full-Order Filter
The 89 error-state full-order filter model is used to es-
tablish a baseline for comparison to any reduced-order filters developed. As a reminder, this filter is composed of
41 INS error-states, 26 RRS error-states, and 22 DGPS
error-states. Baro-altimeter, RRS range, and DGPS pseudorange measurements are used in this EKF to provide accurate position estimates. At this time, no velocity aiding measurements are incorporated into this filter, although research is continuing in this area and eventually RRS range-rate and DGPS delta-range measurements will also be used to provide accurate velocity aiding as well [9].

The Reduced-Order Filter

In order to ensure 24-hour post-processing time of a typical flight profile used to test INSs, CIGTF requested that the final EKF be less than 70 error-states. This means that at least 19 error-states have to be eliminated. More could be eliminated if position (and eventually velocity) accuracy can be maintained. With these objectives in mind, filter order reduction was performed and a new 48 error-state EKF was developed.

The first step in state reduction was taken when 20 DGPS error states were eliminated because their magnitudes were very small compared to the magnitudes of the ABR clock bias and drift states. A typical ABR clock bias error is on the order of 10s ft, while the differentially corrected atmospheric and SV position errors are 1 or 2 ft. The EKF often had trouble accurately estimating the small error-states because of this magnitude difference, so their elimination did not affect filter performance. Of course, the white Gaussian measurement noise variance was increased slightly to compensate for the eliminated states. [9]

As a second step to eliminate more states from the EKF, the recommendations of Lewantowicz and Keen are again followed when 21 more INS error-states are eliminated. The eliminated states are small magnitude gyro and accelerometer errors, and small increases in the dynamics noise variance in some of the remaining states is necessary to compensate for their removal. Note that the removal of these 21 states significantly increased the attitude errors, and slightly increased the velocity errors, but the position errors remain relatively close to the values of the full-order filter. At this point, enough states have been eliminated to ensure quick analysis while producing accurate estimates of the navigation solution. The EKF has been reduced to 48 error-states; 20 INS, 26 RRS, and 2 DGPS, and the results of filter performance are now presented.

Results

This section presents and discusses the results of the filter performance of the full-order and reduced-order filters. The current CIRIS 1σ position and velocity accuracies are used as a baseline to judge the filters performance. Also, a 46 error-state filter called CIRIS-46 is included in this comparison. This filter is composed of the 20 error-state reduced-order INS model and the 26 error-state RRS model and uses baro-altimeter and RRS range measurements. This filter gives an indication of the performance gained by the increase in INS and RRS states modeled, and will lend itself for a better comparison in the performance increase of DGPS alone. Each filter’s performance is tested with MTOFE [2] utilizing 25-run Monte-Carlo analysis of a 2-hour fighter flight profile. Figure 2 shows the latitude, longitude, and altitude information for this fighter flight profile. As seen, it incorporates several turns and dives to simulate a true flight. Figure 3 shows typical latitude, longitude, and altitude plots utilizing the data provided by MTOFE. These particular plots show the relationship between the true 1σ errors and the filter’s estimate of the errors. The single dashed line plots the mean error, or the difference between the true latitude error and the filter’s estimate of that error. The two dotted lines are plots of the mean error ± the true 1σ value. Finally, the solid outermost lines are the filter’s estimate of the 1σ errors. Again these plots are an average over 25 Monte-Carlo runs and represents an adequately tuned filter.
The results of the performance analysis on the 48-state reduced-order ENRS EKF are shown in Table 2. There is a slight decrease in position and velocity performance; however, this filter performs much better than either CIRIS or CIRIS-46. Notice that the reduction in filter states (gyro and accelerometer bias error-states) did affect the filter's ability to estimate the velocity error (and attitude error, though not shown). It is again safe to assume that DGPS measurements dramatically increase the accuracy of the navigation solution. It is obvious from the results that DGPS pseudorange measurements do indeed increase the accuracy of CIRIS and the post-processing 89- and 48-state ENRS filters increase the navigation solution position accuracy by one order of magnitude.

Table 2: Temporal Average of the Ensemble Average of True Errors for the 89-State and 48-State ENRS Filters

<table>
<thead>
<tr>
<th>Filter</th>
<th>Latitude (ft)</th>
<th>Longitude (ft)</th>
<th>Altitude (ft)</th>
<th>East Vel (fps)</th>
<th>North Vel (fps)</th>
<th>Up Vel (fps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIRIS-89</td>
<td>3.12</td>
<td>6.84</td>
<td>18.09</td>
<td>0.046</td>
<td>0.026</td>
<td>0.100</td>
</tr>
<tr>
<td>ENRS-48</td>
<td>0.90</td>
<td>1.32</td>
<td>3.05</td>
<td>0.027</td>
<td>0.020</td>
<td>0.044</td>
</tr>
</tbody>
</table>

Conclusions

The ENRS post-processing filters developed in this research increase the accuracy of the CIRIS system over one order of magnitude in position, so the objectives of this research are met. It is hoped that further research incorporating the velocity aiding measurements (both RRS and DGPS) will also increase the velocity estimates one order of magnitude. Even though it was not shown in this paper, the decrease in INS states from 41 to 20 decreased the attitude states estimation accuracy [9]. It is prudent to increase the INS states back up to 41 so that this accuracy can be maintained. This would increase the reduced-order filter to 69 error-states, still meeting the objective of having less than 70 filter error-states but increasing attitude estimation accuracy.
References


