Robust Discrete Controller Design For An Unmanned Research Vehicle (URV) Using Discrete Quantitative Feedback Theory

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Abstract

The application of non-minimum phase (nmp) \( w' \)-plane discrete MIMO (multiple-input-multiple-output) Quantitative Feedback Theory (QFT) to the design of a three-axis rate-commanded automatic flight control system for a URV is presented. The URV model used is a seven input three output state-space system derived from the small-angle perturbation equations of motion. Plant parameter uncertainty consists of six flight conditions derived from variations in the aircraft equations of motion. Center of gravity, airspeed, and gross weight. A weighting matrix \( \Delta \) is used to post-multiply the plants for blending the seven effector inputs into three effective rate-command inputs and resulting in an effective plant \( P_e = P \Delta \). Second order effector models and first order feedback sensor models are included in the plant. A time-scaled recursive algorithm is used to transform the continuous plant models to the \( w' \)-plane thereby avoiding the numeric problems associated with an intermediate \( z \)-plane representation. All continuous URV plant elements are minimum phase (mp). The discrete-plane transformation, however, produces a sampling nmp zero and one other nmp zero due to the three pole excess actuator/sensor model elements. These nmp elements limit the available loop bandwidth \( (l(p)) \). Standard QFT design is used, with plant templates \( P = (P(z)) \) which quantitatively express the plant uncertainty. Due to the loop bandwidth limitations, only stability bounds are derived. The loop transmissions \( (l(p)) \) are then shaped to achieve the maximum levels subject to the stability bounds. This is followed in the usual QFT manner with design of the prefilters. The design performance verification results are hybrid (amplitude limiting) simulations up to and including limiting of all surfaces. Some flight conditions are open-loop unstable so, in these cases, limiting induces instability. In this design, instability results only when all surfaces are at or very close to limiting. Hard limiting and nominal performance is shown.

Background

The Lambda URV, owned and operated by the Flight Dynamics Directorate (WL/FIGL) at Wright-Patterson AFB, OH, is a research vehicle used to subject flight control system components and control algorithms to the rigors of actual flight. It is a small mono-wing pusher propeller driven aircraft with a 14 foot wing span. It sports a conventional control surface configuration with ailerons and flaps on the main wing and elevators and rudder on a conventional tail. Presently the pilot directly manipulates the seven control surfaces (effectors) individually to attain the desired vehicle response. This study will enhance Lambda’s utility by designing an implementable noninteracting rate-commanded automatic flight control system. The controllers and prefilters resulting from this study will be flight tested on Lambda.

This design study is part of a wider ranging philosophy at the Air Force Institute of Technology (AFIT) to investigate and develop any and all possible flight control system design methods and theory. QFT is one of the design techniques receiving attention at AFIT and has been applied quite successfully to many different aircraft and design scenarios. It is used in this study as part of that wide ranging philosophy, and this paper describes the discrete QFT design of an automatic flight control system for Lambda. A detailed description of all aspects of this design and the aircraft data used is contained in [11]. MIMO QFT is found in references [3, 7, 4, 6, 5, 8, 1, 9].

Aircraft Model

Linearized perturbation equations of motion are derived from the aerodynamic stability and control derivative data generated by the Digital DATCOM CAD package used to design Lambda. The standard state space form is used:

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) \\
y(t) &= C x(t) + D u(t)
\end{align*}
\]

with conventional aircraft longitudinal and lateral system perturbation states \( x(t) \) of \( \theta, u, \alpha, \phi, \beta, p \) and \( r \) and effector inputs \( u(t) \) of \( \delta_{el}, \delta_{mb}, \delta_{fl}, \delta_{fb}, \delta_{a}, \delta_{r} \). The effector inputs \( (\delta) \) are in units of radians and each effector is independently controllable. The state system are in units of radians and radians/second except for perturbed forward speed \( u \) in feet/second. The controlled outputs are \( y(t) = [g(t) p(t) r(t)]^T \) (pitch-rate, roll-rate, and yaw-rate respectively).

QFT design uses the \textit{a priori} bounded range of plant parameter variation to shape unity feedback loop transmissions such that the \textit{a priori} specified system performance specifications are met. Robustness is built into the design through the use of plant templates which quantitatively express the bounded plant variation. Six level flight conditions provide the plant conditions.
variation for this design. Table 1 shows the important elements of each flight condition used. Flight condition 5 and 6 appear to be the same (I), however the stability derivatives are different in those cases.

The seven effectors are used by blending them to form an effective three input system plant \( P_e \) for each of the six level flight conditions. A weighting matrix \( \Delta \) is used such that \( P_e = P \Delta \). Elements of \( \Delta \) are selected so that the "natural" effectors for a given output are weighted more heavily and so that \( |P_e(s)| \) is minimum phase (mp). Optimality in weighting matrix selection is not considered in this design.

Seven identical second order effector actuator models \((324/s^2 + 25.4s + 324)\) for each of the seven effectors and three identical first order rate sensor models \((50/s + 50)\) for the three-axis rate sensor feedback gyro signals are included in \( P_e \) such that:

\[
P_e(s) = P(s)\Delta \begin{bmatrix} 324 \\ s^2 + 25.4s + 324 \end{bmatrix} \begin{bmatrix} 50 \\ s + 50 \end{bmatrix}
\]

The system models provided for Lambda for the six flight conditions are longitudinally \((1 \times 1)\) and laterally \((2 \times 2)\) decoupled.

**Discrete QFT**

Discrete QFT design is accomplished readily in the \( w' \)-plane when \( w' \approx s \) for the frequency band of concern and thus takes full advantage of the well-developed \( s \)-plane QFT design methods. The six plants are transformed to the \( w' \)-plane with a time-scaled recursive algorithm [2]. Data hold assumptions are used so that for each plant case:

\[
P_e(s) \rightarrow \text{Hofmann Algorithm} \rightarrow [G_{ZOH}P_e](w')
\]

A 60 Hz sample rate is used.

The Lambda MIMO sampled-data QFT system is a pre-filtered (F) unity feedback cascade compensated (G) system [11, Fig 3.2] with sampled control inputs and sampled input and output controller signals. For this configuration, transformed to the \( w' \)-plane, the Lambda MIMO system is expressible as:

\[
T(w') = [I + [G_{ZOH}P_e](w')G(w')]^{-1} \cdot \frac{[G_{ZOH}P_e](w')G(w')F(w')}{[G_{ZOH}P_e](w')G(w')F(w')}
\]

(1)

The controller matrix \( G(w') \) and the prefilter matrix \( F(w') \) are design variables, however, the above MIMO formulation of control ratios is not practical for design. Rather, a MISO (multiple-input-single-output) equivalent formulation of the MIMO system is used [4, 1, 11]. Separate "channel" design problems result from that formulation of the MIMO system since interchannel responses in the MIMO system are equivalently expressed as disturbances to the MISO equivalent channels.

The MISO form is derived from (1) with \( G_{ZOH}P_e = P_{x0} \) as follows [4]:

\[
T = [I + P_{x0}G]^{-1} P_{x0}G
\]

For a nonsingular \( P_{x0} \):

\[
[I + P_{x0}G] T = P_{x0}G \\
[P_{x0}^{-1} + G] T = GF
\]

(2)

\( P_{x0}^{-1} \) is partitioned so that:

\[
P_{x0}^{-1} = A + B_p
\]

(3)

is substituted into (2) to form the MISO equivalent system:

\[
T = [A + G]^{-1} [GF - B_pT]
\]

(4)

\( Q(w') \) is the MISO equivalent form of the plant \( P_e \) and its elements \( q_{i,j}(w') \) are related to the plant such that \( q_{i,j}(w') = 1/p_{i,j}(w') \) where the \( p_{i,j}(w') \)'s are the elements of \( P_{x0}^{-1} \).

Fully expanded, \( T \) from (4) above is a matrix whose elements represent the MISO equivalent loops of the original MIMO system [11, pp 3-15,16]. The factor \( q_{i,j}(1 + g_iq_i) \) appears in each element of row \( i \) in \( T \). Thus, \( q_i \) is synthesized to attenuate the coupling disturbances and attain the tracking performance for the elements of row \( i \) of \( T \). \( g_i \) is synthesized to position the loop transmission magnitude properly for specified tracking performance. Three separate single-loop equivalent design synthesis problems result from the MISO equivalent URV system as opposed to the nine interrelated design problems represented by the MIMO system in (1).

Two conditions are required for application of QFT to a MIMO system and its MISO equivalent form. First and most obvious, \( P_{x0}^{-1} \) must exist. Second, *diagonal dominance* must exist [4]. For the 2x2 case, *diagonal dominance* exists if:

\[
|p_{1,1}p_{2,2}| \geq |p_{1,2}p_{2,1}| \quad \text{as} \quad w \Rightarrow \infty
\]

where \( p_{i,j} \) are elements of a 2x2 plant matrix \( P_e \).
The discrete MISO equivalent plant elements $g_{i,j}(w')$ are formed and standard QFT templates constructed, however, only stability bounds are derived for the subsequent channel loop shaping. The transformation to the $w'$-plane produces only stability bounds are derived for the subsequent channel. The Lambda plants $P_i(s)$ are all mp, however, the plant elements all have a four pole excess due primarily to the three pole effector actuator/sensor models. Two nmp zeros result in the $g_{i,j}(w')$'s for Lambda. These nmp zeros introduce the phase lag that limits the maximum achievable loop transmission in a discrete design. Thus, it is usually much easier to extend the phase margin or stability bound for a range of frequencies $\omega$ ($w' = m + p\omega$). Plant templates are used to extend the phase margin or stability bound for $\omega$ by constructing a loci of the nominal plant points as the $\omega$ plant template is moved around the $M$ contour so that no part of the template penetrates the contour. The nominal loop transmission $T_\omega$ is then shaped so that these boundaries are not penetrated.

The Lambda design results in three separate MISO loop design problems where $l_i = g_i q_{i,j}$ ($i = 1, 2, 3$ pitch, roll, yaw). For a particular loop $i$, the nominal loop transmission is $l_i = g_i q_{i,j}$. $l_i$ is then shaped as desired and $g_i$ is derived by dividing $l_i$ by the plant $g_\omega = l_i / g_\omega$. This is fine if the order of $g_i$ is not a concern and $g_\omega$ is stable and mp. The phase contribution of unstable elements of $g_\omega$ are easily accounted for by including them in the loop shaping. By the same token, nmp elements of $g_\omega$ are included in $l_i$ to ensure that their phase contribution is also accounted for. Since lowest-order compensation is a concern for Lambda, $l_i$ is shaped by including all elements of $g_\omega$ so that elements of $g_i$ are selected directly during shaping.

These separate MISO designs involve some knowledge due to the fact that the MISO equivalent equations are developed with worst case channel disturbance input assumptions. The overdesign is reduced by applying the improved QFT design method [5]. This improved method makes use of the correlation that exists between the MISO equivalent loops. Loop shaping and selection of $g_i$ constitutes knowledge gained for a particular MISO channel. Due to the channel correlation, this knowledge is available to further refine the disturbance contributions to the other MISO channels. Incorporating this knowledge in subsequent loop designs reduces the worst case disturbance input assumptions thereby reducing the overdesign associated with the original MISO formulation.

The improved method is used on Lambda’s lateral 2x2 system. The smallest bandwidth loop is designed first with the original MISO equations. The $g_i$ of this loop is then inserted into the loop two modified MISO equations thereby incorporating the loop one knowledge for the second loop shaping. The 2x2 modified design equations are given below without derivation. The subscripts on the elements of these equations denote the order of loop synthesis. 1 is the first loop synthesized, 2 the second, etc.

\[
I_{2,1} = \frac{g_2 q_2}{1 - g_2 q_2}
\]

\[
I_{1,2} = \frac{g_1 q_1}{1 - g_1 q_1}
\]

$g_{2,1}$ is the notation used to denote the effective design transfer function of loop two modified by the known $g_1$.

Channel prefilter design ($f_i$) is accomplished in the usual manner as described in [1, 11, 4]. After all $g_i$ and $f_i$, channel elements are designed, they are transformed to the $z$-plane by the Tustin transformation and expressed as difference equations for implementation in the Lambda flight control computer.

Design

The r-loop (yaw-rate) design in the lateral Lambda system is described below. It is the first loop designed of the lateral system. Improved QFT is applied to the p-loop (roll-rate), the second loop synthesized in the lateral system. Since the lateral and longitudinal aircraft modes are completely decoupled, q-loop (pitch-rate) is a SISO (single-input-single-output) system without lateral channel disturbances. Once the templates are generated and the nominal plant selected, the loop designs are all very similar and are all presented in detail in [11].

A minimum degree-of-freedom design is desired and thus diagonal $G$ and $F$ are used. Nondiagonal $G$ and/or $F$ can be used when more degrees-of-freedom are needed to meet performance specifications. Also, a noninteracting rate-commanded system is specified for Lambda so that:

\[
T(w') = \begin{bmatrix}
\frac{g(w')}{r_{cmd}(w')} & 0 & 0 \\
0 & \frac{g(w')}{r_{cmd}(w')} & 0 \\
0 & 0 & \frac{g(w')}{r_{cmd}(w')}
\end{bmatrix}
\]

Time-domain response models to specify the desired closed-loop system performance are synthesized for the elements of (5). The figures-of-merit for the r-loop are listed in Table 2. The response model data for all loops including the transfer functions used are found in [11, Appen D]. The $w'$-plane frequency bounds resulting from these models are shown in [11, Appen D] and in Figures 3 and 4 as $B_\omega$ and $B_\omega$.

Plant templates are constructed for the r-loop and are shown in Figure 1. Plant case 1 is selected as the nominal plant for the loop shaping. The subscripts in the equations below denote the original system output order and not the
order of loop synthesis. The nominal loop transmission is:

\[ l_{03}(w') = g_3(w') q_{03,3}(w') \]

\[ q_{03,3}(w') = \frac{1.29995 \times 10^{-4}}{(-0.06613 x 120 x -140.9997 x 135.5963) \times (-0.4265 x 1.4601 x 12.8473 x 12.6911) \times (-935.7121 \times -47.3705)} \]

The design is verified by generating the time response of the channel for all plant cases.

These are shown with the response model bounds in Figure 5. The r-loop transmission is sufficient to meet the tracking and disturbance bounds had they been constructed. This is
not the case, however, with the $g$ and $p$ loops. In those cases, low frequency uncertainty (see [11, Appen E]) is very large unlike the $r$-loop (see Fig 1). Tracking bounds at these frequencies are extremely high on the Nichols chart since the allowable variation in magnitude derived from the response models at these frequencies is very small. The nmp elements of those loops limit the maximum loop transmission and thus make it impossible to achieve enough gain and meet the low frequency tracking bounds. This situation results in sagging time responses in those channels which, for all practical purposes, may be unnoticeable to the Lambda pilot.

**Simulation and Results**

Each of the MISO channel designs are simulated as in Figure 5 to verify proper execution of the design synthesis. Hybrid MIMO system simulations are performed with Matrix[10] to verify the complete MIMO system performance. The prefilter $F(s)$ and the controller $G(s)$ are implemented in discrete-time blocks and drive the continuous linear perturbation plant $P(s)$. A zero-order-hold device is incorporated between the controller output and the plant input. The actual amplitude limits on Lambda are used in the simulation to limit the deflection of the effectors. Effector actuator rate limits are not specified for Lambda's actuators and are thus not used in the simulation. Block diagrams of the simulation setup are found in [11].

A moderate and realistic $45^\circ$/sec, one second duration rate input is applied to each channel of the design individually. The longitudinal input does not affect the lateral channels and vice versa due to the decoupling in the model so only the $g$ response to a $q_{cmd}$ input is shown for the longitudinal simulations. In contrast, the lateral system ($2x2$) is highly coupled. In the uncompensated $F(s)$, ailerons can produce nearly as much $r$ response as the rudder and the rudder can produce nearly as much $p$ response as the ailerons. For the lateral simulations, $r$ and $p$ responses are shown for $r_{cmd}$ and $p_{cmd}$ channel inputs. Note that the numbers in the following plots denote plant cases and only the more interesting plant cases are labeled.

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however, for simulation purposes, it is necessary to investigate
the system response in the nonlinear case when all effectors
are quickly driven to limits. In these simulations, the effectors
are driven to amplitude limits within 1 second.

The results derived from these simulations are shown in Table 3. Plots of all state variables and effectors are found in

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Channel</th>
<th>q</th>
<th>p</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-Loop Q(w')</td>
<td>2, 4</td>
<td>2</td>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Closed-Loop 45°/sec</td>
<td>none</td>
<td>1</td>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Closed-Loop 500°/sec</td>
<td>4</td>
<td>1</td>
<td>none</td>
<td></td>
</tr>
</tbody>
</table>

Plants corresponding to flight conditions 2 and 4 are open-
loop unstable in the q channel, but, instability in the closed-
loop q channel only occurs for Plant 4 and only when the
effectors are forced to hard limit with a large command input.
Plant 2 is open-loop unstable in the p channel, however, Plant
1 is closed-loop unstable in the p channel for both simulations
shown. Plant 1 is a minimum controllable flight condition
and thus large effector deflections are needed to produce the
desired responses. For the p channel, the effectors for Plant 1
limit quickly and induce the instability. The flight conditions
are all stable in the r channel even with hard rudder limiting.

Generally, a control system with effectors operating at limits
will respond nearly as it would in open-loop operation. Table
3 shows that the q channel performs better than that, the r
channel performs as expected, and in the p channel there is a
kind of tradeoff. In this case, plant 2 is open-loop unstable and
closed-loop stable, but Plant 1 is open-loop stable and closed-
loop unstable. Note that in any case instability occurs only
when all effectors reach amplitude limits. Again bear in mind
that the nonlinear operation of limiting is never addressed
in the design synthesis although QFT techniques do exist for
this. A linear feedback system design is accomplished and
demonstrates the very beneficial nature and power of feedback.

Some extended (60 sec) coordinated turn simulations found
in [11] add more credence to the design. The steep bank
turns (45°) are accomplished with no effector limiting and no
instability. Further, a successful wing leveler autopilot func-
tion is designed and added to the new Lambda rate-command
system design. It's design and simulation is found in [11, Chap
6].

Summary

The controllers and prefilters designed provide a three-axis
noninteracting rate-commanded automatic flight control law
implementation on the Lambda URV. The control laws are
directly implementable and will be flight tested in the near
future.

The effective plants $P_i(s)$, with a three pole actuator/sensor
model, have an excess of four poles. This produces two npm
zeros in $[P_i'(w')]$. The phase of these npm zeros is accounted
for by including them in the template generation and in the
nominal loop transmission shaping for each MISO channel. In this way, these amp characteristics are handled directly and successful loop transmissions are synthesized for each MISO channel.

Hybrid nonlinear (effector amplitude limited) simulations on Matrix, of the completed design verify the successful application of discrete QFT. The yaw-rate channel meets all specifications. Due to the large uncertainty at low frequencies in the pitch-rate and roll-rate MISO channels, some low frequency tracking bounds are quite high and are not met by the respective loop transmissions. In a strict sense, a priori performance specifications are not met by the pitch-rate and roll-rate channels since the low frequency tracking bound violations result in minor sagging of the time responses. It is reasonable to assert that this may not be very noticeable or even detectable to the Lambda pilot. Pilots tend to rate aircraft on transient performance, and so, in that sense, the QFT designs generated in this study meet all specifications.

References


