MULTIVARIABLE CONTROL DESIGN FOR THE
CONTROL RECONFIGURABLE COMBAT AIRCRAFT (CRCA)

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Abstract
Multivariable output feedback digital control laws are designed for the Control Reconfigurable Combat Aircraft (CRCA). The design incorporates the high-gain error-actuated Proportional plus Integral (PI) controller developed by Professor Brian Porter of the University of Salford, England. Control law development and simulation are performed using the computer aided design program called MATRIX. Typically, control law analysis and design for an aircraft include separating the longitudinal and lateral equations of motion and designing control laws for each separate motion. The simplifying assumptions are often valid and do not adversely affect the analysis and design when aerodynamic cross-coupling is minimal. The CRCA design includes an all-flying canard with 30 degrees of dihedral angle which prevents the normal separation of lateral and longitudinal equations because of high aerodynamic cross-coupling. Consequently, developing a satisfactory controller for all aircraft motion must include all of the control surfaces and is more complicated. The three control surfaces on each wing are operated together, so they are treated in this paper as one control effector. Thus, the five CRCA control inputs for this design consist of two canards, left trailing edge flaperon, right trailing edge flaperon, and rudder. The aircraft dynamics are linearized about three flight conditions ranging from high speed, low altitude, to high speed, high altitude. Fixed gain PI controllers are designed at each flight condition for both the healthy aircraft and with a failed left canard and left trailing edge flaperon. Decoupling of the output variables is achieved and demonstrated by two maneuvers (pitch rate tracking and a 45 degree banked turn). The simulation indicates that the controller is very robust and output responses are fully satisfactory.

1 Introduction
Preliminary digital flight control tracking system designs for the CRCA are presented in this paper. The design technique implements a Proportional plus Integral (PI) controller using singular perturbation theory based on the mathematics of linear algebra [2, 6, 7, 8, 9, 10]. An error signal is formed by comparison of the command input vector, \( r(t) \), and the outputs, \( y(t) \), which are to be controlled. Continuous and discrete-time digital controllers implement proportional plus integral control and the closed-loop systems are able to track a constant command input vector with zero steady-state error. Figures 1 through 3 show the structure of the control systems.

Figure 1: PI Controller Regular Plant Design

Figure 2: PI Controller Irregular Plant Design

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Controllers are designed for a variety of flight conditions, including control surface failures. Performance of the controller is demonstrated by execution of a 45 degree banked coordinated turn and a pitch tracking maneuver. The simulation responses include actuator dynamics and the rate and position limits which may be encountered, especially during reconfiguration of failed control surfaces.

2 Controller Theory

The design technique requires describing the plant to be controlled as a set of input and output equations expressed in the state-space form of Equations 1 and 2

\[
\dot{x} = Ax(t) + Bu(t) \quad (1)
\]
\[
y(t) = Cz(t) \quad (2)
\]

where,

A = the continuous time plant matrix (n x n)
B = the continuous time control matrix (n x m), with the rank of B = m
C = the continuous time output matrix (p x n)
\( x \) = the state variable vector with \( n \) states
\( u \) = the control input vector with \( m \) inputs
\( y \) = the output vector with \( p \) outputs

Equations 1 and 2 may be transformed into a form

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
B_2
\end{bmatrix} u(t) \quad (3)
\]

\[
y(t) =
\begin{bmatrix}
C_1 & C_2
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} \quad (4)
\]

where,

\( z_2 \) has dimension \( m \times 1 \)
\( B_2 \) has dimension \( m \times m \) and has rank \( m \)
\( C_2 \) has dimension \( m \times m \) and has rank \( m \)

The design method used for the controller in this paper requires the number of controlled outputs \( y(t) \) be equal to the number of control inputs \( u(t) \), that is, \( m = p \). Furthermore, the design of the system is dependent on the rank of the first Markov parameter, the matrix product \( CB \). When \( CB \) has full rank \( m \), the plant is defined as regular and the PI controller is implemented as illustrated in Figure 1.

When \( CB \) is rank deficient, the plant is defined as irregular. Irregular plant controller design requires either the augmentation of an inner-loop which provides extra measurements for control purposes, Figure 2, or a Proportional plus Integral plus Derivative (PID) control law implementation. The configuration of the system in this paper provides for an irregular design with minor loop feedback. The high-gain controller implementing the proportional plus integral (PI) control law, Figure 2, is expressed in the continuous time domain by the relationship

\[
u(t) = g[K_1 \epsilon(t) + K_2 z(t)] \quad (5)
\]

where,

\( g = \) scalar gain
\( K_1 = \) proportional gain matrix (\( m \times m \))
\( K_2 = \) integral gain matrix (\( m \times m \))
\( \epsilon(t) = \) error vector between the input \( r(t) \) and output \( y(t) \)
\( z(t) = \int_0^t \epsilon(t) \, dt \)

For the discrete PI controller, Equation 5 is expressed as

\[
u(kT) = (1/T)[K_1 \epsilon(kT) + K_2 z(kT)] \quad (6)
\]

where,

\( T = \) sampling period (\( 1/f \))
\( K_1 = \) proportional gain matrix (\( m \times m \))
\( K_2 = \) integral gain matrix (\( m \times m \))
\( \epsilon(kT) = \) error vector between the \( r(kT) \) and \( y(kT) \)
\( z(kT) = \) digital integral of the error \( \epsilon(kT) \)

approximated by the difference equation

\[
z[(k+1)T] = z(kT) + (1/T)\epsilon(kT) \quad (7)
\]

Figure 3 illustrates the implementation of Equation 6 with the appropriate gain factor and zero order hold (soh) so \( u(t) \) is piecewise constant. The closed-loop system tracks a constant command input vector, that is:
\[ \lim_{k \to \infty} e(kT) = \lim_{k \to \infty} [r(kT) - y(kT)] = 0 \quad (8) \]

The form of the \( B \) matrix in Equation 3 and an investigation of the control law representation of Equation 5 as \( g \) becomes large, or asymptotically approaches infinity, \( g \to \infty \), develops the performance of the closed-loop system under high gain operation.

In Figure 2, the minor loop feedback described by
\[ w(t) = y(t) + M z_1 \quad (9) \]
enables the irregular system to be controllable. Inserting the values obtained from Equations 3 and 4 into Equation 9 yields a new output equation
\[
\begin{bmatrix}
    z_1(t) \\
    z_2(t)
\end{bmatrix} = 
\begin{bmatrix}
    C_1 & C_2 \\
    A_11 & A_12
\end{bmatrix}
\begin{bmatrix}
    z_1(t) \\
    z_2(t)
\end{bmatrix} + 
\begin{bmatrix}
    F_1 \\
    F_2
\end{bmatrix} r(t)
\quad (10)
\]
where
\[ r(t) = \text{vector of command inputs, (m x 1)} \]
and,
\[
y(t) = 
\begin{bmatrix}
    0 & C_1 & C_2
\end{bmatrix}
\begin{bmatrix}
    z_1(t) \\
    z_2(t)
\end{bmatrix}
\quad (11)
\]

The closed-loop transfer function for Equations 13 and 14 is
\[
G(\lambda) = 
\begin{bmatrix}
    0 & C_1 & C_2
\end{bmatrix}
\begin{bmatrix}
    \lambda_p & F_1 & F_2 \\
    0 & \lambda_{n-p} - A_{11} & -A_{12} \\
    -gB_2K_1 & A_{11} & \lambda_p - A_{12} + gB_2K_1F_2 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
    I_p \\
    0 \\
    gB_2K_1
\end{bmatrix}
\quad (12)
\]

Equations 13 and 14 are transformed into the block diagonal form
\[
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix} = 
\begin{bmatrix}
    A_f & 0 \\
    0 & A_f
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix} + 
\begin{bmatrix}
    B_f \\
    B_f
\end{bmatrix} r(t)
\quad (13)
\]

where the subscript "p" designates association with the slow modes of the system and the subscript "f" corresponds to the fast modes of the system. As \( g \to \infty \) the components of Equation 16 and Equation 17 yield
\[
A_s = 
\begin{bmatrix}
    -K_{11}^{-1}K_2 \\
    A_{12}F_2^{-1}K_1K_2 + A_{11} - A_{12}F_2^{-1}F_1
\end{bmatrix}
\quad (18)
\]
\[
B_s = 
\begin{bmatrix}
    0 \\
    A_{12}F_2^{-1}
\end{bmatrix}
\quad (19)
\]
\[
C_s = 
\begin{bmatrix}
    C_2F_2^{-1}K_1K_2 + C_1 - C_2F_2^{-1}F_1
\end{bmatrix}
\quad (20)
\]
\[
A_f = -gB_2K_1F_2
\quad (21)
\]
\[
B_f = gB_2K_1
\quad (22)
\]
\[
C_f = C_2
\quad (23)
\]

The measurement matrix \( M \) in Equations 9 through 12 is selected with as few non-zero elements as possible, containing only enough elements so that \( F_2 \) has rank \( m \). Reference [3] gives an approach to selecting a measurement matrix for optimum decoupling.

The asymptotic closed-loop transfer functions, as \( g \to \infty \), is given by,
\[
\Gamma(\lambda) = \Gamma_s(\lambda) + \Gamma_f(\lambda)
\quad (24)
\]

The slow transfer function, determined from Equations 18 through 20, is
\[
\Gamma_s(\lambda) = [C_1 - C_2F_2^{-1}F_1][A_p - A_{12}F_2^{-1}F_1]^{-1}A_{12}F_2^{-1}
\quad (25)
\]

and contains only the transmission zeros.

The fast transfer function, determined from Equations 21 through 23, is
\[
\Gamma_f(\lambda) = C_2F_2^{-1}[A_{n-p} + gF_2B_2K_1]^{-1}gF_2B_2K_1
\quad (26)
\]

For the discrete controller, the asymptotic closed-loop transfer functions, as \( f \to \infty \), is given by,
\[
\Gamma(\lambda) = \Gamma_s(\lambda) + \Gamma_f(\lambda)
\quad (27)
\]

where \( \Gamma_s(\lambda) \) contains the "slow" modes and \( \Gamma_f(\lambda) \) contains the "fast" modes. Thus,
\[
\Gamma_s(\lambda) = (C_1 - C_2F_2^{-1}F_1)\times
\quad (28)
\]

\[
\Gamma_f(\lambda) = C_2F_2^{-1}(A_p - I_p + F_2B_2K_1)^{-1}F_2B_2K_1
\quad (29)
\]

The proportional matrix, \( K_1 \), from Equation 5, is selected to make the fast transfer function, Equation 26, diagonal. This is accomplished by selecting a diagonal matrix \( Y \) such that

\[
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix} = 
\begin{bmatrix}
    A_f & 0 \\
    0 & A_f
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix} + 
\begin{bmatrix}
    B_f \\
    B_f
\end{bmatrix} r(t)
\quad (16)
\]

\[
y(t) = 
\begin{bmatrix}
    C_s & C_f
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix}
\quad (17)
\]
The integral gain matrix, \( K_2 \), is assigned using
\[
F_z B_z K_1 = \begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_m
\end{bmatrix} = \Sigma
\] (30)
\[
K_1 = (F_z B_z)^{-1} \Sigma
\] (31)
The integral gain matrix, \( K_2 \), is assigned using
\[
K_2 = \lambda K_1
\] (32)
where,
\[
\lambda = \text{proportional to integral proportionality constant (adjusted in the simulation to give good transient response)}
\]
The ultimate goal of the design process is to select the weighting matrix \( \Sigma \) and proportionality constant \( \lambda \) that yield a satisfactory performance over the entire flight envelope. The designs are accomplished with the basic model defined as the nominal flight condition. Actuator dynamics are included in the design process to insure the utmost validity in the output responses.

3 Aircraft Description

The CRCA studied in this paper is a hypothetical high-performance fighter aircraft that is used to investigate fault detection and reconfiguration strategies of modern day aircraft [11, 1]. The aircraft consists of two trailing-edge flaperons and an elevator per wing, a vertical rudder, and right and left canards with 30 degrees of dihedral. The four flaperons and two elevators are primarily used in providing pitch and roll control, but also supplement the canards with independent movement when stability is needed should canard saturation occur during a maneuver. Symmetrical movement of the trailing edge surfaces enables longitudinal trim and airflow control, pitch control, and airfoil lift during landing and air combat maneuvers. Differential deflection of the elevators and flaperons permits precise lateral trim and roll control. The primary function of the vertical rudder surface is to provide directional trim and yaw control. Canard surfaces may also be operated symmetrically to produce pitching motion or differentially to produce yaw or roll moments. The 30 degrees of dihedral angle significantly increases the effect of the canards both in the differential and symmetric deflection categories, but the dihedral angle reduces directional stability. For the designs in this thesis, the trailing edge flaps and elevators are commanded simultaneously to yield "effective" left and right trailing edge control surfaces. This arrangement is consistent with existing control law design and provides preservation of laminar air flow over the wing during control surface movement. The number of controlled surface deflections are reduced to five: two forward canards, left trailing edge flaperon, right trailing edge flaperon, and rudder. Therefore, the control surface deflections are defined as follows:
\[
\begin{align*}
\delta_c &= \text{left canard} \\
\delta_c &= \text{right canard} \\
\delta_{tel} &= \text{left trailing edge flaperon} \\
\delta_{ter} &= \text{right trailing edge flaperon} \\
\delta_{rud} &= \text{rudder}
\end{align*}
\]
The CRCA flight control system is designed to perform satisfactorily in four mission segments, or flight conditions. Although design is at four set points, the control law is valid over a range of set points in terms of maneuvers. These flight conditions are chosen to feature the aircraft's control power during static and dynamic controllability analyses. This paper considers only the design of fixed gain proportional plus integral (PI) controllers for the flight condition Air Combat Maneuvering Entry (ACM Entry). This flight segment involves considerable maneuvering normally associated with air-to-air weapons delivery or enemy aircraft evasion. Aircraft speeds are usually in the transonic region with confrontation altitudes between 10,000 and 30,000 feet.

4 Nominal Flight Condition Design

In the ACM Entry flight segment, the healthy aircraft is wings level and moving at Mach 0.9 at 30,000 feet. The eight state, five input, five output linear state space model can be described by Equations 1 and 2.
\[
\dot{x} = Ax(t) + Bu(t) \\
y(t) = Cx(t)
\] The states of the system are
\[
x = \begin{bmatrix}
\begin{array}{l}
u \\
qu \\
\theta \\
\phi \\
p \\
r
\end{array}
\end{bmatrix}
\] (33)
and the commanded outputs are
\[
\begin{align*}
u &= \text{forward velocity} \\
\beta &= \text{side slip angle} \\
\theta &= \text{pitch angle} \\
\phi &= \text{bank angle} \\
r &= \text{yaw rate}
\end{align*}
\]
The selection of output variables requires a measurement matrix \( M \) which is chosen as the most sparse matrix which
produces full rank for the matrix \(F_2\).

Through insight gained from the asymptotic properties of the system and some trial and error, the values chosen for the design parameters at the nominal flight condition are shown in Table 1.

### Table 1: Discrete Time System Design Parameters

<table>
<thead>
<tr>
<th>ACM-ENTRY</th>
<th>(\sigma_1)</th>
<th>(\sigma_2)</th>
<th>(\sigma_3)</th>
<th>(\sigma_4)</th>
<th>(\lambda)</th>
<th>(m_{\sigma_1})</th>
<th>(m_{\sigma_2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACM-ENTRY</td>
<td>0.2</td>
<td>0.095</td>
<td>0.3</td>
<td>0.5</td>
<td>0.35</td>
<td>0.40</td>
<td>0.20</td>
</tr>
</tbody>
</table>

\[
A = 
\begin{bmatrix}
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
-32.3804 & 0.000 & -0.0119 & -0.0186 & -31.2350 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
-1.0634 & 0.000 & -0.0324 & -1.0634 & 894.4548 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
\end{bmatrix}
\]

\[
B = 
\begin{bmatrix}
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
-3.162 & 3.162 & -2.5974 & -2.5974 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.104 & 0.104 & -0.0099 & -0.0099 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.003 & -0.003 & -0.0004 & 0.0004 & 0.0006 & 0.0006 & 0.0006 & 0.0006 & 0.0006 & 0.0006 \\
0.0762 & -0.0762 & 0.5339 & -0.5339 & 0.1144 & 0.1144 & 0.1144 & 0.1144 & 0.1144 & 0.1144 \\
0.0486 & -0.0486 & 0.0071 & -0.0071 & 0.0071 & 0.0071 & 0.0071 & 0.0071 & 0.0071 & 0.0071 \\
\end{bmatrix}
\]

\[
C = 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
M = 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The actuator model used in the simulation is represented by:

\[
\delta_{\text{cmd}} = \frac{20}{s + 20} \delta_{\text{cmd}}
\]

and incorporates rate and position limits appropriate to each control surface [4].

### 5 Simulation Maneuvers

Two maneuvers are chosen for the simulation to evaluate the longitudinal and lateral characteristics of the PI controller. The first is a coordinated turn with a 45 degree bank angle. The second is a pitch tracking command of 2 degrees/second for 3 seconds. The input commands for the coordinated turn and pitch tracking commands are shown in Figures 4 through 6.

![Figure 4: \(\phi_{\text{cmd}}\)](image-url)
6 Closed-Loop Performance

The simulation contained the discrete controller and the linear state space model, with the actuator dynamics included. The MATRIX, computer aided design package (CAD) was used to select the PI controller design parameters and plot simulation results [5]. The System Build capability within MATRIX allows the controller and plant to be symbolized by the familiar block diagram representation of Figures 2 and 3. Furthermore, the designer can easily modify individual blocks during the simulation as adjustments are made. The initial value of $\Sigma = \text{diag}[\sigma_1, \ldots, \sigma_3]$ for Equation 31 was selected as the identity matrix, $\lambda$ for Equation 32 was equal to one, and $m_{31}$ and $m_{42}$ were equal to 0.25. If the system response is unstable, reducing each element of the $\Sigma$ weighting matrix is usually sufficient to get a stable response. Once a stable response is obtained, each weighting element of the $\Sigma$ matrix is adjusted to fine tune the desired tracking output. For instance, $\sigma_1$ adjustments primarily affect the first output chosen in the $C$ matrix, or the velocity output. The $\sigma_2$ weighting element shapes the response of the second output or the $\beta$ output, and so forth. Increasing the value of $\sigma_1$ increases the damping of the response, increasing the value of $\lambda$ tends to reduce the time to reach steady state output values, and increasing the measurement matrix values increases the damping for the corresponding output.

The aircraft responses and control surface movements for the banked turn are shown in Figures 7 through 11 and the pitch rate tracking task outputs are shown in Figures 12 through 16. The discrete system design was simulated at a sampling frequency of 40 Hz and the responses are smooth and fully satisfactory with little or no coupling achieved in the final design. Control surface rates and deflections are well within acceptable limits.

7 Summary

The capability of the controller to maintain the desired tracking response and decoupled outputs in all flight conditions is exceptional! Minimizing the coupling of the outputs ensures that the fast tracking characteristics of the PI controller are optimized. The inclusion of actuator dynamics ensures the utility of the design method. The design contained in this paper is robust and will achieve satisfactory performance at some other flight conditions in addition to ACM Entry [4]. There are, however, some flight conditions in which a revised control law is required for stability and improved performance. In order to cover the entire flight regime, an adaptive control law is required. Reference [4] contains an adaptive controller based on a recursive least squares control law. That design is planned for a subsequent paper.

References


Figure 7: $\phi$ vs $\beta$ (Coordinated Turn)

Figure 8: Yaw Rate ($\omega$) (Coordinated Turn)

Figure 9: Left/Right Canard Deflection (Coordinated Turn)

Figure 10: Left/Right Trailing Edge Deflection (Coordinated Turn)
Figure 11: Rudder Deflection (Coordinated Turn)

Figure 12: Pitch Angle (θ) (Pitch Tracking Command)

Figure 13: Pitch Rate (q) (Pitch Tracking Command)

Figure 14: Normal Acceleration (Pitch Tracking Command)

Figure 15: Left/Right Canard Deflection (Pitch Tracking Command)

Figure 16: Left/Right Trailing Edge Deflection (Pitch Tracking Command)