NOISE FIGURES

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Abstract

In this paper, we present a short tutorial description of noise figures for two-port linear transducers and entire receiver systems. Due to the long history of the use of noise figures to specify noise performance, numerous definitions have evolved. The relationships between the various noise figure definitions found in the literature are specified in this paper and tables are provided as a cross reference to the notation and naming conventions used in the references.

1 Introduction

The concept of a noise figure is a common and useful means of describing the noise performance of receiving systems in communications, radar, and related subjects. The long history of the use of noise figures to describe performance has given rise to a variety of notation and definitions for the quantities involved. The purposes of this paper are to: 1) present a tutorial description of noise figures for two-port transducers and receiver systems, 2) specify the relationship between the different noise figure definitions, and 3) provide a cross reference of noise figure definitions and notation found in the literature. The goal is not to establish yet another set of names and notation, but to provide a point of reference for comparison of the notation and definitions which are in place.

The definitions presented here are not original to this paper. As indicated by the references, the seminal work was published more than forty years ago [40], [19]. An excellent overview of the historical development of noise modeling, description, and measurement is given by Okwit [42].

All noise figures discussed here are average noise figures [26], rather than single frequency or spot noise figures. They are figures of merit for the average noise performance of the device over some operating band of frequencies. In addition, only single response frequency bands are considered. The additional noise which can occur from image frequencies in a heterodyned system is not considered.

2 Noise Figure for a Two-Port Transducer

We begin by defining the noise figure for a linear two-port transducer, such as an amplifier, a cascade of amplifiers, a passive device, or an entire receiver (not including the antenna). The two-port noise figure is a measure of the noise performance of the transducer and there is little variation in the definitions given in the references. Let \( F_n \) be the two-port noise figure as defined in reference [30].

\[
F_n = \frac{N_{0\text{ out}}} {N_{0\text{ in}} \cdot G} = \frac{N_o} {G \cdot k T_0 B_n}
\]

where \( T_0 = 290 \text{K} \) is the standard reference temperature.

2.1 Active Two-Port Transducers

An ideal device is one which adds no additional noise to the signal passing through the device. The only noise at the output of the ideal device is the noise at the input which has been amplified by the gain of the device. Thus, the noise figure is the actual noise power output divided by the noise power output due only to the input noise generated by the input termination at \( T_r \). We assume that the output resistance (real part of the output impedance) of the source and the two-port transducer are both positive. These results are extended to the negative resistance case in reference [24].

Let the available power gain of the device be \( G = S_o/S_i \), where \( S_o \) and \( S_i \) are the available output and input signal power levels, respectively. The available power is the maximum power which can be drawn from a source by arbitrary variation of its terminal current or voltage [24]. The available input noise power to the device, due to the input terminated at temperature \( T_0 \), is \( N_i = k T_0 B_n \) [18], where \( k = 1.38 \times 10^{-23} \text{ Joulies per kelvin} \) is Boltzman's constant, and \( B_n \) is the noise bandwidth. If we let \( N_o \) be the actual noise output power from the real device, then we can write the noise figure for the device as:

\[
F_n = \frac{N_o} {G \cdot N_i} = \frac{N_o} {G \cdot k T_0 B_n}
\]

If we let \( \Delta N \) be the noise at the output of the transducer due to the device itself, we can express the noise figure as:

\[
F_n = \frac{N_o} {G \cdot k T_0 B_n} + \frac{\Delta N} {G \cdot k T_0 B_n}
\]

Note that \( F_n \geq 1 \); the best two-port noise figure a device can achieve is a value of one.

\[
\begin{array}{c}
G, B_n, T_r, F_n \\
\hline
S_i \\
\hline
N_i \\
\hline
S_o \\
\hline
N_o
\end{array}
\]

Figure 1: Block Diagram of Two-Port Transducer

A block diagram model of a two-port transducer is shown in Figure 1. If we represent the noise added by the device, \( \Delta N \), as an additive noise source at an effective noise temperature of \( T_n \) referred to the input of an ideal (noiseless) device, then noise power of this source has a value of \( k T_n B_n \). The equivalent circuit for this representation of the noise source and ideal device is shown in Figure 2.

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From equation (3), we express the noise figure as:

\[ F_n = 1 + \frac{T_\text{to}}{T_o} \]  

(4)

### 2.2 Passive Two-Port Transducers

If the two-port device is purely passive, such as a transmission line or attenuator, the effective input noise temperature, \( T_\text{in} \), is a function of the device's thermodynamic temperature, \( T_\text{d} \), and the available loss, \( L \) [7].

\[ T_\text{in} = T_\text{d}(1 - L) \]  

(5)

The available loss is defined as \( L = S_i/S_o \), which is just the reciprocal of the available gain, \( G \).

Equation (5) gives the effective noise temperature of the transducer referred to the input of the passive device. Several references give the effective noise temperature of the passive device referred to the output of the device, \( T_\text{out} \).

\[ T_\text{out} = T_\text{d}(1 - 1/L) \]  

(6)

### 2.3 Cascade of Two-Port Transducers

If transducers are cascaded, we can calculate the noise figure of the entire cascade from the noise figures and gains of the individual stages. Let \( F_1, F_2, \) and \( F_3 \) be the two-port noise figures for the first, second, and third stages in a cascade of transducers. Let \( G_1, G_2, \) and \( G_3 \) be the power gains of the three stages, respectively, and let \( T_1, T_2, \) and \( T_3 \) be the effective input noise temperatures of the three stages, respectively. From equation (2), we can show that

\[ F_{123} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \]  

(7)

where \( F_{123} \) is the two-port noise figure of the cascade of the three stages. A similar result can be obtained for the equivalent input noise temperature of the cascade, \( T_{123} \).

\[ T_{123} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} \]  

(8)

These results can be extended to \( n \) stages [33].

The definition and notation for the noise figure of a two-port transducer are common and fairly uniform in the references; see Figures 5 through 7. Differences in notation, definition, and nomenclature usually arise when the overall noise performance of the entire receiver system including the antenna is considered.

### 3 Noise Figure for a Receiver System

We now consider describing the noise performance of an entire receiver system, including the antenna, with a system noise figure. The block diagram of the system to be modeled is shown in Figure 3.

All noise, other than that generated internally by the receiver, is modeled by an additive noise source of value \( kT_B n \). The temperature \( T_B \) is called the effective system noise temperature. The effective noise temperature of the entire transmitter system including the antenna is considered.

\[ N_s = kT_B n \]  

(9)

We now consider two definitions for system noise figure. The first definition is for the standard system noise figure, \( F_s \). In this paper, the term "standard" is used, as in reference [51], because this noise figure is referenced to the standard temperature, \( T_s = 290K \). The second definition is for the operating system noise figure, \( F_o \). The term "operating," as used in this paper, indicates that this noise figure definition is referenced to the effective operating temperature at the input of the receiver system. Here is where confusion can set in when comparing definitions from various texts and papers. The standard noise figure defined in this paper is the same as the standard noise figure defined by Skolnik in reference [51]. North calls this the "operating" noise figure in references [40], [41] and other references use further variations on the name. We shall use the names and definitions given in this section for their mnemonic value in specifying the reference temperature.

Each of these system noise figures is derived from the basic definition given in equation (2). Although these two noise figures are different in value, name, and definition, they provide complimentary representations of a receiver's noise performance.
We begin by presenting the definition for each system noise figure and then discuss some points of comparison between the two forms.

3.1 Standard System Noise Figure

The standard system noise figure, $F_s$, is as presented (in some cases under a different name and notation) by North, Skolnik, and others listed in Figures 8 through 10. The "standard" in the name refers to the fact that the noise performance of the system is referenced to the standard temperature $T_0 = 290K$.

The system to be modeled is shown in Figure 4. In terms of this model, the definition for noise figure given by equation (2) can be written as

$$F_s = \frac{k_B G(T_s + T_e)}{k_B G T_e}$$

from which we can obtain

$$F_s T_0 = T_s + T_e = T_s$$

and

$$F_s = \frac{T_s}{T_e} + F_n - 1$$

Note that $F_s \geq 0$, whereas $F_n$ is restricted to values greater than or equal to one.

Now, let us perform a similar development for the operating system noise figure.

3.2 Operating System Noise Figure

The operating system noise figure, $F_o$, presented here as given by Barton, Davenport and Root, and others listed in Figure 11. The "operating" in the name refers to the fact that the noise performance of the system is referenced to the actual effective input temperature of the system, $T_s$.

As in the previous section, the system to be modeled is as shown in Figure 4. The definition for noise figure given by equation (2) can be written as

$$F_o = \frac{k_B G(T_s + T_e)}{k_B G T_e}$$

$$= 1 + \frac{T_s}{T_e} (F_n - 1)$$

Note that the reference temperature at the input has been changed to $T_e$ instead of $T_0$ as presented in equation (2).

This is a fundamental change from the definition provided in reference [30]; however, we shall see that the resulting operating noise figure definition is consistent with the other noise figure definitions presented here.

From equation (14), it can be seen that $F_o$ must be greater than or equal to one, just like $F_n$. The standard system noise figure, $F_s$, is the only one which can have a value less than one when the output resistances are positive.

3.3 $F_o$ in Terms of $F_s$

We can now determine the relationship between the operating system noise figure and the standard system noise figure. From equation (12), we solve for the receiver noise figure.

$$F_n = F_s + 1 - \frac{T_s}{T_e}$$

Similarly, from equation (14), we obtain:

$$F_n = 1 + \frac{T_s}{T_e} (F_s - 1)$$

Set these two equations equal to obtain:

$$F_s = \frac{T_o}{T_e} F_o$$

Equation (17) provides the conversion between the operating system noise figure and the standard system noise figure.

3.4 Discussion

It is useful to consider the functional forms of $F_o$ and $F_n$. Both noise figures depend on the same three variables ($T_o$, $T_e$, and $F_n$); however, the value of $F_o$ increases with larger values of $T_o$ whereas $F_n$ decreases. The apparent noise performance of a system may be manipulated by the selection of the value of $T_o$ at which the system's noise performance is measured. If $F_s$ is used as the figure of merit for the system, an unrealistic value of $T_o = 0$ would imply the best possible noise performance. In this case, $F_s = F_n - 1$, which will be greater than zero for an actual system. If $F_s$ is chosen as the figure of merit, a very large value for $T_o$ (also unrealistic) would force the value of the noise figure to the "perfect system" value, $F_s = 1$ [38].

For a given value of $F_n$, the value of either system noise figure can be manipulated by selection of $T_o$; however, $F_s$ can only be forced to a lower bound value of $F_n - 1$, but a large value of $T_o$ will force $F_s$ to the "perfect" value.

4 Notation Cross Reference

Figures 5-11 provide a cross reference between the notation and naming conventions used in the references. In each figure, the top entry contains the notation used in this paper and each reference is listed in separate row.

5 Concluding Remarks

In this paper, we have presented the definition for the noise figure of a two-port linear transducer and the definitions for two system related noise figures.

The definitions for the two-port transducer noise figure, $F_n$, and the standard system noise figure, $F_s$, follow naturally from the basic definition given in reference [30]. The definition of the operating system noise figure, $F_o$, requires a change in the reference temperature from that given in reference [30].

The noise performance of a receiver system can be described by either the standard system noise figure or the operating system noise figure. The results are equivalent, and the noise performance description can be transformed from one noise figure to the other.

Given the long historical record of literature, and the wealth of references, it is probably not feasible to standardize the notation and naming conventions for noise figures. However, the cross reference tables should ease the comparison of receiver and transducer noise performance when variations in symbols and names are encountered.
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<th>$T_s$</th>
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<td>Present an extension of the definition to the case of negative resistance input termination.</td>
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<td>$T_s$</td>
<td>$T_d$</td>
<td>$T_N$</td>
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<td>The symbol used in this reference is actually closer to a &quot;cursive T&quot; rather than the calligraphic $T$ shown here.</td>
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Figure 5: References that only Define a Two-Port Noise Figure.

Figure 6: References that only Define a Two-Port Noise Figure, Continued.
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<td>Schwarz [47]</td>
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<td>$F$</td>
<td>Schwarz defines the two-port noise figure as $F = 1 + F$ (his notation). Note that $F$ is referenced to the effective antenna temperature, not $T_e$ as in this paper and the other references.</td>
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<td>Difranco &amp; Robin [14]</td>
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<td>$F_i$</td>
<td>$F_i$ is called the &quot;receiver noise figure.&quot;</td>
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**Figure 7:** References that only Define a Two-Port Noise Figure, Continued.

**Figure 8:** References that Define a Standard System Noise Figure.

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Goldman does not use separate notation to refer to the standard system noise figure, although the quantity is identified. $F_n$, is called the "system noise figure." "F* is called the "modified noise figure." $F_{op}$ is called the "operating noise figure." $F_{oper}$ is called the "operating noise factor." Okwist presents an excellent historical perspective on the development of noise performance modeling and description. $F_{op}$ is called the "operating or effective noise figure of the receiving system."
This article by Barton [4], Davenport & Root [11], Peel7les [14], and Shea [43] refers to a paper by I. J. Bussgang et al., 1959. The interpretation depends on how the statement "F' is the absolute temperature..." is construed in the paper.

F' is called the "average standard noise figure" and F, is called the "average operating noise figure." F is called the "operating noise factor."

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<td>From the paper &quot;Range Performance of CW, Pulse, and Pulse Doppler Radar&quot; by J. J. Bongang et al., 1959. This interpretation depends on how the statement &quot;...F' is the absolute temperature...&quot; is construed in the paper.</td>
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Figure 11: References that Define an Operating System Noise Figure.

References


