Modeling and Experimental Test of Effective Dielectric Constant of Multilayer Substrate with Periodic Metal Inclusion

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Abstract — This paper applies a novel integral equation approach to model the multilayer substrate with periodic metal inclusion. The approach combines the equivalence principle with connection scheme to avoid the sophisticated multilayered periodic Green’s function. Effective media theory is adopted to extract the effective dielectric constant. Good agreement has been achieved between our approach and analytical result. A printed circuit board was designed, fabricated and tested to validate our model.

Index Terms — dielectric measurement, integral equation, periodic structure.

I. INTRODUCTION

The multilayer substrate structure has very wide applications in modern semiconductor industry, such as in the interconnects, packaging and printed circuit boards. In these applications, the substrates usually are embedded with metal. The metal inclusions change the electromagnetic performance of the substrate. An efficient and accurate model of the structure will substantially benefit the design stage. Integral equation (IE) approach in spatial domain is one of the numerical methods to build such electromagnetic models [1]. The IE approaches for periodic structures need efficient evaluation of periodic Green’s function. The technique we used in this paper combines equivalence principle with connection scheme (EPACS) to avoid calculation of the multilayered periodic Green’s function [2]-[4]. This approach also features the modeling of 3D real metal objects with finite high conductivity. The formulation for EPACS is presented in the next section. In section III, numerical results as well as the experimental results are provided to validate the EPACS. The conclusions are summarized in section IV.

II. FORMULATION

Fig. 1 illustrates the multilayer substrate with periodic metal inclusions investigated in our research. The substrate is assumed to be infinitely large in the XY plane while the metal inclusions are doubly periodic in this plane as well. The parameters of each layer, such as the dielectric constant, thickness, and permeability can be different. One period is divided into N cells based on number of layers of substrate. With the periodic boundary condition and connection scheme, our computation domain can be limited to just one cell.

Fig. 1. The structure of multilayer substrate with periodic metal inclusion and unknown distribution in one cell.

Applying the equivalence principle, the electric field integral equation (EFIE) and magnetic field integral equation (MFIE) on the outer surface of cell1 So1, can be derived as follows [3], [4]:

$$
\mathbf{E}_{oi} = \mathcal{L}_i \left( \eta_i \mathbf{J}_{oi} \right) - \mathcal{K}_i \left( \mathbf{M}_{oi} \right)
$$

$$
\mathbf{H}_{oi} = \frac{1}{\eta_i} \mathcal{L}_i \left( \mathbf{M}_{oi} \right) + \mathcal{K}_i \left( \mathbf{J}_{oi} \right)
$$

where $\mathbf{J} = \mathbf{J}_0$ and $\eta_0 = \eta_1/\eta_0$.

For equations on the inner surface, the PMCHWT formulation is applied [4]. The equations are expressed as:

$$
\left[ \mathcal{L}_i \left( \eta_i \mathbf{J}_{oi} \right) + \mathcal{L}_{1,m} \left( \eta_{1,m} \mathbf{J}_{oi} \right) - \mathcal{K}_i \left( \mathbf{M}_{oi} \right) \right]_i = \left[ \mathcal{L}_i \left( \eta_i \mathbf{J}_{oi} - \mathbf{K}_i \left( \mathbf{M}_{oi} \right) \right) \right]_i
$$

$$
\left[ \mathcal{L}_{1,m} \left( \mathbf{M}_{1,i} / \eta_i \right) + \mathcal{L}_{1,m} \left( \mathbf{M}_{1,i} / \eta_{1,m} \right) + \mathcal{K}_i \left( \mathbf{J}_{oi} \right) + \mathcal{K}_{1,m} \left( \mathbf{J}_{oi} \right) \right]_i = \left[ \mathcal{L}_i \left( \mathbf{M}_{oi} / \eta_i - \mathbf{K}_i \left( \mathbf{J}_{oi} \right) \right) \right]_i
$$

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The definitions of operators $\mathcal{L}_{i,(m)}$ and $\mathcal{K}_{i,(m)}$ for Cell i are defined as follows[4]:

$$\mathcal{L}_{i,(m)}(\mathbf{X}) = jk_{i,(m)} \int \left[ \mathbf{X}(r') + \frac{\nabla \mathbf{X}(r')}{ik_{i,(m)}} \right] G_{i,(m)}(r',r) dS'$$

$$\mathcal{K}_{i,(m)}(\mathbf{X}) = \int \mathbf{X}(r') \times \nabla G_{i,(m)}(r',r) dS'$$

(5)

Then the integral equations are discretized. The unknowns on the inner surface and the four sides of outer surface are eliminated by applying the periodic boundary condition. After that, the connection scheme is adopted to enforce the tangential continuity of neighboring cells so that the relations between the top layer and bottom layer is determined [3], [4]. Finally, the tangential fields on the topmost and bottommost surface are matched to get the matrix equation as:

$$\begin{pmatrix} A^{(x)} - W \end{pmatrix} \begin{pmatrix} J_t^{(1)} \\ M_t^{(1)} \\ J_b^{(X)} \\ M_b^{(X)} \end{pmatrix} = \begin{pmatrix} E_{\text{inc}} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(6)

The coefficient matrix $A^{(x)}$ is obtained by a recursive way when one performs the connection scheme to the structure, as discussed above. The matrix $W$ is the coefficient generated when matching the tangential fields of the topmost and bottommost layer. Solving the matrix equation for the equivalent source and the reflection/transmission coefficients are calculated in terms of the equivalent sources.

Based on the same reflection and transmission coefficients, the effective media model can be built to get the effective dielectric constant of the whole structure [5].

III. NUMERICAL ANALYSIS AND EXPERIMENTAL VALIDATION

A. Numerical Analysis of Finite Conductivity

The EPACS is capable of modeling the real metal which has finite high conductivity by modeling the inclusion as general dielectric which has complex dielectric constant and permeability. This enables us to explore how the effective dielectric constant and the permeability change due to the change of inclusion conductivity. The numerical result for complex dielectric constant and complex permeability of one layer substrate with sphere metal inclusions are shown in Fig.2 and Fig. 3, respectively. The results are compared with Maxwell-Garnett (MG) formula [6], an approximated analytical formulation for the sphere inclusion case. The MG formula results are also plotted with respect to the real and imaginary parts, using red ‘+’ and dot, respectively. The black dashed line marked as “PEC limit” in the figure is the results for PEC inclusion. The equation for PEC limit is in [6].

![Fig. 2. Effective dielectric constant as a function of the conductivity of the sphere in one layer of 3D sphere inclusion, radius of sphere is 2.4μm; periodic cell dimension: 10 μm × 10 μm × 10 μm; real($\varepsilon_r$) of the sphere: 2; $\varepsilon_r$ of the substrate: 3.9; frequency: 20.1GHz. Red circle and solid line: real and imaginary parts of EPACS results. Blue ‘+’ and dot: real and imaginary parts of MG formula. Long black dashed line: results for PEC limit.](image)

![Fig. 3. Effective relative permeability as a function of the conductivity of the sphere in one layer of 3D sphere inclusion. Simulation specifications are the same as the case in Fig. 2. Red circle and solid line: real and imaginary parts of EPACS results. Blue ‘+’ and dot: real and imaginary parts of MG formula. Long black dashed line: results for PEC limit.](image)
limit. Meanwhile, the imaginary part of the complex dielectric constant shows a negative peak in the transition region but stay close to zero when the results converge to PEC limit.

Fig. 3 shows that the effective relative permeability stays in one when conductivity of the inclusion is low: below $10^6$/m in our case. When the inclusion’s conductivity goes higher than that, the results approaches to PEC limit and the effective relative permeability is slightly less than one.

B. Experimental Validation

To verify our theory, a printed circuit board (PCB) is designed, fabricated and measured. The PCB had 4 layers. The top metal and the bottom ground form traditional transmission line (TML) while the inner two layers are embedded with periodic metal patches. A photo of the fabricated PCB and the structure illustration are presented in Fig. 4. The traditional TMLs without periodic metal embedded in the substrate were also fabricated and measured. This was for comparison and to show the effect caused by the metal inclusion.

![Image](image_url)

Fig. 4. The photo of fabricated PCB (left) and the structure illustration of TML with periodic metal inclusion (right).

The design utilizes transmission line pairs with different lengths to measure the dielectric constant, or effective dielectric constant of the substrate [7]. By measuring the phase difference of forward transmitting coefficient $S_{21}$, one can calculate the effective dielectric constant based on the property of matched TML. The detailed formulation can be found in [2], [7].

Table I lists the measurement results as well as results from EPACS model. In Table I, $\varepsilon_{r_{\text{sub}}}$ is the dielectric constant of the substrate without metal inclusion while $\varepsilon_{r_{\text{eff}}}$ is the effective dielectric constant of periodic metal embedded substrate. Several observations can be made through the measurement results. First, the dielectric constant of FR4 decreases when the frequency increases. The same dispersive trend is reported in [7]. Second, with the periodic metal embedded, the effective dielectric constant rises, but the dispersive trend is still the same. Comparing with the measurement, our EPACS model gives very accurate prediction. The differences are around 2%.

<table>
<thead>
<tr>
<th>TABLE I: EFFECTIVE DIELECTRIC CONSTANT, MEASUREMENT AND EPACS</th>
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<tbody>
<tr>
<td><strong>Measurement Results</strong></td>
</tr>
<tr>
<td>Frequency(GHz)</td>
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</tr>
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IV. CONCLUSION

A novel integral equation approach is adopted in this paper to calculate the effective dielectric constant of the multilayer substrate with periodic metal inclusion. This approach features in combining the equivalence principle with connection scheme (EPACS) to avoid the sophisticated multilayered periodic Green’s function thus reducing the computation cost. The approach is able to model real 3D metal inclusions with finite high conductivity. Effective media theory is adopted to extract the effective permittivity of the structure of interest. The numerical results show a good agreement between our approach and the analytical MG formula. A printed circuit board (PCB) was designed, fabricated and measured to validate our model. The test was performed in radio frequency range and the results support our model’s accuracy.

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REFERENCES


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