CHANNEL SHARING IN COGNITIVE RADIO NETWORKS

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Abstract: In this paper, we consider a cognitive radio network in which the secondary users are allowed to share the spectrum with the primary users as long as the interference caused by the secondary users to the primary users is below a specified level. Both the primary users and secondary users access the common channel by way of transmission schedules. The channel model includes realistic features such as receiver noise, fading, and multiuser interference. Our primary performance measure is network throughput, which is the average number of packets that are successfully received per time slot. For a given level of guaranteed performance for each primary user, our goal is to determine the transmission schedule for the secondary users that maximizes their throughput. Our method exploits the multi-packet reception capability to improve throughput performance. We show that our method for scheduling can allow significant additional throughput for the secondary users, while keeping the impact of interference to the primary users to the specified level.

1 INTRODUCTION

We study a cognitive radio network that includes two groups of users: the primary group consisting of users that have higher priority on the usage of the channel, and the secondary group consisting of lower-priority users. A major goal of cognitive radio networking is to provide methods for making efficient use of the existing spectrum [2], [4].

A common method is to allow the secondary users to use the spectrum at times when the primary users are idle. Thus, there are two key issues. First, there is a need for the secondary users to reliably detect when the primary users are idle. This results in extra system complexity and costs. The detection becomes more difficult if the user traffic is highly dynamic, e.g., when the primary users alternate quickly between active and idle modes. This becomes more problematic when the network operates under heavy fading, shadowing, and barriers. Further, the detection is subject to undetected probability and false-alarm probability. Proposed detection techniques include matched filter detection, energy detection, cyclostationary detection, and wavelet detection [4]. The second issue is how the secondary users use the channel, after the primary users are detected to be idle. It is important for the secondary users to use efficiently the newly available channel.

In this paper, we consider a cognitive radio network that consists of two groups: the primary group of \( K \) source nodes transmitting data to destination \( D \), and the secondary group of \( K' \) source nodes transmitting data to destination \( D' \). The network operates in the presence of detrimental effects such as channel fading, receiver noise, and other-user interference. Fig. 1 shows such a network with \( K = 6 \) and \( K' = 4 \).

Our goal is to study channel-access methods for a cognitive radio network that uses scheduling methods for accomplishing the transmissions between the source nodes and their destinations. In our model, we assume that the primary users are active (i.e., non-idle) all the time, and we allow the secondary users to share the channel with the primary users, as long as their transmissions do not cause excessive additional interference to the primary users. Because our protocol operates under the assumption that the primary users are always active, the problem of detection of idle channel is not an issue, resulting in lower system complexity and costs. We show by numerical calculation that, by carefully coordinating the transmissions of the secondary users, our method can allow significant additional throughput for the secondary users, while keeping the impact of interference to the primary users to the specified level. As a consequence, the throughput for the secondary users is even greater if the primary users actually turn out to be idle. Our model can be classified as an “underlay” cognitive radio model, as surveyed in [2].

Fig. 1 A cognitive radio network consists of 6 primary sources \( S_i \) intended for destination \( D \), and 4 secondary sources \( S'_i \) intended for destination \( D' \).

2 NETWORK MODEL AND ASSUMPTIONS

We consider a stationary cognitive radio network that consists of two groups. The primary group has \( K \) source nodes, denoted by \( S_1, S_2, \ldots, S_K \), that transmit their traffic to a destination, denoted by \( D \). The secondary group has \( K' \) source nodes, denoted by \( S'_1, S'_2, \ldots, S'_K \), that transmit their traffic to another destination, denoted by \( D' \). An example network with \( K = 6 \) and \( K' = 4 \) is shown in Fig. 1. We assume the following:

- The nodes, whose locations are known and fixed, are equipped with omnidirectional antennas.
- Each destination can receive more than one successful transmission at a time, i.e., it has multiple reception capability.
- Each source node can communicate directly with its destination. Routing is not discussed in this paper. However, our model can be extended to include multi-hop communication by allowing some nodes to forward traffic.
- Each source node always has traffic to transmit, i.e., its transmission queue is never empty.
• Time is divided into slots. The traffic is expressed in terms of fixed-size packets such that it takes one time slot to transmit one packet. A frame consists of $M_{\text{frame}}$ consecutive time slots.

• Our primary performance measure is sum throughput, which is the average number of packets that are successfully received by the destination in a time slot. We do not address issues such as time delays and stability analysis in this paper.

• Nodes transmit according to a schedule, i.e., a node can transmit only in an assigned time slot.

• Each primary source node transmits exactly once in each frame, and that the schedule repeats from frame to frame. Thus, it is sufficient to study the performance in any one frame.

• The primary and secondary groups share the same channel. Thus, a transmission from one group will interfere with the transmission of the other group.

• With the additional interference caused by transmissions from some secondary users, the original throughput of the primary users is reduced. The primary group has higher priority, and the secondary group has lower priority. That is, a certain level of performance is guaranteed for each primary user, and a secondary user is allowed to transmit only if this level is met (see Section 5).

**Definition 1** A schedule is a tuple

$$(H_1, H_2, \ldots, H_{M_{\text{frame}}})$$

where $H_k$ is the set of source nodes that simultaneously transmit in time slot $k$.

Later in the paper, we present algorithms for constructing schedules, in which the frame length $M_{\text{frame}}$ and the sets $H_k$ are determined, $k = 1, 2, \ldots, M_{\text{frame}}$.

The network operates according to the principle of power capture, i.e., a packet is successfully received, even in the presence of interference and noise, as long as its signal-to-interference-plus-noise ratio (SINR) exceeds a given threshold. More precisely, suppose that we are given a set $H$ of source nodes that transmit in the same time slot, and $S \in H$. Let $P_{tx}(S, D)$ be the signal power received from node $S$ at node $D$, and let $\text{SINR}(S, D)$ be the SINR determined by node $D$ due to the transmission from node $S$, i.e.,

$$\text{SINR}(S, D) = \frac{P_{tx}(S, D)}{P_{\text{noise}}(D) + \sum_{U \in H \setminus \{S\}} P_{tx}(U, D)}$$

where $P_{\text{noise}}(D)$ denotes the receiver noise power at node $D$. We assume that a packet transmitted by $S$ is successfully received by $D$ if

$$\text{SINR}(S, D) > \beta$$

where $\beta > 0$ is a threshold at node $D$, which is determined by application requirements (such as data rates and required BER). When $\beta < 1$ (e.g., in spread-spectrum networks), it is possible for two or more transmissions to satisfy (1) simultaneously.

The wireless channel is subject to fading, as described below. Let $P_{tx}(S)$ be the transmit power at node $S$, and $r(S, D)$ be the distance between nodes $S$ and $D$. When node $S$ transmits, the power received by node $D$ is modeled by

$$P_{tx}(S, D) = A(S, D)g(S, D)$$

where $A(S, D)$ is a random variable that incorporates the channel fading. We refer to $g(S, D)$ as the “received power factor,” which depends on $r(S, D)$ and $P_{tx}(S)$. For far-field communication (i.e., when $r(S, D) \gg 1$), we have

$$g(S, D) = P_{tx}(S) r(S, D)^{-\alpha}$$

where $\alpha$ is the path-loss exponent whose typical values are between 2 and 4. A simple approximate model for both near-field (i.e., when $r(S, D) < 1$) and far-field communication is

$$g(S, D) = P_{tx}(S) |r(S, D) + 1|^{-\alpha}$$

where the expression $r(S, D) + 1$ is used to ensure that $g(S, D) \leq P_{tx}(S)$. Under Rayleigh fading, $A(S, D)$ is exponentially distributed.

Our goal is to study methods for accomplishing the communication between the sources and destinations, and to evaluate the resulting performance. Under the well-known traditional TDMA method, each source node is given a turn to transmit in each frame, i.e., there is exactly one transmission and no other-user interference in each time slot. In this paper we also consider multi-packet reception approaches, as described in the following sections, under which more than one transmission is allowed in a time slot.

### 3 THROUGHPUT EVALUATION

Consider a transmission schedule $(H_1, H_2, \ldots, H_{M_{\text{frame}}})$, where $H_k$ is the set of source nodes that transmit in time slot $k$ (see Definition 1). For a given time slot $k$, let $C_{H_k}(S, D)$ be the probability that a packet from source node $S$ is successfully received by destination $D$, given that all the nodes in $H_k$ simultaneously transmit in this time slot. Let $C_{\text{success}}(k)$ be the average total number of successful transmissions in time slot $k$. We then have

$$C_{\text{success}}(k) = \sum_{S \in H_k} C_{H_k}(S, D)$$

We now define throughput $T$ to be the average number of packets that are successfully received by the destination in a time slot. Recall that there are $M_{\text{frame}}$ time slots in a frame. Using (4), the throughput is then

$$T = \frac{1}{M_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} C_{\text{success}}(k)$$

$$= \frac{1}{M_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} \sum_{S \in H_k} C_{H_k}(S, D)$$

For the case of Rayleigh fading, the following result (whose proof is given in [3, 5]) provides the exact formula for $C_{H_k}(S, D)$, which depends on the receiver noise, channel fading, receiver threshold, and other-user interference.

**Theorem 1** Suppose that the fading between a transmitting node $S$ and a receiving node $D$ is modeled as a Rayleigh random variable $Y_S$ with parameter $\nu(S, D)$. For $S \neq U$, assume that $Y_U$ and $Y_U$ are independent. Let $g(S, D)$ denote the received power factor, which depends on the distance and the transmit power, e.g., $g(S, D) = P_{tx}(S) |r(S, D) + 1|^{-\alpha}$. 

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Given that all the nodes in $H_k$ simultaneously transmit in time slot $k$, the probability that a packet from $S$ is successfully received by $D$ is

$$C_{H_k}(S, D) = \exp \left( -\frac{\beta P_{\text{noise}}(D)}{v(S,D)g(S,D)} \right) \prod_{U \in H_k \setminus \{S\}} \left[ 1 + \beta \frac{v(U,D)g(U,D)}{v(S,D)g(S,D)} \right]$$

where $\beta$ and $P_{\text{noise}}(D)$ are the required SINR threshold and the receiver noise power at $D$, respectively.

4 TRANSMISSION SCHEDULES FOR PRIMARY USERS

Recall from Definition 1 that a schedule is specified in terms of a frame. Each frame has $M_{\text{frame}}$ time slots. The set of source nodes that transmit in time slot $k$ is denoted by $H_k$. Similar to the traditional TDMA method, our capture-based method also require that each source node transmits once in each frame. However, our method allows the possibility of more than one transmission in a time slot, i.e., we may have $|H_k| > 1$ for some $k$. Under the TDMA method, we have $M_{\text{frame}} = K$ and $|H_k| = 1$ for all $k$, where $K$ is the number of source nodes. Under the capture-based method, we have $1 \leq M_{\text{frame}} \leq K$ and $|H_k| \geq 1$ for all $k$.

Let us consider an arbitrary schedule ($H_1, H_2, \ldots, H_{M_{\text{frame}}}$). Because we require that each source node transmits once in each frame, we must have $\{S_1, S_2, \ldots, S_K\} = \bigcup_{k=1}^{M_{\text{frame}}} H_k$ and $H_k \cap H_l = \emptyset$ for $k \neq l$. Thus, the schedule is associated with a partition of the set of the $K$ source nodes. The number of possible schedules is then the number of different partitions of the set of the $K$ source nodes. This number, called the Bell number $B_K$ [1], obeys the recursion

$$B_{n+1} = \sum_{i=0}^{n} \binom{n}{i} B_i$$

with $B_0 = 1$. The Bell numbers grow rapidly, e.g., $B_2 = 2$, $B_3 = 5$, $B_7 = 877$, $B_{10} = 115975$, and $B_{13} = 27644437$.

To summarize, we can compute the throughput $T$ in (5) for each of the $B_K$ schedules. Thus, our model and formulation naturally lead to the following schedule optimization problem: Find an optimal schedule that maximizes the throughput $T$.

In the following we briefly present centralized algorithms for constructing schedules used by the $K$ primary sources for transmitting their packets to their destination (see [6] for more details). In this section, we focus on the transmissions of a network consisting of only the primary group (i.e., there is no secondary group in the network). The coexistence of both the primary and secondary groups is considered in the next section.

Optimal Algorithm (OPT) Under OPT, we perform an exhaustive search to compute the throughput values for all $B_K$ possible schedules, and then choose an optimal schedule that yields the maximum throughput. Here, $B_K$ is the Bell number, which is also the number of different partitions of the set of the $K$ source nodes [see (6)]. This number is very large, even for moderate values of $K$, e.g., $B_{30} \approx 8.467 \times 10^{21}$.

Although OPT yields the best possible throughput, it has the disadvantage of high computational complexity. It is shown in [6] that the overall complexity of OPT is $O(B_K) \times O(K^2)$.

Because of the high complexity of OPT, heuristic suboptimal algorithms that have polynomial complexity are desirable. One of these heuristic algorithms is the following [6].

Algorithm 1 This algorithm has $K$ steps, where $K$ is the number of source nodes. At step 1, source node $S_1$ is scheduled for time slot 1. At step $i$, source node $S_i$ is scheduled for time slot $m$ that will result in the maximum throughput (computed up to this step). Note that $m$ can be a slot constructed in a previous step (i.e., $S_i$ can share the slot with some other previous nodes) or $m$ can be a new slot. The algorithm stops at step $K$ in which the final source node $S_K$ is scheduled. It is shown in [6] that the overall complexity of Algorithm 1 is $O(K^3)$. In this algorithm, for simplicity, the source nodes are scheduled one by one in the natural order $S_1, S_2, \ldots, S_K$. However, any other form of ordering can also be used.

We now compare the throughput performance for OPT, Algorithm 1, and TDMA for a network with $K = 10$ primary sources that transmit packets to destination $D$. Results for larger networks are provided in [6]. We assume the following:

- The path-loss exponent is $a = 3$.
- The wireless channel is subject to Rayleigh fading with Rayleigh parameter $v(S,D) = 1$.
- The received power factor $g(S,D)$ is given by (3).
- The transmit power is $P_{tx}(S) = 1$ for all source nodes $S$.
- The receive noise power at destination $D$ is $P_{\text{noise}}(D) = 10^{-3}$.
- The sources are located randomly in the circle centered at destination $D$ and of radius $r = 10$.

Fig. 2 shows the throughput $T$ versus the receiver SINR threshold $\beta$. The values of throughput are averaged over 100 randomly generated network instances. The figure shows that, as expected, smaller values of $\beta$ result in higher throughput $T$, and OPT (which is computationally expensive) outperforms both TDMA and the heuristic Algorithm 1 (which has polynomial-time complexity). Further, both OPT and Algorithm 1 (which exploit the multi-packet reception capability) significantly outperform TDMA, especially for $\beta < 1$.

![Fig. 2](image-url)
5 CHANNEL SHARING FOR EFFICIENT USE OF SPECTRUM

Recall that we consider a cognitive radio network that consists of two groups: the primary group of $K$ sources that are intended for destination $D$, and the secondary group of $K'$ sources that are intended for destination $D'$. The two groups share the same channel. Thus, a transmission from one group will interfere with the transmission of the other group. The primary group has higher priority, and the secondary group has lower priority. That is, a certain level of performance is guaranteed for each primary source, and a secondary source is allowed to transmit only if this level is met.

Due to the additional interference caused by the transmissions from the secondary users, the original throughput of the primary users is reduced. Our objective is to schedule the transmissions for the secondary users so that the reduced throughput of each primary user exceeds 90% of its original throughput. For example, by letting $h = 0.9$, only transmissions from certain secondary sources are allowed so that the new (reduced) throughput of each primary user exceeds the chosen threshold. Otherwise, it is called inadmissible in the time slot. Our goal here is to schedule the admissible secondary users for sharing the channel with the primary users. Note that, to maximize the sum throughput for the secondary users, some admissible secondary users may not be allowed to transmit.

The threshold is formally defined as follows. Consider a time slot $k$ of the transmission frame. Let $H_k$ be the set of primary sources that transmit in slot $k$. Then $C_{H_k}(S, D)$ denotes the probability that a packet from source $S$ is successfully received by destination $D$. Let $H'_k$ be the set of secondary sources that transmit in the same slot. Thus, the set of both primary and secondary sources that transmit in slot $k$ is $H_k \cup H'_k$. Because of the additional interference caused by the secondary transmissions, we must have $C_{H_k \cup H'_k}(S, D) \leq C_{H_k}(S, D)$. To guarantee that the performance of each primary source in slot $k$ exceeds a threshold factor $h$ of the original performance, $0 \leq h \leq 1$, we require

$$\frac{C_{H_k \cup H'_k}(S, D)}{C_{H_k}(S, D)} \geq h$$

We define $h$ to be the performance guarantee factor (PGF). For example, by letting $h = 0.9$, only transmissions from certain secondary sources are allowed so that the new (reduced) throughput of each primary source exceeds 90% of its original throughput.

5.1 Protocol for Channel Sharing

When the primary group does not share the channel with the secondary group, the throughput of the primary group is [see (5)]

$$T_{\text{unshared}} = \frac{1}{M_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} \sum_{S \in H_k} C_{H_k}(S, D)$$

where $H_k$ is the set of primary users that transmit in time slot $k$, and $M_{\text{frame}}$ is the transmission frame, i.e., the number of time slots in each frame.

When the primary group shares the channel with the secondary group, the throughput of the primary group becomes

$$T_{\text{shared}} = \frac{1}{M_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} \sum_{S \in H_k \cup H'_k} C_{H_k \cup H'_k}(S, D)$$

where $H'_k$ is the set of secondary users that transmit in time slot $k$. The throughput of the secondary group is then

$$T' = \frac{1}{M_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} \sum_{S' \in H'_k \cup H'_k} C_{H'_k \cup H'_k}(S', D')$$

In the following we present a centralized algorithm that allows the secondary group to share the channel with the primary group. The main idea is, for a given PGF $h$ as defined in (7), to allow a secondary source to transmit in a time slot, as long as (i) its transmission will guarantee that the new throughput of each primary user is within the required factor $h$ of the original throughput, and (ii) the total throughput of the secondary sources is maximized in the time slot.

Algorithm for Channel Sharing For each of the $M_{\text{frame}}$ time slots, keep adding admissible secondary sources to the slot until the throughput of the secondary users in this slot cannot increase further, subject to the performance guarantee for each primary source. More specifically, the following algorithm is used for each time slot $k$, $1 \leq k \leq M_{\text{frame}}$.

- For each $i$, $1 \leq i \leq K'$, perform the following procedure:
  - Initial step: Add admissible secondary source $S'_i$ as the initial node to slot $k$, provided $S'_i$ satisfies (7).
  - Intermediate step: Add to slot $k$ the admissible secondary source that yields the maximum secondary throughput computed at the current step, provided (7) is satisfied for this source.
  - Final step: Stop adding admissible secondary sources to slot $k$ when the total secondary throughput in slot $k$ does not increase further. Let $A_i$ be the set of all admissible secondary sources that are added to slot $k$, and $T'_i$ be the resulting total secondary throughput for slot $k$.

- Choose the set of admissible secondary sources that yield the maximum secondary throughput to share slot $k$ with the existing primary sources, i.e., the chosen set is $A_i$, where $i^*$ is such that $T'_{i^*} \geq T'_i$ for all $i$, $1 \leq i \leq K'$.$\square$

It can be shown that the complexity for each time slot is $O(K'^3)$. Thus, the overall complexity of the algorithm for the entire frame of $M_{\text{frame}}$ time slots is $O(M_{\text{frame}}K'^3)$. Note that, when both the primary and secondary groups coexist, the structure of the original frame of the primary transmission schedule (e.g., the frame size and the order of transmission) remain unchanged. The main difference is the additional interference (caused by the secondary transmissions), which reduces the throughput of the primary group. In each frame, in general, because different sets of primary sources transmit in different time slots, different sets of secondary sources transmit in different time slots. To maximize the total sum throughput of the secondary users, some may be given numerous opportunities to transmit, while others none at all, i.e., we do not address fairness issues associated with the secondary users.
5.2 Performance Evaluation

In this section, we evaluate the throughput performance when both the primary and secondary groups coexist. We also show the impact of channel conditions, receiver noise level, other-user interference, network topology, and schedules on performance. We now study a two-group network as shown in Fig. 3. The first group consists of \(K\) primary sources \(S\) intended for destination \(D\). The primary sources are located randomly in the circle centered at \(D\) and of radius \(r\). The second group consists of \(K'\) secondary sources \(S'\) intended for destination \(D'\). The secondary sources are located randomly in the circle centered at \(D'\) and of radius \(r'\). We assume the following:

- The path-loss exponent is \(a = 3\).
- The wireless channel is subject to Rayleigh fading with Rayleigh parameter \(e(S, D) = 1\).
- The received power factor \(g(S, D)\) is given by (3).
- The transmit power is \(P_{tx}(S) = 1\) for all source nodes \(S\), and \(P_{\text{noise}}(D) = P_{\text{noise}}(D') = P_{\text{noise}}\).
- The two destinations \(D\) and \(D'\) are separated by distance \(d\), \(D\) and \(D'\) have the same \(x\)-coordinate, and \(r = r' = 10\).
- \(K = K' = 10\).
- The primary users transmit their packets according to the optimal schedule OPT described in Section 4. The secondary users transmit their packets according to the Algorithm for Channel Sharing described in Section 5.1.

In the following, we show the throughput \(T\) versus the SINR threshold \(\beta\) for various topology configurations. We compare the three types of throughput values: the unshared throughput computed from (8) for the primary group \((T_{\text{unshared}})\), the shared throughput computed from (9) for the primary group \((T_{\text{shared}})\), and the throughput computed from (10) for the secondary group \((T')\). The throughput values are averaged over 100 randomly generated network instances. We will present the results for \(T \in \{T_{\text{unshared}}, T_{\text{shared}}, T'\}, P_{\text{noise}} \in \{0, 10^{-3}\}, d \in \{10, 20\},\) and \(h \in \{0.5, 0.9\}\).

First, consider the case of zero receiver noise, i.e., \(P_{\text{noise}} = 0\). The results are shown in Figs. 4 - 7. Fig. 4 shows the throughput performance for \(d = 10\) and \(h = 0.9\). That is, the two destinations \(D\) and \(D'\) are 10 units apart, and the new (shared) throughput of each primary source is maintained at least 90% of its original (unshared) throughput. Fig. 4 shows that, although the reduction in throughput of the primary group is negligible, significant additional throughput is obtained for the secondary group, especially for \(\beta < 2\). When the required SINR threshold becomes more stringent (e.g., when \(\beta > 10\)), the additional throughput for the secondary group becomes negligible, as expected.

Fig. 5 shows the throughput performance for the same topology configuration (i.e., \(d = 10\)), but now the PGF is reduced to \(h = 0.5\). That is, the new (shared) throughput of each primary source is only maintained at least 50% of its original (unshared) throughput. As expected, there are tradeoffs between the loss in primary throughput and the gain in secondary throughput. As shown in Fig. 5, the shared primary throughput is lower (and the secondary throughput is higher) than that for the case \(h = 0.9\) of Fig. 4.

Figs. 6 and 7 show the throughput results when the distance between the destinations \(D\) and \(D'\) is increased to \(d = 20\). The interference between the two groups is now reduced, which results in higher shared throughput than that for the case \(d = 10\) for both groups, as expected.

Next, consider the case of non-zero receiver noise, i.e., \(P_{\text{noise}} = 10^{-3}\). The results are shown in Figs. 8 - 11. The SINR is now reduced, which results in lower throughput than that for the case of zero receiver noise shown in Figs. 4 - 7.
6 SUMMARY

As shown in this paper, with proper design, it is possible for both the secondary and primary users to share the same channel at the same time in a cognitive radio network. Our network model does not require the assumption of idleness or “white spaces” in the spectrum, and in fact demonstrates that significant throughput of the secondary users can be achieved even when the primary users always have packets to transmit, while maintaining the throughput of primary users at an acceptable value. This contrasts with the dominant existing models that detect and then use white spaces, i.e., allowing either the primary users or the secondary users (i.e., not both) to use the common channel at any particular time.

REFERENCES