Abstract—The effect of pulse noise jamming and phase noise on a coherent RAKE receiver with convolutional coding is analyzed. The effect of additive white Gaussian noise (AWGN) is also included in the analysis. The maximum-likelihood RAKE receiver for the combination of pulse noise and AWGN is derived. Pulse jamming usually has a significant effect when soft decision decoding is used, however, we show that the maximum-likelihood RAKE receiver effectively mitigates the effects of the pulse noise jammer. Phase noise causes an irreducible error floor when soft decision decoding is used. The maximum-likelihood RAKE receiver with hard decision decoding reduces the effect of pulse noise jamming and can provide performance below the irreducible error floor obtained with soft decision decoding.

1. INTRODUCTION

The effects of phase estimation errors in RAKE receivers have recently been analyzed in the IEEE journals. A lower bound on the probability of bit error for a RAKE receiver with imperfect phase estimates is presented in [1] and an upper bound is in [2]. In [3] the performance of a RAKE receiver with phase noise was analyzed, however, this analysis did not include convolutional coding or pulse noise jamming. They also did not include the effect phase noise would have on the RAKE tap weight estimation. In this paper, the performance of a coherent RAKE receiver with pulse jamming, phase noise, and frequency selective fading is examined. The maximum-likelihood receiver for a coherent receiver with both Gaussian noise and pulse jamming is derived, and the performance of the maximum-likelihood RAKE receiver with a constraint length 9, rate 1/2 convolutional code is analyzed. Both hard and soft decision decoding will be considered. A block diagram of the system to be analyzed is shown in figure 1. Assuming that a pilot tone is transmitted on a separate frequency allows us to show the effect that jamming the data will have when the pilot tone has a relatively low level of phase noise. The effect of a noise-like jamming signal increasing the phase noise on the pilot tone is also considered. The performance of our receiver will also be compared with the performance of a RAKE receiver optimized for signals received with additive white Gaussian noise (AWGN). This comparison is of interest because commercially available RAKE receivers currently in use are optimized for AWGN channels, and their resistance to jamming is important for military communication.

II. MAXIMUM-LIKELIHOOD RAKE RECEIVER FOR PULSE NOISE AND GAUSSIAN NOISE

We will now consider a BPSK signal which is received with both AWGN and a noise like pulse jamming signal. It will be assumed that the jamming signal is either on or off for a full bit period and jams an integer number of bits. The interleaver randomizes the bits that are jammed, so it will be assumed that the jammed bits and the unjammed bits occur independently at the input to the RAKE receiver.

The maximum-likelihood receiver maximizes the ratio of the joint probability density functions for soft decision receiver outputs when bit “1” is sent and the soft decision receiver outputs when bit “0” is sent

\[ \frac{f_1(y)}{f_0(y)} \]

(1)
where y is the received sequence of soft decision receiver outputs. If the ratio of the joint density functions is greater than 1, the receiver assumes bit “1” was sent. If the ratio is less than 1, the receiver assumes bit “0” was sent. Since soft decision decoding uses a sequence of soft decision receiver outputs to make a bit decision, we will assume that the joint density functions are a combination of d soft decision receiver output values where d is an arbitrary number. The marginal density function for the soft decision receiver output assuming bit “1” was sent and received with only AWGN can be written as

\[ f_1(y) = \left( \frac{1}{2\pi\sigma_0^2} \right)^{\frac{d}{2}} \exp\left(\frac{-1}{2\sigma_0^2} (y(m) - s_1)^2\right) \]  

(2)

where \(\sigma_0^2\) is the AWGN power and \(s_1\) is the baseband signal that represents a binary “1”. If the jamming signal is present then the variance of the marginal density function is \(\sigma_0^2 + \sigma_j^2\) where \(\sigma_j^2\) is the power of the jamming signal. Since it is assumed that the bits arrive independently at the RAKE receiver, the joint density function is the product of the marginal density functions. Hence, the joint density function for the soft decision receiver output assuming a binary “1” was sent and that i of them are jammed and d-i have only AWGN can be written

\[ f_i(y) = \left( \frac{1}{2\pi\sigma_0^2} \right)^{\frac{d-i}{2}} \exp\left(\frac{-1}{2\sigma_0^2} \sum_{m=1}^{d-i} (y(m) - s_1)^2\right) \times \left( \frac{1}{2\pi(\sigma_0^2 + \sigma_j^2)} \right)^{\frac{i}{2}} \exp\left(\frac{-1}{2(\sigma_0^2 + \sigma_j^2)} \sum_{m=im+j+1}^{d} (y(m) - s_j)^2\right) \]  

(3)

the joint density function for the soft decision receiver output assuming a binary “0” was sent can be written by changing \(s_1\) to \(s_0\) in (3). Substituting these joint density functions into (1) and simplifying, we get

\[ \exp\left(\frac{-1}{2\sigma_0^2} \sum_{m=1}^{d-i} (y(m)s_0 - 2y(m)s_1 - s_0^2 + s_1^2)\right) \times \exp\left(\frac{-1}{2(\sigma_0^2 + \sigma_j^2)} \sum_{m=im+j+1}^{d} (y(m)s_0 - 2y(m)s_1 - s_0^2 + s_1^2)\right) = 1 \]  

(4)

taking the natural log of both sides of (4) and rearranging terms we obtain

\[ \frac{-1}{\sigma_0^2} \sum_{m=1}^{d-i} (y(m)s_0 - y(m)s_1) + \frac{-1}{\sigma_0^2 + \sigma_j^2} \sum_{m=im+j+1}^{d} (y(m)s_0 - y(m)s_1) \]  

(5)

\[ = \frac{1}{2\sigma_0^2} \sum_{m=1}^{d-i} (s_0^2 - s_1^2) + \frac{1}{2(\sigma_0^2 + \sigma_j^2)} \sum_{m=im+j+1}^{d} (s_0^2 - s_1^2) \]

since BPSK signals are antipodal, \(s_1 = -s_0\) and \((s_0^2 - s_1^2) = 0\). This simplifies (5) to

\[ \frac{1}{\sigma_0^2} \sum_{m=1}^{d-i} (y(m)s_0 - y(m)s_1) \]

(6)

which shows that the optimal receiver correlates the received signal with the signal that represents a binary “0” and subtracts the received signal correlated with the signal that represents a binary “1”. The result is weighted by the inverse of the variance, \(1/\sigma_0^2\) for bits with only AWGN and \(1/(\sigma_0^2 + \sigma_j^2)\) when the jamming signal is present.

This weighting by the inverse of the variance was first evaluated in [4, 5] for non-coherent frequency hopped signals with soft decision convolutional coding. The previous analysis in this section shows that weighting each bit by the inverse of the variance is the optimal maximum-likelihood weighting for a coherent RAKE receiver with convolutional coding and soft decision decoding.

II. EFFECT OF PHASE NOISE ON TAP WEIGHT ESTIMATION

A diagram of tap weight estimator for a BPSK RAKE receiver is shown in Figure 2.

![Figure 2: Tap Weight Estimator for a BPSK RAKE Receiver With Phase Noise.](image)

The received signal \(r(t)\) is assumed to be comprised of a carrier amplitude \(\sqrt{2}A_c\), a Ricean distributed random variable \(\alpha\), a chipping sequence \(c(t)\), and a data
component $d(t)$ which equals $s_i$ when a binary “1” is transmitted and $s_0$ when a binary “0” is sent. The Ricean fading is assumed to vary slowly compared to the bit period, so it will be treated as a constant for the bit duration. For a signal transmitted over a frequency-selective, multipath channel, the received signal can be written as

$$r(t) = \sum_{i=1}^{N} \sqrt{2} A_i \alpha_i c(t - i T_d) d(t) \cos(\omega t)$$  \hspace{1cm} (7)

where each of the $I$ components is a flat fading signal and could have a different fading coefficient $\alpha_i$. The chipping sequence $c(t)$ is assumed to have a chip rate matched to the RAKE finger spacing $T_d$, and it is assumed that

$$\frac{1}{N} \sum_{k=0}^{N-1} c(t - m T_d) \times c(t - k T) = \delta(m - k).$$

This is a good approximation for long chipping sequences. The tap weight estimator is also assumed to be out of phase with the received signal by a random variable $\phi$, which is assumed to be Tikonov distributed. In [6] it was shown that the Tikonov pdf

$$p(\phi) = \frac{\exp(\beta \cos(\phi))}{2 \pi I_0(\beta)}$$  \hspace{1cm} (8)

models the phase error of a first order phase lock loop with a signal and Gaussian noise at the input. In (8), $\beta$ is the loop signal-to-noise ratio and $I_0$ is the modified Bessel function of the first kind and order 0. The output of the first mixer can be written as

$$\sqrt{2} A_i \alpha_i d(t) \cos(\phi)(\cos(2 \omega t) + 1)$$  \hspace{1cm} (9)

This signal is then multiplied by the previous demodulated bit, which replaces $d(t)$ with $d^2(t)$. We will now assume that $s_i = 1$ and $s_0 = -1$ so that $d^2(t) = 1$. Since $\cos(2 \omega t)$ is a high frequency term it will be filtered out by the low pass filter, hence the tap weight estimate will be

$$\sqrt{2} A_i \alpha_i \cos(\phi)$$  \hspace{1cm} (10)

This shows that phase noise reduces the tap weight estimate by $\cos(\phi)$ since without phase noise the estimated tap weight would have been $\sqrt{2} A_i \alpha_i$.

III. Performance Analysis for the Maximum-Likelihood RAKE Receiver With Convolutional Coding

We will now derive the performance for a RAKE receiver with AWGN and pulse noise jamming. The RAKE receiver will be designed according to (6), which was shown to be the optimal maximum-likelihood receiver for signals received with a combination of AWGN and pulse noise jamming. A diagram of a RAKE receiver designed according to (6) is shown in Figure 3.

![Figure 3: Maximum-Likelihood RAKE Receiver for Signals With Pulse Noise Jamming and Gaussian Noise](image)

This RAKE receiver is similar to the one shown in [7] except that the fingers of the RAKE receiver are weighted by the inverse of their variance $1/\sigma_i^2$ which equals $1/\sigma^2_0$ when only AWGN is present and $1/(\sigma^2_0 + \sigma^2_j)$ when pulse noise jamming is present. The fingers are also multiplied by the complex conjugate of the tap weight estimates $c_i^*$, which can be written as $\sqrt{2} A_i \alpha_i^* \cos(\phi)$. Note that the reference signal is out of phase with the received signal by a random variable $\phi$, just like the tap weight estimator. We will assume that the data bits are equally probable and will assume that a sequence of binary “0’s” was sent. This way the probability of bit error will simply be the integral from 0 to $\infty$ of the probability density function (pdf) at the RAKE receiver. The received signal $r(t)$ will be assumed to be a Gaussian signal with each finger having a mean value $\sqrt{2} A_i \alpha_i$, which is a Ricean distributed random variable. Each finger will multiply the received signal by $\sqrt{2} A_i \alpha_i^* \cos^2(\phi)$. As shown in [8], when a Gaussian random variable is multiplied by $C$ the result is another Gaussian random variable with the mean multiplied by $C$ and the variance multiplied by $C^2$. Hence, the mean and variance of the $i^{th}$ finger just prior to the summer can be written as
A. Soft Decision Decoding

The bit error rate for a system with soft decision decoding is upper bounded by [7]

\[ P_b \leq \sum_{d=1}^{\infty} B_d P_d \]

(17)

where \( B_d \) is the total number of nonzero information bits in all weight \( d \) paths and \( P_d \) is the probability of selecting a code word that is a Hamming distance \( d \) away from the correct code word. The interleaver randomizes the jammed bits so \( P_d \) can be written [1]

\[ P_d = \sum_{i=0}^{d} \left( \begin{array}{c} d \\ i \end{array} \right) r^i (1-r)^{d-i} P_d(i) \]

(18)

where \( r \) is the duty cycle of the pulse noise jammer, and \( P_d(i) \) is the probability of selecting a code word a Hamming distance \( d \) from the correct code word given that \( i \) of the \( d \) receptions were jammed. For a rate \( \frac{1}{2} \) constraint length 9 convolutional code the first three terms of the series are \( 33P_{12} + 281P_{14} + 2179P_{16} \). To calculate \( P_d(i) \) the Laplace transform

\[ \Phi_i(s | \phi) = \left( \prod_{i=1}^{d} \Phi_i(s | \phi, B_i) \right) \times \left( \prod_{i=1}^{d} \Phi_i(s | \phi, B_i) \right)^{d-d} \]

(19)

where

\[ B_1 = \frac{2E_b \cos^2(\phi) \left[ 1 - \frac{s}{2} \cos^2(\phi) \right]}{\sigma_0^2 + \sigma_j^2} \]

(20)

and

\[ B_2 = \frac{2E_b \cos^2(\phi) \left[ 1 - \frac{s}{2} \cos^2(\phi) \right]}{\sigma_0^2} \]

(21)

must be inverted and integrated from 0 to \( \infty \). This can be done using the Gauss-Chebyshev quadrature method from [3,10], which allows us to approximate the probability of bit error as

\[ P_b = \frac{1}{n} \sum_{k=1}^{n} \Re \left[ \Phi(s = c + j\mathbf{c} \tau_k) \right] \]

(22)

\[ + \tau_k \Im \left[ \Phi(s = c + j\mathbf{c} \tau_k) \right] + E_n \]

where

\[ \tau_k = \tan \left[ \left( 2k - 1 \right) \pi / 2n \right] \]

(23)

c is a constant and \( E_n \) is an error term which goes to zero as \( n \) gets large. Methods for choosing \( c \) are discussed in [10]. To converge to the low probability of bit error for this powerful convolutional code, values of \( n \) from 200-
800 were used. For systems with higher bit error rates, such as in [3] where coding was not considered, values of \( n \) from 50-100 were sufficient. Since (19) is dependent on the random phase offset \( \phi \), we numerically integrated over the product of (19) and the Tikonov pdf for every value of \( s \) required in (22). We used a three finger RAKE with, \( E_b/N_0 = 15dB \), assumed the pilot tone was received with 15 dB SNR, used a 10dB ratio of direct to diffuse power and calculated the performance of the maximum-likelihood RAKE receiver against a jamming signal with one percent duty cycle (\( r=0.01 \)), ten percent duty cycle (\( r=0.1 \)) and a jamming signal that was on all the time (\( r=1 \)). The results are shown in figure 4.

![Figure 4: Performance of Maximum-likelihood RAKE Receiver With Pulse Noise Jamming](image)

We found that the bound given in equation 8 did not converge well for \( E_b/N_0 \) below the cutoff rate for the convolutional code and at high values of \( E_b/N_0 \) the phase noise on the received pilot tone causes \( P_{12} = P_{14} = P_{16} \) and the bound diverges. However, several important conclusions may be drawn from figure 4. It is clear that the maximum-likelihood RAKE is not affected by pulse noise jamming. The worst case performance occurs when the jammer is on all the time. Phase noise on the received pilot tone causes an irreducible probability of bit error. Since soft decision decoding makes bit decisions on a sequence of soft decision receiver outputs, any phase noise on the received pilot tone causes a higher probability of bit error for the entire sequence. This means that jamming the pilot tone should be an effective method for attacking this communication system since it will effect every soft decision. In figure 5 we show the performance of the maximum-likelihood RAKE receiver with a pulse jammer that has a ten percent duty cycle and various amounts of phase noise on the received pilot tone. We assumed that the phase noise on the pilot tone is constant and show how increasing the phase noise dramatically increases the bit error rate.

![Figure 5: Effect of Increasing the Pilot Tone Phase Noise](image)

The bottom trace shows the performance when the pilot tone is received with a 15dB SNR, the middle on has 10dB SNR and the top trace has 5dB SNR. When the pilot tone is received with 15dB SNR, the effect of phase noise is negligible and the coded bit error rate is suitable for military communications. However, when the pilot tone has a 10dB SNR, the coded bit error rate is not acceptable for military communications. When the received pilot tone has 5dB SNR, the error rate is 50 percent. Clearly, jamming the pilot tone is an effective strategy.

### B. Hard Decision Decoding

If hard decision Viterbi detection is utilized, \( P_d \) in (18) is [5]

\[
P_d = \sum_{i=d+1}^{d} \left( \frac{d}{i} \right) \cdot p^i \cdot (1 - p)^{d-i}
\]

when \( d \) is odd, and

\[
P_d = \frac{1}{2} \cdot \left( \frac{d}{2} \right)^{d} \cdot p^\frac{d}{2} \cdot (1 - p)^\frac{d}{2} + \sum_{i=d+1}^{d} \left( \frac{d}{i} \right) \cdot p^i \cdot (1 - p)^{d-i}
\]

when \( d \) is even. In (24) and (25), the channel transition probability \( p \) is the inverse Laplace transform integrated from 0 to \( \infty \) of...
\[ \Phi_i(s | \phi) = r \left( \Phi_i(s | \phi, B = B_1) \right) + (1-r) \left( \Phi_i(s | \phi, B = B_2) \right) \]

where \( r \) is the duty cycle of the jammer, \( B_1 \) is defined in (20) and \( B_2 \) is defined in (21). In figure 6 we show the probability of bit error for a three finger RAKE with hard decision decoding when \( E_b/N_0 = 15dB \), the pilot tone is received with 15dB SNR, the ratio of direct to diffuse power is 10dB and the jamming signal has a one percent duty cycle (\( r=0.01 \)), ten percent duty cycle (\( r=0.1 \)) and a jamming signal that was on all the time (\( r=1 \)). From Figure 6 we see that hard decision decoding does not perform as well as the soft decision maximum-likelihood RAKE receiver at low \( E_b/N_0 \), however, at high values of \( E_b/N_0 \) the hard decision decoding performed better than soft decision decoding.

Soft decision decoding uses a sequence of soft decision outputs to make a bit decision, in which case phase noise causes an irreducible error floor for the sequence. Hard decision decoding, however, makes bit decisions on a bit by bit basis, hence, phase noise causes an error floor on the bit decision prior to decoding. As a result, hard decision decoding can perform below the soft decision irreducible error floor.

IV. Performance of Conventional RAKE Receiver With Convolutional Coding

A RAKE receiver optimized for AWGN is shown in [7] and is similar to figure 3 except that it does not scale the received signal by the inverse of the variance. In this case the mean and variance for the \( i^{th} \) finger just prior to the summer can be written as

\[ m_i = 2E_b \sigma_i^2 \cos^2 (\phi) \]
\[ \sigma_i^2 = 2E_b \sigma_i^2 \cos^2 (\phi) \]

where \( \sigma_i^2 = \sigma_i^2 + \sigma_j^2 \) for bits with only AWGN and \( \sigma_i^2 = \sigma_i^2 + \sigma_j^2 \) when the jamming signal is present. In (28) we see that the variance of the \( i^{th} \) finger increases when the received bit is jammed. Hence, a pulse noise jammer should have a devastating effect against this receiver. We will use (17) and (18) for our performance evaluation. To calculate \( P_b(i) \) for this case, the Laplace transform

\[ \Phi_i(s | \phi) = \left( \Phi_i(s | \phi, B = B_1) \right)^i \times \left( \Phi_i(s | \phi, B = B_4) \right)^{d-i} \]

where

\[ B_3 = 2E_b \cos^2 (\phi) \left( 1 - \frac{5}{2} (\sigma_0^2 + \sigma_j^2) \cos^2 (\phi) \right) \]

and

\[ B_4 = 2E_b \cos^2 (\phi) \left( 1 - \frac{5}{2} \sigma_0^2 \cos^2 (\phi) \right) \]

must be inverted and integrated from 0 to \( \infty \). Once again, we use a three finger RAKE with \( E_b/N_0 = 15dB \), assume the pilot tone is received with 15dB SNR, use a 10dB ratio of direct-to-diffuse power and calculated the performance of the conventional RAKE receiver against a jamming signal with one percent duty cycle (\( r=0.01 \)), ten percent duty cycle (\( r=0.1 \)) and a jamming signal that was on all the time (\( r=1 \)). The results are presented in figure 7.
noise jammer has a one percent duty cycle, the conventional RAKE receiver requires greater than 15dB Eb/Nj to achieve a bit error rate below $10^{-6}$. The maximum-likelihood RAKE receiver provides significantly better performance against pulse noise jamming.

For hard decision decoding the conventional RAKE receiver provides the same performance as the maximum-likelihood RAKE receiver. Since hard decision decoding makes bit decisions on a bit by bit basis, scaling the receiver output has no effect on the SNR. Soft decision decoding, however, uses a sequence of soft decision outputs to make a bit decision. If there is a jamming pulse in any of these soft decision receiver outputs, then the variance for the sequence of bits will be high and the conventional RAKE receiver will have a high bit error rate. The maximum-likelihood RAKE receiver minimizes the effect of a jamming pulse by scaling the soft decision receiver output by the inverse of the variance so that when a jamming pulse is present, the variance of the soft decision receiver outputs will not be affected so dramatically.

V. Conclusions

In this paper, the performance of a coherent RAKE receiver with pulse jamming, phase noise and frequency selective fading was examined. The maximum-likelihood receiver for a coherent system with both AWGN and pulse jamming was derived. We found that the maximum-likelihood RAKE receiver effectively mitigates the effects of pulse noise jamming when a constraint length 9 rate ½ soft decision convolutional coding is used. However, when the received pilot tone has 10dB SNR, the error rate was too high for a reliable communication system. When the pilot tone has 5dB SNR the soft decision error rate was 50 percent. Hard decision decoding does not perform as well as soft decision decoding when the pilot tone has 15dB SNR for low values of Eb/Nj, however, at high values of Eb/Nj it is able to perform below the irreducible error floor that phase noise impose on the soft decision decoder. The conventional RAKE receiver with soft decision decoding is very susceptible to pulse noise jamming. Decreasing the duty cycle of the pulse jammer significantly increases the bit error rate. The maximum-likelihood RAKE receiver limits the effect of pulse jamming by weighting each bit by the inverse of the variance. This requires the variance to be measured on a bit by bit basis, which greatly compiles the receiver. However, it also greatly improves the performance against pulse noise jamming.

REFERENCES