PERFORMANCE SIMULATION OF A TRANSFORM DOMAIN COMMUNICATION SYSTEM FOR MULTIPLE ACCESS APPLICATIONS

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Abstract — A previously proposed interference-avoiding Transform Domain Communication System (TDCS) is shown capable of operating successfully in a Multiple Access Environment (MAE)[1]. The TDCS uses phase coding (mapping) generated from a Linear Feedback Shift Register (LFSR) configured to output a maximal-length binary pseudorandom sequence (m-sequence). Quasi-orthogonal Basis Functions (BFs) are used in a Code Division Multiple Access (CDMA) scheme to provide private communication channels to independent user pairs in the MAE. An existing single channel TDCS model is augmented to simulate MAE interference effects on bit error performance (PB). The proposed TDCS system is simulated using MATLAB® for system capacities up to eight channels and Eb/N0 values ranging from 0 to 9 dB. Simulated MAE TDCS bit error performance closely approximates estimated results; the error for eight channels has a mean value less than $1.7 \times 10^{-3}$ and a standard deviation less than $1.3 \times 10^{-5}$. The analysis of acquisition-related performance metrics and bit error performance through computer simulation provides a good measure of TDCS operational capabilities in a MAE.

I. INTRODUCTION

Modern communication networks enable simultaneous communication between multiple transmitter-receiver (user) pairs over private channels — “private” implies individual pairs within the network induce minimal, or manageable, interference into other user channels. Such Multiple Access (MA) schemes better facilitate information interchange and often use available resources more efficiently while sharing the Communication Resource (CR) — CR refers to the “medium” through which user pairs communicate, e.g., coaxial cable, optical fiber, free-space, etc.

CDMA schemes generally allow multiple user pairs to simultaneously use part or all of the CR. CDMA may be implemented using traditional Spread-Spectrum (SS) techniques such as Frequency Hopping (FH) or Direct Sequence (DS) modulation. In these cases, privacy is ensured by exploiting orthogonality properties of either the hopping-codes (FH) or the spreading codes (DS), i.e., the spectral and/or temporal cross-correlation of two distinct waveforms within the MAE approaches zero [2]. Although each SS technique has unique implementation issues, i.e., complexity, synchronization, etc., they both provide a certain degree of anti-jam capability.

This paper presents research results aimed at characterizing TDCS performance in a MAE. As supported by analysis, modeling and simulation, the MA TDCS implementation provides comparable performance with other MA schemes. A brief introduction of TDCS techniques is provided in the background section followed by the analytic development, modeling, and simulation approach. Optimal LFSR phase mapping characterization and bit error performance results are then presented in support of declaring the TDCS a viable alternative for MA applications.

II. BACKGROUND

Transform domain filtering involves operating on a transformed version of the received signal rather than the signal itself. One common transform domain filtering technique simply notches out (removes) signal frequency components containing interference [3]. This process effectively combats narrowband interference and multiple tone jamming while causing minimal communication performance degradation. Such filtering also removes desired signal components at excised frequency locations, effectively decreasing the Signal-to-Noise Ratio (SNR) as compared to the case where no jamming exists while increasing the probability of bit error. If a transmitter-receiver system avoids using jammed frequencies altogether, i.e., the modulated information-bearing signal has no spectral components at jammed frequency locations, the SNR is unaffected by transform domain filtering. This interference-avoiding concept serves as the basis for recent TDCS design and simulation research using a single transmitter-receiver pair and assuming both users observe identical spectra, thus avoiding the same frequencies [4].

Fig. 1 represents a TDCS transmitter which samples the electromagnetic environment over the system’s operating bandwidth ($\mathcal{W}$). Once sampled, the spectral content is
estimated using any available technique, e.g., periodogram, autoregressive linear predictive filtering, etc. A thresholding process is applied to the spectral estimate, creating a magnitude vector containing zeros and ones; spectral regions exceeding the threshold are assigned a value of zero (0) and other regions a value of unity (1). Effectively, this process yields a vector representing an ideal rectangular spectrum which has been selectively “notched,” as denoted by $A'(a)$ in Fig. 1.

Next, a multi-valued complex pseudorandom (PR) phase vector is generated, identical in length to $A'(a)$ and containing components of the form $e^{j\phi}$, and multiplied element-by-element with $A'(a)$ in a process called phase coding, producing spectral vector, $B(a)$, with known amplitude and PR phase characteristics. The spectral vector is subsequently amplitude-scaled by constant $C$ to ensure communication symbols have the desired signal energy. The resultant spectral vector, $B(a)$, is inverse Fourier transformed to produce a time-domain Basis Function (BF), $b(t)$, which is subsequently stored and modulated in accordance with data, $d(t)$. For TDCS implementation, Cyclic Shift Keying (CSK) modulation has been shown to provide extremely low cross-correlation, even for relatively large spectral notches using up to 16 communication symbols (M-ary signaling) [4].

Prior to signal synchronization, the TDCS receiver performs continuous correlation of the received signal with $M$ possible communication symbols derived from a locally generated BF reference. Receiver detection circuitry determines when one of $M$ correlator outputs exceeds a preset synchronization threshold, at which time signal “presence” is declared and demodulation begins. To minimize false synchronization, i.e., a threshold crossing resulting from noise and/or other interference, it is desirable to select TDCS operating parameters such that the likelihood of false synchronization is minimized while the likelihood of correct synchronization is maximized. Here it is assumed the transmit-receive pair observe identical spectra, thus-notch out identical spectral components during BF generation. The correlator outputs (test statistics) are then compared and a symbol estimate produced based on relative weighting.

III. ANALYTIC DEVELOPMENT, MODELING, AND SIMULATION

Binary PR sequences have been studied at length for their applicability to SS and MA communication systems. Simple inexpensive LFSR configurations may be used to produce maximal-length PR sequences ($m$-sequences), such as commonly used for generating DS spreading codes and FH hopping sequences. Privacy in a CDMA environment may be achieved by assigning individual user pairs a unique signal which is quasi-orthogonal (not strictly orthogonal) to other user pairs in the system. A TDCS achieves desired orthogonality characteristics by generating a PR phase vector via a LFSR phase mapping process.

A LFSR may be configured to output several different binary maximal length sequences ($m$-sequences), depending on the feedback tap configuration and the number of stages $n$. The period (length) of an $m$-sequence is $2^n - 1$ [2]. As shown in Fig. 2, the phase mapper receives $r$ LFSR stage outputs which may include any combination of the $n$ LFSR stages, but for this research were selected to be contiguous.

A “snapshot” of the LFSR contents is mapped to one of $2^r$ possible complex numbers as represented by points in the complex plane. To simplify analysis, without loss of generality, these points were evenly spaced around the unit circle. The LFSR contents are shifted $s$ stages and another “snapshot” is mapped to the next phase value. The process repeats until all $N$ elements of the complex phase vector are determined – optimal selection of $s$ is provided later.

The final parameter to be specified for the mapping process is the number of shifts, $s$, between phase mappings. Noting that $\gcd(a, m) = 1$ implies that $a$ and $m$ are relatively prime by definition [5], selecting a value $s$ that is relatively prime to the $m$-sequence period, as shown in (1) results in all possible $2^r$ phase values being produced over one full period of the $m$-sequence.

$$\gcd(s, 2^r - 1) = 1$$

This is a consequence of $m$-sequence randomness properties caused by all possible register states, except the all-zeros state, being produced in a single $m$-sequence period [2].

![Fig. 2. TDCS Phase Mapping Process.](image)
Several modulation methods may be used with the TDCS implementation, including antipodal signaling and a nearly orthogonal, M-ary CSK technique [4]. Binary CSK (BCSK) was used in this research where the first symbol, \( s_1(t) \), is the BF and the second symbol, \( s_2(t) \), is generated by executing a circular shift of the BF over exactly one-half the symbol period \( T \). The notation \( x((t - T/N))_T \) is introduced to represent a circular shift of \( x(t) \) by one-Nth of its symbol period – the notation used in (2) to describe BCSK.

\[
s_1(t) = \text{Basis Function (BF)}
\]

\[
s_2(t) = s_1\left(\left\lfloor \frac{t - T}{T} \right\rfloor \right)_T
\]  (2)

Previous research indicates that, under proper parametric conditions, CSK may be regarded as a form of orthogonal signaling [4]. Theoretical symbol error performance for coherently detected M-ary orthogonal signaling is well-established as given by (3) with (4) representing the binary \((M = 2)\) case [2].

\[
P_E(M) \leq (M - 1) Q \left( \frac{E_b}{N_0} \right)
\]  (3)

\[
P_B = Q \left( \frac{E_b}{N_0 + N_I} \right)
\]  (4)

For this research, it is postulated that the cross-correlation mean squared magnitude value gives an accurate estimate of bit error performance degradation resulting from multiple signals in the MAE. The theoretical bit error performance of (4) can be modified to produce (5) which includes an interference term, \( N_I \), the cross-interference due to other users in the MAE [6].

\[
P_B = Q \left( \frac{E_b}{N_0 + N_I} \right)
\]  (5)

\[
N_I = \sum_{j=1}^{c_h} N_{IJ}
\]

\[
N_{IJ} = \frac{E_b}{R_{IJ}} = \frac{1}{N}
\]  (6)

The \( N_I \) term in (5) is the total cross-interference term which may be rewritten as shown in (6) where the cross-correlation \( R_{IJ} \) is defined in (8) and \( c_h \) represents the number of channels in the MAE. The Bit Error Performance Simulations (BEPS) are conducted under the assumption that received power is identical in all channels, i.e., identical channel \( E_b/N_0 \) ratios. This allows for simple addition of all cross-interference terms without accounting for relative weightings due to systemic and environmental factors (aperture patterns, multipath, etc.).

The complex phase vector length, \( N \), is chosen to be one greater than the \( m \)-sequence period as shown in (7).

\[
N = (2^n - 1) + 1 = 2^n
\]  (7)

IV. RESULTS

In an attempt to “optimize” the phase mapping process, i.e., obtain a desired output phase distribution, the following example is offered. An eight stage \((n = 8)\) LFSR is configured to produce an \( m \)-sequence of period \( 2^8 - 1 = 255 \). Three contiguous LFSR register taps \((r = 3)\) are used to produce a complex pseudorandom phase vector of length \( N = 2^8 = 256 \) by the phase mapping process described previously. The feedback tap connections on the LFSR itself are determined by an eighth-order primitive polynomial. The LFSR contents are shifted one stage at a time between successive phase mappings \((s = 1)\). The resulting phase distribution, i.e., the distribution of the phase mapper output, is very nearly uniform given the randomness properties of \( m \)-sequences. Phase distribution histograms for eight different \( 8^\text{th} \)-order \( m \)-sequences are shown in Fig. 3.

Two factors prevent the distributions in Fig. 3 from being perfectly uniform. First, the LFSR never reaches the all-zeros state. Second, because the phase vector is one element longer than the \( m \)-sequence period, two elements of the complex phase vector are always identical.

The phase distribution for all eight \( m \)-sequences changes drastically for \( s = 3 \) as shown in Fig. 4 where the number of occurrences ranges from 12 to 54, a dramatic change from the nearly constant value of 32 seen in Fig. 3. Similar results were obtained for all combinations of the parameters listed in Table 1. For each trial, the value of \( N \) was selected to satisfy (7) and the initial fill of the LFSR was an impulse fill (zeros plus a single one).
These results were used to establish the relationship given by (1) – for \( s \) relatively prime to the \( m \)-sequence period, all possible \( 2^s \) phase values are produced over one period of the \( m \)-sequence.

Using this as a starting point, the maximum periodic cross-correlation values for a variety of parameter ranges were plotted – the results presented in Fig. 5 are “typical” for all cases considered. Fig. 5 represents maximum cross-correlation results for all possible pairs of \( 8^\text{th} \)-order \( (n = 8) \) \( m \)-sequences for \( s = 1 \). The BF lengths \( N \) considered here included all powers of two over the range \([2^5, 2^9]\) and the number of phase mapper taps \( r \) was restricted to integer values in the range \([1, 6]\).

The clear trend in Fig. 5 is toward lower cross-correlation values as \( N \) increases. There are several conclusions that can be drawn from Fig. 5. First, using longer phase vectors, i.e., larger length BFs, decreases the likelihood that false synchronization will occur since cross-correlation values decrease as \( N \) increases. Second, there appears to be minimal advantage, in terms of acquisition performance, in choosing a Larger \( r \) value, especially when \( N \) is greater than six – the response is relatively flat for a given \( N \) value. Finally, the maximum cross-correlation value appears to reach a minimum (approximately 0.2) for \( N \geq 8 \). Increasing \( N \) beyond this point simply adds complexity while providing minimal improvement in performance, i.e., reduction in maximum cross-correlation values.

To aid in predicting TDCS bit error performance in a MAE, maximum cross-correlation data was generated over a wide range of system parameters, given in Table 2. After thoroughly analyzing computed data, it was determined that the mean squared magnitude cross-correlation value for any two BFs of the same order is only a function of \( N \). Specifically, the mean squared magnitude cross-correlation value for various combinations of parameters is as given by (8). Again, this result was only confirmed for a TDCS using the previously described \( m \)-sequence phase mapping process to generate the BFs.

\[
\hat{R}_{ij} = \frac{1}{N} \cdot \frac{1}{\sqrt{N_D + \frac{ch-1}{N}}} \cdot \left( \frac{1}{E_b} \right)
\]

This result, as appearing in (6) and substituted into (5), is used to obtain the Estimated Bit Error Probability given by (9), a function of the total number of MAE channels, \( ch \), and BF length, \( N \).

A comparison was made between this estimated value and simulated results and for three different TDCS scenarios and an eight channel MAE. The scenarios included values of \( E_b/N_0 = 1 \) to 9 dB, \( r = 3 \), \( s = 1 \), and BF lengths of \( N = 256, 512, \) and 1024. A direct comparison of results shown in Fig. 6 reveals close agreement between the two – the mean absolute difference is no greater than 1.7 x 10^{-3} with a maximum standard deviation of 1.3 x 10^{-3}.
V. CONCLUSIONS

This research expands previous knowledge of interference-avoiding TDCSs by exploiting TDCS techniques to allow multiple user pairs to successfully operate on private channels within a MAE. Parameters of interest include: 1) the number of LFSR shifts between successive phase mappings (s), 2) the number of LFSR taps input to the phase mapper (r), 3) the Basis Function (BF) length (N), and 4) the number LFSR stages (n). The communication channel is assumed to be an AWGN channel with no multipath interference. The time domain BFs, used for distinguishing users via a CDMA technique, were generated using a phase mapping process and LFSR configured to output an m-sequence. The BFs were modulated using CSK, a form of orthogonal signaling.

Cross-correlation statistics on a multitude of possible BF implementations were analyzed to characterize the predicted performance of a TDCS in a MAE. The maximum cross-correlation values between the locally generated BF reference and other BFs used in the MAE were examined and particular parametric values suggested which guarantee good acquisition performance.

Extensive computer calculations show that the maximum cross-correlation value between BF pairs decreases as BF length increases, thus longer BFs are encouraged for use in a TDCS to optimize acquisition performance. Likewise, the maximum cross-correlation value decreases as the parameter s increases for "small" values of N (N = 32). For larger values of N (N = 256 or 512), changing s generally has no effect on the maximum cross-correlation value. The mean-squared magnitude of the cross-correlation between the local BF reference and other BFs in the MAE was examined and a bit error performance estimate derived as a function of the number of channels in the MAE, as well as the BF length.

The proposed TDCS multiple access system was simulated using MATLAB® for an assortment of system capacities over a wide range of $E_b/N_0$ values. The simulation results were compared to derived MAE estimates, and it was found that simulated bit error performance results compare favorably with theoretical performance estimates (within a mean value of $1.7 \times 10^{-3}$).

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