ABSTRACT
The performance of a Direct Sequence Differential Phase Shift Keying (DS-DPSK) spread spectrum system employing time diversity and self-normalization over a Rician frequency nonselective, slowly fading channel in the presence of pulsed noise interference and additive white Gaussian noise (AWGN) is considered. An interference model which considers the possibility that either both, one or none of the information bits experience jamming is employed. Worst case performance under severe and moderate fading conditions are analyzed. Results indicate that the self-normalized receiver is effective in mitigating the effects of pulse noise jamming for all fading conditions considered.

INTRODUCTION
We consider the performance of a Direct Sequence Differential Phase Shift Keying (DS-DPSK) spread spectrum system with self-normalization over a Rician frequency nonselective, slowly fading channel in the presence of pulsed noise interference and additive white Gaussian noise (AWGN). The system employs L-fold time diversity with i of L channels, i = 1, 2, ..., L experiencing interference at any given point in time. It is assumed that each of the diversity receptions are received in an independent fashion. The following analysis assumes that the received signal is perfectly “despread.” That is we assume bit and code synchronization between the transmitter and receiver. The general form of a DPSK signal may be written as

\[ s(t) = \sqrt{\frac{2}{\pi}} A \left[ c_0 p_T(t) + c_1 p_T(t - T) \right] \cos \omega_c t \]  

for 0 ≤ t ≤ 2T, with c_0c_1 = 1 representing bit 0 and c_0c_1 = -1 representing bit 1. The function p_T(t) represents a rectangular pulse of unit amplitude on the bit interval 0 ≤ t ≤ T . The carrier frequency is denoted by \( \omega_c \). We also note that the signals representing bit 0 and bit 1 are orthogonal to each other over the 2-bit interval, 2T. The optimum receiver for noncoherent detection of orthogonal signals in AWGN according to the Bayes criterion is an envelope or square-law detector [1]. We consider the outputs of a square-law detector with the output \( V_1 \) as the bit 0 detector and \( V_2 \) as the bit 1 detector. Assuming a bit 0 was sent, the random variable \( V_1 \) has a noncentral Chi-squared distribution with 2 degrees of freedom and the random variable \( V_2 \) has a central Chi-squared distribution with 2 degrees of freedom. The probability density function for \( V_1 \) may be expressed in the following form

\[ f_{V_1}(v_1|0) = \frac{1}{2(4\sigma_a^2 + \sigma_n^2)} \times \exp \left( -\frac{1}{2(4\sigma_a^2 + \sigma_n^2)}(v_1 + 4\alpha^2) \right) \times I_0 \left( \frac{2\alpha \sqrt{v_1}}{(4\sigma_a^2 + \sigma_n^2)} \right) u(v_1) \]  

where the average signal energy over a 1-bit interval, denoted by \( E_b \), is \( \alpha^2 + 2\sigma_a^2 \) [1], \( \sigma_n^2 \) is the noise power due to AWGN, \( I_0(x) \) is the modified Bessel function of the first kind of zero order and \( u(x) \) is the unit step function. \( \alpha^2 \) represents the direct path signal energy while \( 2\sigma_a^2 \) represents the diffuse signal energy. The probability density function for \( V_2 \) is

\[ f_{V_2}(v_2) = \frac{1}{2(4\sigma_a^2 + \sigma_n^2)} \times \frac{1}{2\pi} \]  

for 0 ≤ v_2 ≤ \( 2\pi \), with \( E_b = \alpha^2 + 2\sigma_a^2 \) [1].
The random variables $V_1$ and $V_2$ serve as inputs to the self-normalized receiver. Before describing the self-normalized receiver, we first describe the pulse noise interference model in the next section.

**PULSE NOISE INTERFERENCE MODEL**

We consider an interferer who jams a fraction $p$ of the information bits ($0 \leq p \leq 1$). The fraction of bits not jammed is equal to $(1-p)$. Pulse noise jamming is explicitly defined for the case of $p < 1$. We model the interfering signal as white Gaussian noise whose PSD is $N_1/2p$ when the jammer is on and 0 when the jammer is off. The total average PSD is then equal to $N_1/2$. We allow for the possibility that either 2 consecutive bits, 1 bit or no bits of the DPSK signal may be jammed and define the following event space for these three cases:

- **$I_1$** Event that either the first bit contains interference and the second bit does not, or that the second bit contains interference and the first bit does not ($\{I, NI\} \cup \{NI, I\}$).

- **$I_2$** Event that the first bit and the second bit both contain interference ($\{I, I\}$).

- **$I_3$** Event that neither the first bit nor the second bit contain interference ($\{NI, NI\}$).

We define the probabilities of the three events as $Pr(I_1) = p_1$, $Pr(I_2) = p_2$ and $Pr(I_3) = (1-p_1-p_2)$ where $0 \leq p_1, p_2 \leq 1$. It is assumed that the jammer noise component is present equally in branches $V_1$ and $V_2$. The jammer noise variances at the receiver branches just prior to the squaring operation for the three cases are $\sigma_{I_1}^2 = N_1/2p$, $\sigma_{I_2}^2 = N_1/p$, and $\sigma_{I_3}^2 = 0$. The total conditional noise variance is defined as $\sigma_j^2 = \sigma_n^2 + \sigma_{I_j}^2$ for $j = 1, 2, 3$. The total conditional variance for branch 1 of our receiver is $\sigma_{I_1}^2 = (4\sigma_n^2 + \sigma_j^2)$ and for branch 2, $\sigma_{I_2}^2 = \sigma_j^2$ for $j = 1, 2, 3$. The new density function for $V_1$ conditioned on the three jammer events, $f_{V_1}(v_1|0, I_j)$ may be expressed from equation 2 by replacing $(4\sigma_n^2 + \sigma_j^2)$ with $\sigma_{1_j}^2$. The new density function for $V_2$ conditioned on the three jammer events, $f_{V_2}(v_2|0, I_j)$ may be expressed from equation 3 by replacing $\sigma_n^2$ with $\sigma_{2_j}^2$.

**SELF-NORMALIZED RECEIVER**

It has been shown that the use of the self-normalized receiver shown in Figure 1 can improve the worst case performances of a frequency-hopped BFSK signal under partial band jamming interference over a Rician channel [2]. In an analogous way, we seek to improve the performance of a DPSK system employing time diversity under pulse noise interference over a Rician channel. For the L-fold diversity receiver shown, a constant bit rate system is assumed. The inputs to the self-normalizing receiver for the $k^{th}$ diversity reception are $V_{1k}$ and $V_{2k}$. The density functions for $V_{1k}$ and $V_{2k}$ are the same as derived for $V_1$ and $V_2$ before, with the exception that $E_b$ is now replaced with $E_b/L$. The random variables $Z_{1k}$ and $Z_{2k}$ are defined as

$$Z_{1k} = \frac{V_{1k}}{V_{1k} + V_{2k}}$$

$$Z_{2k} = \frac{V_{2k}}{V_{1k} + V_{2k}}$$

and

$$Z_{1k} + Z_{2k} = 1$$

Summing both sides of equation 6 over $L$ and rearranging terms yields

$$Z_2 = L - Z_1$$

From this we can see that each diversity reception has a weight of 1 out of $L$ in the decision process. This prevents any particular diversity reception from dominating the final decision process. The probability of bit error as a function of the diversity $L$ is

$$P_e(L, i_1, i_2) = P_r(Z_1 < Z_2|0, I_j, L, i_1, i_2)$$

Substituting equation 7 into equation 8 yields
The density function for $Z_{1k}$ is given by

$$f_{Z_{1k}}(z_{1k}|0, I_j) = \frac{((\gamma + 1)\Gamma)^3}{[(\gamma + 1)\Gamma + 2(1 - z_{1k})]^3} \times \frac{2((\gamma + 1)\Gamma)^2(i + (1 - z_{1k}))}{[(\gamma + 1)\Gamma + 2(1 - z_{1k})]^3} \times \frac{4(\gamma + 1)\Gamma(1 - z_{1k}) + [2i\gamma z_{1k}((\gamma + 1)\Gamma)^2]}{[\gamma + 1)\Gamma + 2(1 - z_{1k})]^3} \times \exp\left(-\frac{2\gamma(1 - z_{1k})}{(\gamma + 1)\Gamma + 2(1 - z_{1k})}\right)u(z_{1k})$$  \hspace{1cm} (10)

where $\gamma = \frac{\sigma^2}{2\sigma^2}$ is the ratio of direct signal power to diffuse signal power and $\Gamma = \left(\frac{E_b}{N_0}\right)^{-1} + \left(\frac{E_0}{\sigma_i^2}\right)^{-1} \cdot \left(\frac{E_b}{N_0}\right)$ is the average bit energy to thermal noise density ratio and $\left(\frac{E_b}{\sigma_i^2}\right)$ is the average bit energy to jammer noise power ratio.

for case $I_j, j = 1, 2, 3$. The conditional density for $Z_1$ may be expressed in the Laplace domain as

$$\mathcal{L}^{-1}\left(\left[\mathcal{L}(f_{Z_{1k}}(z_{1k}|0, I_j))\right]^{i_1}\right) \times \left[\mathcal{L}(f_{Z_{1k}}(z_{1k}|0, I_2))\right]^{i_2} \times \left[\mathcal{L}(f_{Z_{1k}}(z_{1k}|0, I_3))\right]^{(L-i_1-i_2)}$$  \hspace{1cm} (11)

where $\mathcal{L}$ and $\mathcal{L}^{-1}$ denote the forward and inverse Laplace transforms. Since we model each of the $L$ diversity receptions as independent events, the probability distribution for $(L, i_1, i_2)$ is derived from a multinomial distribution and is given as

$$\Pr(L, i_1, i_2) = \frac{L!}{i_1!i_2!(L-i_1-i_2)!} \times \rho_1^{i_1}\rho_2^{i_2}(1-\rho_1-\rho_2)^{(L-i_1-i_2)}$$  \hspace{1cm} (12)

We may obtain the density for $Z_1$ conditioned on bit 0 being sent by averaging the conditional density for $Z_1$ in
equation 11 over the probability distribution in equation 12. The expression is given as

\[ f_{Z_1}(z_1|0) = \sum_{i_2 = 0}^{L-1} \sum_{i_1 = 0}^{L-1} f_{Z_i}(z_1|0, i_1, i_2) \times Pr(L, i_1, i_2) \]

(14)

Finally the expression for probability of bit error follows from equation 9 and is

\[ P_b = \int_0^{L/2} f_{Z_1}(z_1|0)dz_1 \]

(15)

This last expression must be evaluated numerically.

**NUMERICAL RESULTS**

In the following analysis, we will be interested in determining receiver worst case performance. Worst case performance represents a composite performance by obtaining the value of the jamming fraction, \( \rho \) that produced the highest probability of bit error as a function of the bit energy to interference noise density ratio, \( E_b/N_1 \).

Worst case performance was determined by numerical search since no analytical solution could be produced. Performance curves of the self-normalized receiver for diversity orders \( L = 1 \) (no diversity) and \( L = 4 \) and jammer fractions \( \rho = 0.01, 0.1, 0.25, 1 \), worst case with \( E_b/N_0 = 15 \) dB and \( \gamma = 0 \) (Rayleigh fading) are shown in Figures 2 and 3 respectively. It is seen that the self-normalized receiver has completely negated the effects of pulse noise jamming since the worst case performance curve coincides with the continuous jamming curve (\( \rho = 1 \)).

Another indicator of the effectiveness of the self-normalized receiver is that for increasing diversity order, the optimum value of \( \rho \) increases forcing the interferer to a more continuous jamming strategy. This result is seen in Figure 4 where the optimum value of \( \rho \) is plotted as a function of the diversity order with \( E_b/N_0 = 15 \) dB, \( \gamma = 5 \) and \( E_b/N_1 \) as a parameter. We also observe that the optimum value of \( \rho \) decreases as \( E_b/N_1 \) increases.

Figure 2: Performance of self-normalized receiver for pulse jamming fractions \( \rho = 1, 0.25, 0.1, 0.01 \) and worst case for diversity order \( L = 1, E_b/N_0 = 15 \) dB and \( \gamma = 0 \).

Figure 3: Performance of self-normalized receiver for pulse jamming fractions \( \rho = 1, 0.25, 0.1, 0.01 \) and worst case for diversity order \( L = 4, E_b/N_0 = 15 \) dB and \( \gamma = 0 \).

Figure 5 shows the optimum value of \( \rho \) as a function of \( \gamma \) with diversity order as a parameter and with \( E_b/N_0 = 15 \) dB and \( E_b/N_1 = 20 \) dB. We observe the overall trend that the optimum value of \( \rho \) decreases for increasing \( \gamma \). From these plots, we can see that there appears to be a lower limit as to the effectiveness of pulse noise jamming. For example, for a diversity order of \( L = 4 \), pulse noise jamming is no longer effective below \( \gamma = 3 \). For \( \gamma \) greater than 3, higher order diversities would be required to render the pulse noise jammer ineffective.
was determined that the self-normalizing receiver was effective in mitigating the effects of pulse noise jamming for all fading conditions considered. It was noticed that higher diversity orders than those considered would be required to render the pulse noise jammer ineffective for less severe fading conditions. This was observed for values of $\gamma$ between 0 and 10.

REFERENCES


CONCLUSIONS

We have considered the performance of a Direct Sequence Differential Phase Shift Keying (DS-DPSK) spread spectrum system over a Rician frequency nonselective, slowly fading channel in the presence of pulsed noise interference and additive white Gaussian noise (AWGN). The receiver employs L-fold time diversity with self-normalization and soft decision equal gain combining. The performance of the self-normalizing receiver under conditions of severe and moderate fading was analyzed. It