ABSTRACT
In line-of-sight (LOS) links which use space or frequency diversity to combat multipath fading, the fading in each diversity branch is uncorrelated provided the antenna spacing or frequency separation is sufficiently large. Therefore, the availability or outage probability of space and frequency diversity systems is closely described by independent (uncorrelated) Rayleigh fading statistics. In the case of angle diversity, uncorrelated Rayleigh fading does not describe the performance of the angle diversity system. Instead, the fading on the two branches is positively correlated when the fades are shallow and negatively correlated when the fades are deep; i.e., if a deep fade occurs in one branch there is a high probability that the signal in the other diversity branch will not fade. In this paper we discuss briefly the theory of angle diversity and present the results of a short experimental program to compare the performance of angle diversity and space diversity on a fading LOS microwave link.

INTRODUCTION
Two techniques commonly used to combat multipath fading on LOS links are space and/or frequency diversity. Frequency diversity consists of transmitting the same information at two or more frequencies. The probability that the signal will fade simultaneously at all frequencies is smaller than the probability of a fade at a single frequency. Space diversity uses two or more receiving antennas to achieve the same result. We have proposed investigating the viability of an angle diversity technique for LOS links which employs only one receiving antenna with a multi-beam feed to produce two or more received signals with different fading statistics. One advantage of using angle diversity on LOS links over frequency or space diversity is bandwidth savings or tower height, respectively. Frequency diversity requires twice the bandwidth of angle diversity systems and antenna towers must be about 30 to 40 feet higher for dual vertical space diversity in order to provide the same earth curvature clearance for the lower space diversity path as the single angle diversity path. Angle diversity can also be used to augment existing space or frequency diversity systems to provide additional fade margins on long and troublesome links.

In this paper we discuss the theory of angle diversity and present the results of a short experimental program which compares the performance of angle diversity and space diversity under flat-fading conditions.

THEORY
Consider a receiving antenna with a vertical plane monopulse feed so that the two angle diversity beams are the sum and difference beams of the monopulse antenna as shown in fig. 1. When multipath propagation occurs on LOS paths the received signal consists of three or more rays, one of which can be identified with the LOS ray which would be received under normal (non-fading) conditions and two or more of which are rays which are totally reflected by a ducting layer (ref. 1). Ducting layers occur whenever the air temperature increases with height instead of decreasing as is the case under normal conditions. If the terrain is smooth, as in over water paths, a surface reflected ray can also occur. For the purpose of illustration let us assume that only three rays occur and that each ray arrives at a different angle as shown in fig. 1. Let us also assume that the 3-dB beamwidth of the sum pattern of the monopulse antenna is 2°.

Let the arriving rays have amplitudes \(A_1\), \(A_2\), and \(A_3\) not including the receive antenna gain. Then \(G_s(\theta)\) is the voltage gain pattern of the sum beam and \(G_d(\theta)\) is the voltage gain pattern of the difference beam, then the amplitude of the \(i\th\) ray at the output of the sum port is \(G_s(q_i)A_i\), and at the output of the difference port it is \(G_d(q_i)A_i\) as illustrated in fig. 2a. From fig. 1, it can be seen that for angles of arrival within the 3-dB beamwidth of the sum pattern, \(-1° < \theta < 1°\), the difference beam pattern, \(G_d(\theta)\), has a steep slope while the sum pattern, \(G_s(\theta)\), is nearly constant.

Let \(\phi_1\), \(\phi_2\), and \(\phi_3\) be the phase shifts of the three rays due to propagation through the medium. A deep fade will occur at the output of the sum port if the amplitudes and phases of the three rays are such that.
\[
\sum_{i=1}^{3} A_i G(s_i) e^{j\phi_i} = 0
\]  

which is equivalent to saying that the phasors \( A_i G(s_i) \exp(j\phi_i) \) form a triangle as shown in fig. 2b. Even though the same rays are received in the difference port, a deep fade will not occur simultaneously at the output of the sum and difference ports unless the angles of arrival of the three rays are such that

\[
\frac{G(s_1)}{G(s_2)} = \frac{G(s_2)}{G(s_3)} = \frac{G(s_3)}{G(s_1)}.
\]

However, this is not possible in general because the sum and difference patterns have different slopes. Therefore, a deep fade is not likely to occur simultaneously in both the sum and difference ports as illustrated in fig. 2b.

In order for angle diversity to provide a diversity gain for medium fades, the spread in the angle of arrival of the rays must be comparable to the 3-dB beamwidth of the sum beam pattern. To understand this point consider the situation depicted in fig. 1 where the 3-dB points of the sum pattern are at \( \pm 35^\circ \) off-boresight and the multipath arrivals are at angles within \( \pm 0.35^\circ \) off-boresight. In this case, the strongest signal at the output of the difference port is about 12 dB below the average sum port output signal level. This implies that the sum port signal has to fade by more than 12 dB before the anti-correlated fading in the difference port can provide a diversity gain. On the other hand if the multipath rays arrived within \( \pm 0.8^\circ \) off-boresight then the signal in the sum port would only have to fade by 5 dB before the signal in the difference port could provide a diversity gain.

Under flat fading conditions, diversity is one of the most effective ways to reduce the fade margin requirements. Because of the anti-correlated nature of the fading, angle diversity can potentially reduce the fade margin required (relative to space or frequency diversity) by systems whose outage probability must be small. When the fading is frequency selective, equalization is needed in addition to angle diversity to combat the irreducible outage (i.e., bit error) probability due to intersymbol interference.

Since there is no concrete empirical data which indicates what typical spreads in multipath angle of arrival on LOS links are, other than those determined from theoretical models such as that due to Parl (ref. 1), we proposed an experiment to determine the effectiveness of angle diversity relative to space diversity as well as the antenna beamwidths needed for angle diversity to work. The experiment was limited to RSL measurements. A second experiment proposed and conducted by the Institute of Telecommunications Sciences consisted of bit error measurements using a switched diversity radio. The results of this second set of measurements are discussed in a companion paper (ref. 2).

**EQUIPMENT SET-UP**

The test link used for the experiment was a link between San Nicolas Island and Laguna Peak at Pt. Mugu, California. The path length is approximately 65 miles (104.6 km) and is almost all over water. The link is operated on a dual frequency diversity mode using vertical polarization to transmit approximately 45 MHz RF bandwidth (as defined by FCC Docket 19311) using 8-PSK modulation. The two frequencies are 7.17 GHz and 7.47 GHz. The experiment made use of the 7.47 GHz signal to determine the statistics of the received signal amplitude under multipath fading conditions.

The transmit site elevation at San Nicolas Island is 912 ft (278 m) above mean sea level and the transmitting antenna is a 10 ft parabolic reflector at a height of 70 ft (21.3 m) above the surface. The receive site elevation at Laguna Peak is 1400 ft (426.7 m) above mean sea level. Two 4 ft parabolic dish reflectors were used to receive the 7.47 GHz signal. The lower receive antenna was at a height approximately 38 ft (11.6 m) above the ground while the higher space diversity antenna was about 100 ft (18.9 m) above the ground. That is, the vertical spacing between the antennas was about 62 feet.

The experiment consisted of recording the instantaneous received signal level (RSL) measured on three calibrated receivers, two of which correspond to the two output ports of the angle diversity feed of the lower antenna and the third corresponding to the higher antenna (space diversity) output port. The three calibrated receivers and RSL detectors were provided by the Institute of Telecommunication Sciences (ITS) and had a dynamic range between -30 dBm and -100 dBm. The 3-dB bandwidth of the receivers was 12 MHz and their noise figure was about 4-dB which implies that the receiver noise level was about -100 dBm.

The outputs of the 3 RSL detectors were recorded in analog form on 1/2 inch wide magnetic tapes using a multichannel instrumentation recorder. Eight tapes of data were collected each containing roughly 8 hours worth of data.

The angle diversity system was implemented by using a 4 ft reflector with a vertically polarized single plane (elevation) monopulse feed especially designed for the intended angle diversity application. The two angle diversity output ports are the sum and difference ports of the monopulse feed. The elevation plane sum pattern and the difference pattern are shown in fig. 3.

**RSL DISTRIBUTIONS**

Only those periods during which the signal faded by 5 dB or more were used to determine the statistics of the angle and space diversity signals; that is roughly 72.5% of the data obtained. Fig. 4 shows a 14 minute segment during which the signal faded by about 30 dB in the sum channel (middle trace). This figure also shows the signals on the difference channel (upper trace) and the elevated antenna (lower trace) during the same interval. Intervals in which the signal did not fade by more than 5 dB were not included in the final statistics.

19.5.2
The RSL distribution of the sum (S₁), difference (D₁), elevated antenna (S₂), maximal ratio-combined angle diversity (S₁+D₁), and maximal ratio-combined space diversity (S₁+S₂) signals are shown in fig. 5. Only data from 8.5 minute periods where at least one 10 dB fade occurred in the sum channel were used to generate these distributions. The distributions are based on 3040 minutes (50.7 hours) of data collected on 10 different days during the months of November, December, January, February and March. Most of the deep fading data was collected on 3 days in the month of March however.

The RSL distributions of fig. 5 show that the median RSL in the sum port of the monopulse antenna was -57.4 dBm during periods in which the signal faded at least 10 dB below its average (mean level). The median RSL in the difference port of the monopulse antenna was 12 dB lower than in the sum port which indicates that the various multipath rays arrived in a narrow cone of angles much smaller than the 3-dB beamwidth (2.3°) of the receiving antenna. From the difference port pattern of fig. 3, we may actually deduce that the multipath arrived in a cone of rays of width not exceeding 0.6°. The median RSL in the elevated (spaced about 62 ft higher) antenna was 5 dB lower than the median of the sum port signal in the monopulse antenna. This was due to the additional cable loss.

MEASURED ANGLE AND SPACE DIVERSITY GAINS

The diversity gain depends on the availability or outage probability required during a period of multipath fading (worst hour availability requirement). Table 1 compares the diversity gain of the maximal-ratio-combined sum and difference ports (angle diversity) to the diversity gain of the maximal-ratio-combination of the sum port and the elevated antenna signals (space diversity). The measured spaced diversity gain at an outage probability of 10⁻⁴ is 8 dB smaller than that for an ideal dual diversity system with maximal-ratio-combining partly because of the 5-dB smaller power received on the elevated antenna. The 5 dB lower received power in the elevated antenna can only account for a loss in diversity gain of at most 2.5 dB (half of the dB difference in received power). The additional loss is presumed to be due to non-Rayleigh statistics caused by fluctuations in the mean signal level.

Table 1 shows that angle diversity does provide a diversity gain which increases more rapidly than that of space diversity as the outage probability decreases, although the angle diversity gain is smaller than the space diversity gain for the antenna size used. When the required worst hour outage probability is 10⁻⁴, the angle diversity gain is 7.4 dB smaller than the space diversity gain. As the outage probability decreases, the difference between the angle and space diversity gains decrease. At an outage probability of 10⁻⁵, the angle diversity gain is 5.8 dB smaller than the space diversity gain.

PROJECTED ANGLE DIVERSITY GAINS WITH LARGER ANTENNAS

The measurements made with 4 ft antennas at 7.4 GHz showed that the angle diversity gain is not as large as that of space diversity. The reason for this is that the spread in angle of arrival of the multipath rays was about one quarter of the beamwidth of the 4 ft antenna. If we were to increase the antenna size by a factor of 3 (thus decreasing the beamwidth by a factor of 3) and then the instantaneous RSL in the difference port of the monopulse antenna would increase by 9.5 dB. Similarly, the instantaneous RSL in the difference port would increase by 19 dB, that is by 9.5 dB relative to the sum port. Therefore, the angle diversity gain that could be achieved with a 12 ft antenna can be determined from the RSL data obtained with the 4 ft antenna by increasing the instantaneous RSL measured in the difference port of the 4 ft antenna by 9.5 dB, and combining it with the RSL measured at the output of the sum port to generate the RSL distribution of the maximal-ratio combined signal that would have been measured with a 12 ft antenna. This RSL distribution is shown in fig. 6 as a dashed line (curve labeled S₁+S₂) and the difference in RSL between this curve and the sum port RSL distribution (curve labeled S₄) at 19 dB, a given probability of occurrence (outage probability) is the projected angle diversity gain at 7.4 GHz for a 12 ft antenna. The assumptions made in arriving at the angle diversity gain of the 12 ft antennas are valid as long as the antenna beamwidth of the 12 ft antennas is greater than the spread in angle of arrival of the multipath rays (estimated to have been not more than 0.6°). The angle diversity gain for antenna sizes between 4 ft and 12 ft could be generated by increasing the measured RSL in the difference port by the ratio of the antenna gains. This is strictly a consequence of the fact that the "slope" of the voltage gain pattern of the difference beam is inversely proportional to the sum beam beamwidth.

Table 2 compares the angle diversity gains for a 12 ft antenna with those for a 4 ft antenna at 7.4 GHz at various worst-hour availabilities or outage probabilities. This table also gives the space diversity gain for comparison purposes. Note that the latter would be the same for both 4 ft and 12 ft antennas. The projected angle diversity gain of the 12 ft antenna is seen to be significantly greater than that of the 4 ft antenna and almost equal to the space diversity gain. If we were to extrapolate the RSL distributions of fig. 6 for smaller outage probabilities we would also get that the 12 ft antenna projected angle diversity gain would actually be greater than the space diversity gain for worst-hour outage probabilities smaller than 10⁻⁵.

SUMMARY AND CONCLUSIONS

We have presented the theory of angle diversity for LOS microwave links and preliminary results from a short experimental program to verify the theory under flat fading conditions. Based on the limited amount of data collected, we
have determined that antenna beamwidths of the order of 0.75° or less are required in order for angle diversity to provide the same or larger fade margins afforded by dual space or frequency diversity systems under flat fading conditions. More extensive measurements are needed, however, as the results presented here are based on 50 hours of fading data.

ACKNOWLEDGMENT

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REFERENCES


Fig. 4. Typical Received Signal Levels During Deep Fading Periods.

Fig. 5. RSL Distributions for Periods in Which the Sum Port Signal Faded by 10 dB or More.

Fig. 6. Measured (Solid Lines) RSL Distributions for 4' Monopulse Antenna and (Relative) Projected (Dashed Lines) RSL Distributions for 12' Monopulse Antenna.

Table 1. Measured Diversity Gain with 4' Antenna

<table>
<thead>
<tr>
<th>WORST HOUR AVAILABILITY (%)</th>
<th>WORST HOUR OUTAGE PROBABILITY</th>
<th>DIVERSITY GAIN (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>1x10^{-2}</td>
<td>1.3</td>
</tr>
<tr>
<td>99.9</td>
<td>1x10^{-3}</td>
<td>3.5</td>
</tr>
<tr>
<td>99.99</td>
<td>1x10^{-4}</td>
<td>6.0</td>
</tr>
<tr>
<td>99.999</td>
<td>1x10^{-5}</td>
<td>10.7</td>
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Table 2. Diversity Gain with 12' Antennas at 7.4 GHz

<table>
<thead>
<tr>
<th>WORST HOUR AVAILABILITY (%)</th>
<th>OUTAGE PROBABILITY</th>
<th>4 PT ANGLE DIVERSITY GAIN</th>
<th>12 PT ANGLE DIVERSITY GAIN</th>
<th>SPACE DIVERSITY GAIN</th>
</tr>
</thead>
<tbody>
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<td>99</td>
<td>10^{-2}</td>
<td>1.3 dB</td>
<td>4.9 dB</td>
<td>7.0 dB</td>
</tr>
<tr>
<td>99.9</td>
<td>10^{-3}</td>
<td>3.5 dB</td>
<td>8.0 dB</td>
<td>10.9 dB</td>
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<tr>
<td>99.9</td>
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<tr>
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<td>10^{-5}</td>
<td>10.7 dB</td>
<td>15.5 dB</td>
<td>16.5 dB</td>
</tr>
</tbody>
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19.5.5