Abstract—This paper presents an on-line algorithm that provides accurate heading predictions for Unmanned Ground Vehicles (UGVs). The algorithm uses cross-correlation of SICK laser scans to improve the heading predictions from GPS. It was tested on our Centaur vehicles in outdoor urban environments and verified to provide accurate smooth heading predictions.

I. INTRODUCTION

The MITRE Corporation has a pair of Ontario Drive and Gear (ODG) DM950 Centaur turbo diesel off-road utility vehicles (Figure 1). As skid steer vehicles, they are well suited as models of the vehicles used in combat. Centaurs run an operating system architecture called wombat [1]–[3]. Heading (i.e., yaw, referred to here as \( \eta \)) is the clockwise deviation from true North on the X-Y plane. Heading can be obtained from a variety of sources such as a Micro-electromechanical System (MEMS) based inertial measurement units (IMU), Global Positioning System (GPS), Inertial Measurement System (IMS), or magnetic compass. In the case of Centaurs, there is significant vibration present, and magnetic compass and MEMS-based IMUs do not produce accurate predictions. GPS can produce spikes and requires line-of-sight to multiple satellites. It also produces unreliable headings at low speeds. Our vehicles operate at slow speeds (2-5 MPH) for safety concerns. Also, as skid-steer vehicles, they are capable of very large heading changes with very little, or no forward motion. This exacerbates GPS as it requires forward motion before a valid heading can be derived.

There are a number of methods used to increase the accuracy of the heading prediction. For example, odometry and range data can be combined to predict the heading [4]. There has been extensive work using cameras as a vision compass to detect the rotational motion of robots [5]–[7]. However, these algorithms usually require camera calibration parameters, and significant computational resources that may tax on-board computing power. Additionally, vision systems often have problems with changes in lighting and in shadowy environments.

There is also a great body of work in localizing the vehicle using Iterative Closest Point (ICP) algorithms (i.e., scan matching). Among these, [8], [9] are closest to the algorithm presented in this paper. [8] uses an iterative minimization of sum of squares in polar coordinates to localize the vehicle. This makes it more expensive than our algorithm, which does not require iteration or minimization. [9] uses cross-correlation function but it does not perform distance correction. We found distance correction essential to the performance of the algorithm. Both algorithms were tested indoors and it is not known how they would perform in an outdoor, dynamic environment.

Currently, Centaurs have one GPS each and two SICK Laser Measurement Sensors (LMSs), one scanning horizontally, and the other vertically. The primary function of the SICKs is obstacle detection. The objective of this paper is to present a fast and inexpensive on-line algorithm to obtain the vehicle heading from the cross-correlation of SICK LMS sweeps when GPS is unreliable or noisy. The algorithm takes advantage of data used for obstacle detection thus no additional sensor is required.

The outline of the paper is as follows. Section II presents the algorithm’s derivation and parameterization. Section III presents the tests of the algorithm and syncing it with GPS, and Section IV presents our conclusions.

II. METHODS

A. Cross-Correlation Function

The cross-correlation of two signals \( f[m] \) and \( g[m] \) in an arbitrary dimension measures the degree of similarity.
between them as a function of the lag \( n \) in that dimension:
\[
(f \ast g)[n] = \sum_{m=-\infty}^{\infty} f[m] g[n + m]. \tag{1}
\]
\( \max(f \ast g) \) corresponds to the best fit between \( f \) and \( g \) while, \( \arg \max(f \ast g) \) corresponds to that value of \( n \) that will produce the best fit.

B. Transformation for Distance Correction

Centaurs can move both forward and backward. However, we utilize them only in the forward mode. As Centaurs move forward, objects get larger from one laser sweep to another. It was critical to correct for this effect by transforming the laser scan at time \( t \) to time \( t - dt \), which is the time that the last scan was performed. We define \( X(t, \Theta(t)) \) as a vector of laser recording at time \( t \). This vector has an associated vector of angular sweep \( \Theta(t) \). Figure 2 shows the trigonometric relations of the transformation. When the vehicle turns, it moves on an arc of a circle with radius \( r = \frac{d}{2 \sin(d\eta/2)} \), where \( d \approx vdt \). \( d\eta \) is the heading change during \( dt \); \( dt \) is the time difference between the two consecutive sweeps \( X(t - dt, \Theta(t - dt)) \) and \( X(t, \Theta(t)) \). \( dt \) is obtained from the encoding time of laser messages; \( v \) is the speed of the vehicle, which is obtained from the average speed of the left and right tracks of Centaur. In Figure 2,
\[
\omega = \frac{d\eta}{2} + \pi + \Theta(t), \tag{2}
\]
\[
x'(t, \sigma) = \sqrt{x(t, \theta(t))^2 + d^2 - 2dx(t, \theta(t)) \cos(\omega)}, \tag{3}
\]
\[
\beta = \sin^{-1}\left(\frac{x(t, \theta(t)) \sin(\omega)}{x'(t, \sigma)}\right), \tag{4}
\]
\[
\sigma = -\left(\beta + \frac{d\eta}{2}\right). \tag{5}
\]
The formulas transform \( X(t, \Theta(t)) \) on \( X'(t, \Sigma) \). \( \Sigma \) is non-uniform and does not correspond to the original vector of sweeps, \( \Theta(t) \). To obtain the transformation on \( \Theta(t - dt) \), polynomial interpolation is employed to interpolate \( \Theta(t - dt) \) on \( \Theta(t - dt) \) from \( \Sigma \). The interpolation yields two new vectors, \( \Theta'(t) \) and \( X'(t, \Theta'(t)) \). \( X'(t, \Theta'(t)) \) is cross-correlated with \( X(t - dt, \Theta(t - dt)) \) to obtain the heading change between the two laser sweeps.

This transformation is non-linear. After the transformation and interpolation, it is possible for the range of \( \Theta'(t) \) to be different from \( \Theta(t - dt) \). Only segments of \( X'(t, \Theta'(t)) \) and \( X(t - dt, \Theta(t - dt)) \) corresponding to \( \Theta(t - dt) \cap \Theta'(t) \) \( (\theta_{\min} : \theta_{\max}) \) are used in the cross-correlation. In the Appendix, we show that this transformation is a first-order approximation and the error is independent of \( d\eta \). Therefore, our algorithm does not need any iterations to improve performance.

In [9], it is stated that the cross-correlation of two laser scans “is roughly independent from the position where the scan was taken, but not from the orientation”. It should be mentioned that they tested their algorithm indoors. However, we found this statement is not true for outdoor environments. Figure 3 shows the difference between heading prediction from GPS and our algorithm from 30 s to 140 s of the run shown Figure 6B with (green) and without (blue) distance correction. Clearly, without distance correction, the drift between the two is much larger.

\[
\text{Fig. 2: Trigonometric relations for the transformation of one ray, } x(t, \theta(t)) \text{ to } t - dt, x'(t, \sigma). \quad \text{The green rectangle is the vehicle at time } t - dt, \text{ the cyan rectangles is the vehicle at time } t, \text{ the red rectangle is the vehicle at time } t \text{ transformed to time } t - dt, \text{ and the blue rectangle is an object. The dotted lines represent the } x-\text{axis of the vehicle (direction of motion), } d \text{ is the distance traveled between the two sweeps, and } d\eta \text{ is the heading change between the two sweeps.}
\]

\[
\text{Fig. 3: The difference (in degrees) between GPS heading } \eta_{\text{GPS}}(t) \text{ and laser compass heading } \eta_{\text{LC}}(t) \text{ for the run shown at Figure 6B from } t = 30 \text{ s to } t = 140 \text{ s. } \eta_{\text{LC}}(t) \text{ was reset to } \eta_{\text{GPS}}(t) \text{ once at } t = 30 \text{ s and then allowed to drift. The blue line shows the drift without distance correction; the green line shows the drift with distance correction.}
\]
No-returns throw off the correlation sum. When there is a significant number of no-returns present in two consecutive laser scans, the correlation function will try to align them as opposed to points with lower values. In an outdoor environment, a great number of recordings can be no-returns as shown in the sweep histogram in Figure 4A. Removing no-returns from the sweeps produced low-quality algorithm performance, as did removing them and interpolating between the remaining points with a number of different interpolation methods. We believe this is because no-returns provide information about the environment; if they correspond to the horizon or a large black car, they keep their relative location on two consecutive scans.

The following procedure proved successful. First, we removed all isolated no-returns (e.g., the recording at $\theta = 55$ degrees in Figure 4A). We calculated the histogram of the sweep by dividing the recordings in 10 m bins (Figure 4B) and setting all no-returns to the median of the bin with the largest distance after no-returns that is not empty (e.g., for the histogram in Figure 4B, the largest non-empty bin after no-returns is the 6th bin with the median of 55 m). We set all recordings with a value grater than 55 m to 55 m.

![Figure 4: A) An example of a laser sweep before and after scaling no-returns. The red line is the original laser sweep. The green line is the same sweep after cleanup. B) The histogram of the original laser sweep. The median of the largest bin after no-returns is 55 m.](image)

**Algorithm 1: Outline of Laser Compass Algorithm**

**Data:** $X(t - dt, \Theta(t - dt))$, $X(t, \Theta(t))$, $dt$, $v$

**Result:** Differential heading $\Delta_{\eta_{LC}}(t)$

**begin**

RemNoReturns($X(t - dt, \Theta(t - dt)), X(t, \Theta(t)))$

$|X'(t, \Theta'(t)), \Theta'(t)| = \text{Transform}(X(t, \Theta(t)));

\text{FillBlindSpots}(X'(t, \Theta'(t)))$

$[\theta_{min} : \theta_{max}] = \Theta(t - dt) \cap \Theta'(t)$

\text{Interpolate} ($X(t - dt, \Theta(t - dt)), X'(t, \Theta'(t)))$

$C = X(t - dt, \Theta(t - dt)) \ast X'(t, \Theta'(t))$

$j = \arg\max C$

$\Delta_{\eta_{LC}}(t) = j \Delta \theta_{int}$

**return** $\Delta_{\eta_{LC}}(t)$

**end**

The obstacle than they did at time $t - dt$. When the sweep at time $t$ is transformed to $t - dt$, it will have a wider field-of-view around the obstacles, but no information about the environment. The same is true at the boundaries of the laser scan. However, the boundaries are already corrected for by using $\Theta(t - dt) \cap \Theta'(t)$. The blind spots around the obstacles, not at the boundaries of the scans, were treated as no-returns and scaled accordingly.

Each laser scan covers 180° with a default lateral bin size of 0.5°. To achieve higher resolution for heading prediction, we interpolated the laser sweeps using polynomial interpolation to ($\Delta \theta_{int}$) to 0.05°. Note that this interpolation is in addition to the one performed in Section II-B. The previous interpolation transformed $X'(t, \Sigma)$ on $X'(t, \Theta'(t))$; this interpolation increases the resolution of the heading prediction. Note also that this interpolation only increases the resolution of the prediction, not its accuracy.

The cross-correlation formula (Equation 1) is quadratic ($O(N^2)$, where $N = |X(t)|$). The cross-correlation can be obtained from the Fourier transform, which can be implemented in $N \log N$ time. This brings the running time of the algorithm to $N \log N$, making it very suitable for real-time implementations. The layout of the algorithm is shown in Algorithm 1.

**III. RESULTS**

**A. Statistics**

The algorithm uses a number of parameters to calculate the heading. We performed a statistics-based study of these parameters to make sure that they were reliable. The statistics were calculated from two hours of combined runs from both Centaurs.

The laser compass uses the average speed of the Centaur’s tracks to calculate the speed. To determine how accurate the average speed was, we compared it to the GPS speed. In general, we found a very good correlation between the GPS and the average track speed (data not shown), except at isolated points where the GPS speed made large excursions. As expected, most of these excursions happened at low speeds. It should be noted that the vehicles were operating...
on a paved road. If they were operating in sandy terrain or another surface with high track slip, the correlation might not be this high, and the algorithm needs a more reliable source for measuring ground speed.

GPS heading does not degrade linearly with the vehicle’s decrease in speed. Due to its internal correction models, GPS maintains a relatively high signal-to-noise ratio for heading before suddenly breaking down. To determine this breakpoint, we calculated the mean and standard deviation of GPS heading and GPS speed over a 20-point data window. Since we needed the speed to be very accurate, we removed the data points where the standard deviation of speed was more than 0.1 m/s from our samples. Figure 5 shows that once the speed falls below 0.5 m/s, GPS heading is not reliable. We did not merge GPS with the laser compass when the speed was less than 1 m/s (see Section C).

B. Performance Evaluation

Algorithm was evaluated on runs in and around the MITRE campus at daytime, due to safety constraints and without visible precipitation. The vehicle was operated around stationary objects such as parked cars, as well as moving objects such as pedestrians and vehicles in crossing intersections.

The laser compass is a feed-forward algorithm and suffers from drift. When GPS signal was available and reliable, we used it to correct for the drift. In this section, we evaluate the algorithm’s performance without any GPS corrections. We collected two short runs in the MITRE parking lot and compared the GPS heading (as the ground truth) with the laser compass headings (Figures 6A and B). As expected, the drift between laser compass and ground truth increased with time. However, the compass reproduced the ground truth’s features remarkably well. To determine the performance of the algorithm for the dynamic environments, we performed a long run starting in the MITRE parking lot and driving for 20 minutes on the MITRE campus, as well as nearby streets and highways (Figure 6C). We reset the laser compass heading twice to GPS at 410 s and 500 s to make comparing the two graphs easier. In Figure 6C, it is evident that the laser compass at times underestimates the turns significantly. For example, at around 550 s, it underestimates the turn by 125°. The reason for the poor performance during this segment was due to a lack of features across consecutive laser sweeps. At the time, the robot was passing an intersection to cross a highway and there were few features within the laser’s recording range. A camera snapshot of the Centaur’s field-of-view during this time (Figure 7A) shows that there was nothing within the recording range. The lack of features means that the consecutive laser sweeps were very similar although the environment changed significantly (Figure 7B).

Another problem for the algorithm are large dynamic features that register significantly on SICK lasers, especially when Centaurs themselves are stationary. For example, when a Centaur is stopped at a red light in front of a busy highway, large trucks and buses result in a heading change. To alleviate this effect, we employed two measures (see Section C for implementation details). We did not update the heading when the average speed of the vehicle was zero, and we put a maximum threshold ($\Delta_{\text{max}} = 10^\circ$) on the amount of heading change we would allow. Usually, a fast-moving object results in a large heading derivative (on the order of 20°) in two consecutive sweeps (in about 0.2 s). We discarded these values and used the last best-known heading estimate ($\Delta_{\text{good}}$, see Section C).
F. I G. 6: (A and B) Two short runs comparing the performance of laser compass (red line) and GPS (blue line). At the end of each, Centaur is parked. C) A 20-minute run during which the laser compass was reset to GPS twice at 410 s and 500 s to make comparison easier. The algorithm suffers from lack of features in the laser sweeps, especially when the robot is taking sharp turns (for example at 550 s).

C. Merge with GPS

The laser compass produces much less noisy heading predictions than GPS. When merging the two, it would be desirable to keep this feature of the laser compass in merged headings. To synchronize the two, we used a construct similar to an $\alpha$-$\beta$ tracker [10]. We first calculated the derivative of GPS

$$\Delta_{GPS}(t) = \eta_{GPS}(t) - \eta_{GPS}(t - dt)$$

(6)

and used $\sigma(\Delta_{GPS})$ (i.e., the standard deviation of the derivative of GPS) as the criteria for merge. From our statistics-based study, we chose the following parameters for our filter:

$$\alpha = \begin{cases} 0 & \sigma(\Delta_{GPS}) > 5^\circ \text{ or } v < 1 \text{ m/s} \\ \frac{1}{20} & \text{otherwise.} \end{cases}$$

and

$$\beta dt = \begin{cases} 0 & v = 0 \\ \Delta_{\eta_{LC}}(t) & \Delta_{\eta_{LC}}(t) < \Delta_{max} \\ \Delta_{good} & \text{otherwise} \end{cases}$$

We calculated the merged heading ($\hat{\eta}$) using the three iterative equations for the $\alpha$-$\beta$ filter:

$$\eta(t) = \hat{\eta}(t - dt) + \beta dt$$

(7)

$$r(t) = \eta_{GPS}(t) - \eta(t)$$

(8)

$$\hat{\eta}(t) = \eta(t) + \alpha r(t)$$

(9)

Figure 8 shows two relative long runs, comparing merged heading with GPS heading. The first row corresponds to the run of Figure 6C. The spike in the GPS heading at $t=385$ s is due to GPS error and does not get merged. The second row corresponds to a run of the Centaur in the MITRE’s parking lot.

IV. CONCLUSIONS

In this paper, we have presented a fast on-line algorithm that can predict the vehicle heading by the cross-correlation of SICK LMS. Apart from being used as a source to predict heading, our algorithm can also be used to improve the performance of motion models. In most motion models, the predicted x and y displacements are determined by projecting the heading onto the x and y coordinates. If no heading prediction is available, the heading is drawn from a random distribution, which ties the motion model to the type of distribution and whether or not the assumption is valid. Using the laser compass as a source of heading information as opposed to a random sampling from a probability distribution can improve the quality of motion models significantly. We are in the process of implementing this concept for Centaurs.

Our algorithm does not need to search for point associations, which are usually needed in ICP methods; it is faster than the iterative closest-point search methods and does not require iteration to improve its performance.
Assuming that $d$ is small, Equation 3 can be rewritten as:

$$x'(t, \sigma) \simeq x(t, \theta(t)) \left(1 - \frac{d \cos(\omega)}{x(t, \theta(t))}\right)$$  \hspace{1cm} (10)

Substituting for $\omega$, performing a Taylor series expansion around $\theta$, and assuming that the term $d\theta$ is negligible, we obtain:

$$x'(t, \sigma) \simeq x(t, \theta(t)) + d \cos(\theta(t)).$$  \hspace{1cm} (11)

Similarly, a Taylor series expansion of Equation 4, around $\sin(\omega)$, yields:

$$\beta = \sin^{-1}(\sin(\omega)) + \frac{1}{\sqrt{1 - \sin^2(\omega)}} \left(\frac{x(t, \theta(t))}{x'(t, \sigma)} \sin(\omega) - \sin(\omega)\right).$$  \hspace{1cm} (12)

Since $d$ is small, we can assume that:

$$\frac{x(t, \theta(t))}{x'(t, \sigma)} \simeq 1 + \Delta.$$  \hspace{1cm} (13)

After substitution in Equation 13 and substituting for $\omega$, we obtain:

$$\tan(d\eta/2 + \pi + \theta(t)) \simeq -(\theta(t) + d\eta/2 + \Delta \tan(d\eta/2 + \theta(t))).$$  \hspace{1cm} (14)

After a Taylor series expansion of the second term around $\theta$, removing all quadratic terms, and assuming that $\Delta d\eta/2$ is negligible, we obtain:

$$\beta \simeq - (\theta(t) + d\eta/2) + \Delta \tan \theta(t).$$  \hspace{1cm} (15)

and from Equation 5, we obtain:

$$\sigma \simeq \theta(t) - \Delta \tan \theta(t)$$  \hspace{1cm} (16)