Parity-Time Acoustic Metamaterials and Unidirectional Invisible Sensors

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Abstract – We theoretically and experimentally investigate the unique scattering properties of parity-time acoustic systems. We observe acoustic invisibility of airborne sound at audible frequencies in a parity-time symmetric meta-molecule made of perfectly balanced gain and loss subunits. This is achieved in a waveguide configuration using electro-acoustic resonators loaded with non-Foster elements, which are tailored to induce invisibility at the design frequency. This unidirectional reflectionless system is fully stable and self-sensing, allowing for robust implementation of parity-time acoustic metamaterials for loss compensation, non-invasive sensing and advanced acoustic signal manipulation.

I. INTRODUCTION

Metamaterial-inspired acoustic devices can control sound in surprising ways, as in the case of acoustic cloaks [1], subwavelength focusing lenses [2], zero-index structures [3], and non-reciprocal devices [4]. These systems are generally made out of one or several atom-like acoustic inclusions that can strongly interact with an external wave, resulting in physical properties that are not normally encountered in natural acoustic media. These strong wave interactions are often associated with increased absorption losses, as a consequence of Kramers-Kronig relations, which hold for any passive, linear and causal medium. Because absorption generally limits the performance of metamaterials, strategies for decreasing or compensating these losses are currently under active investigation.

In the seemingly unrelated field of modern quantum physics, there has been a lot of interest in studying the spectral properties of non-Hermitian Hamiltonians that commute with the parity-time (PT) operator [5]. These PT-symmetric systems, surprisingly, can exhibit entirely real spectrum eigenvalues, and define a consistent unitary extension of quantum mechanics. A fascinating property of PT symmetric systems is the notion of spontaneous symmetry breaking, i.e., the possibility for the state of the system to lose its PT symmetry, yielding a particular solution of the physical equations that is less symmetric than the theory itself. This generally happens when a continuous parameter quantifying the non-Hermiticity of the Hamiltonian exceeds a threshold value, and it is accompanied by a phase transition from an entirely real spectrum to a complex, or partially complex one. Interestingly, these properties can be observed in paraxial optical systems possessing an even distribution of the real part of the refractive index along the optical axis, and an odd distribution for its imaginary part, in a balanced loss/gain configuration [6]. The perpendicular direction represents the time axis, on which the time-evolution of the system is effectively observed, yielding unique diffraction effects. Another distinctive aspect of PT symmetric system is their scattering signature [7]. For instance, the scattering matrix S of a PT-symmetric system reflects its phase transition: the S matrix is unimodular in the PT symmetric phase, with each eigenvalues being unimodular, while the broken symmetry state is characterized by pairs of eigenvalues with reciprocal moduli. These unusual scattering properties may be used to our advantage to manipulate waves in novel ways, and build novel loss-compensated metamaterials based on PT symmetry.

One attractive property of two-ports PT systems is the possibility of achieving unidirectional invisibility [8]. A unidirectional invisible system is a reciprocal system having no reflection and unitary transmission when excited from port 1, and non-zero reflection from port 2. While this appealing possibility has been widely studied theoretically, the experimental works on the topic consider either passive optical structures, which can exhibit reflection contrast but not unitary transmission, or PT symmetric schemes in the temporal domain, which are not directly applicable to loss compensation schemes. In contrast, we present the first experimental verification of a unidirectional reflectionless PT symmetric system with spatially distributed gain and loss elements. This is obtained in a fully PT-symmetric acoustic system, opening new possibilities in loss compensation and sound manipulation in a novel class of PT-symmetric acoustic metamaterials with mixed loss and gain inclusions.
II. RESULTS AND DISCUSSION

A. Theory

We consider the simple one-dimensional PT-symmetric density distribution represented in Fig. 1, with an even distribution for the real part \( \rho_r = \rho_0 \) of the density and an odd distribution for the imaginary part \( \rho_i = r \rho_0 (\delta(x - d/2) - \delta(x + d/2)) \). The acoustic bulk modulus is assumed to be constant and equal to \( \kappa_0 \).

Denoting port 1 the passive side of the system, the scattering matrix can be calculated as

\[
S = \begin{pmatrix}
\frac{r(r - 2)\sin(x)}{r^2 \sin(x) + 2i e^{inx}} & \frac{2i}{r^2 \sin(x) + 2i e^{inx}} \\
\frac{2i}{r^2 \sin(x) + 2i e^{inx}} & \frac{r(r + 2)\sin(x)}{r^2 \sin(x) + 2i e^{inx}}
\end{pmatrix}
\]

(1)

where \( x = k_0 d \) is the acoustic distance separating the passive and the active lumped elements. In the case \( r = 2 \), one obtains

\[
S = \begin{pmatrix}
0 & e^{ix} \\
e^{ix} & 2 - 2e^{2ix}
\end{pmatrix}
\]

(2)

which describes a unidirectional reflectionless 2-port system regardless of the separating distance.

B. Experiment

This PT acoustic metamolecule supporting the invisibility condition \( r = 2 \) can be realized at a design frequency using electro-acoustic resonators loaded with active non-Foster circuits. This general technique can, in principle, be used to synthesize any stable acoustic impedance [9]. A picture of our experimental set-up is shown in Fig. 2. Two waveguides are connected to our PT symmetric system. The PT device is made of two transverse loudspeakers loaded with active electrical circuits. These non-Foster circuits completely change the dynamics of the loudspeakers: they are tailored to meet the unidirectional reflectionless condition \( r = 2 \) at a specific frequency, while ensuring that the whole system be stable, by careful poles/zero placement.

Fig. 1. Density distribution for the parity-time symmetric acoustic metamolecule considered in this work.

Fig. 2. Experimental set-up using actively loaded acoustic transducers.
Fig. 3 compares the scattering parameters obtained from analytical modeling (solid lines) to measurements (dashed lines). The theory, which predicts zero reflectance and unitary transmittance at 250 Hz, is in good agreement with the measurements. Our fabricated system is unidirectional reflectionless at 260 Hz.

III. CONCLUSIONS

We have successfully designed and fabricated the first PT-symmetric, unidirectional reflectionless acoustic system. Our study proves the feasibility of balanced loss/gain acoustic structures with parity-time symmetry and opens the way to a novel class of acoustic metamaterials and non-invasive sensors with compensated losses and unique wave manipulation capabilities.

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