Millimeter-Wave Orontron Oscillation—Part I: Theory

RICHARD P. LEAVITT, DONALD E. WORTMAN, AND HERBERT DROPKIN

Abstract—Physical principles of operation of the orotron, a Smith-Purcell free-electron laser in the millimeter and submillimeter wavelength regions, are described. Electron bunching in this device occurs when the electron beam interacts with an evanescent component of the electromagnetic wave that propagates along the metallic development. The bunching produces coherent radiation. The equations of motion are linearized, and the starting current and electron tuning characteristics of the device are calculated in closed form. Numerical calculations of the electron efficiency are described.

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called diffraction radiation generators). Much of the work that led to these devices is reviewed in the monograph by birth to a second generation of orotrons1141 which they based on the Smith-Purcell effect [11]. Modifying the original device of Rusin and thereby gave electron lasers" [3]. Smith-Purcell free-electron laser, or production of millimeter and submillimeter wave radiation. Development of new devices that are intended primarily for the wave tubes, etc.) become extremely inefficient as the wave-length of the radiation approaches the submillimeter range. These restrictions have required the development of new devices and also because manufacturing tolerances become too stringent. Approaching the millimeter region (klystrons, magnetrons, traveling wave tubes, etc.) become extremely inefficient as the wave-length of the radiation approaches the submillimeter range from the other end of the spectrum, the microwave region (klystrons, magnetrons, traveling wave tubes, etc.) become extremely inefficient as the wave-length of the radiation approaches the submillimeter range. These restrictions have required the development of new devices and also because manufacturing tolerances become too stringent. Approaching the millimeter region (klystrons, magnetrons, traveling wave tubes, etc.) become extremely inefficient as the wave-length of the radiation approaches the submillimeter range.

In the orotron, depicted in Fig. 1, a sheet electron beam passes over a metallic reflecting diffraction grating imbedded in a mirror which, with another mirror, forms an open resonator. This resonator reflects the radiation emitted by the beam back onto the beam and causes the beam to bunch. If the proper conditions of synchronism are met between the electron velocity and the phase velocity of a slow wave propagating along the grating surface, the orotron will radiate coherently at a frequency near one of the resonant frequencies of the open resonator.

We have observed oscillation of an orotron in the 53–73 GHz frequency band. The device is described in detail in the following paper [17]. The present paper discusses the theory of the mechanism by which the orotron operates from a theoretical point of view. We describe the open resonator of the device and the effect of the reflecting diffraction grating on its mode structure. A linear theory of electron bunching in the orotron is presented that allows computations of the threshold current of the device and its tuning characteristics. The effects of the finite electron beam temperature on the device characteristics are described within the context of the linear theory. A full nonlinear numerical calculation that predicts electron efficiencies is then presented.

I. INTRODUCTION

CONVENTIONAL sources of electromagnetic radiation in the microwave region (klystrons, magnetrons, traveling wave tubes, etc.) become extremely inefficient as the wavelength of the radiation approaches the submillimeter range [1]. They do so both because of fundamental frequency limitations on these devices and also because manufacturing tolerances become too stringent. Approaching the millimeter–submillimeter range from the other end of the spectrum, the optical, presents difficulties in that the efficiencies of these devices (optically pumped lasers) are restricted by the Manley-Rowe condition [2]. These restrictions have required the development of new devices that are intended primarily for the production of millimeter and submillimeter wave radiation.

Foremost among the latter devices are the so-called "free-electron lasers" [3]–[10]. A promising candidate for generating millimeter and submillimeter wave radiation is the Smith–Purcell free-electron laser, or orotron. This device is based on the Smith-Purcell effect [11]; Smith-Purcell radiation is produced when an electron beam skims the surface of a metallic diffraction grating. The first orotrons were developed independently by Rusin et al. in the Soviet Union [12] and by Mizuno et al. in Japan (that group called the device the ledatron [13]). Another group in the Soviet Union made modifications to the original device of Rusin and thereby gave birth to a second generation of orotrons [14] (which they called diffraction radiation generators). Much of the work that led to these devices is reviewed in the monograph by Shestopalov [15]. A related device, intended for operation in the infrared, was proposed by Wachtel [16].

In the orotron, depicted in Fig. 1, a sheet electron beam passes over a metallic reflecting diffraction grating imbedded in a mirror which, with another mirror, forms an open resonator. This resonator reflects the radiation emitted by the beam back onto the beam and causes the beam to bunch. If the proper conditions of synchronism are met between the electron velocity and the phase velocity of a slow wave propagating along the grating surface, the orotron will radiate coherently at a frequency near one of the resonant frequencies of the open resonator.

We have observed oscillation of an orotron in the 53–73 GHz frequency band. The device is described in detail in the following paper [17]. The present paper discusses the theory of the mechanism by which the orotron operates from a theoretical point of view. We describe the open resonator of the device and the effect of the reflecting diffraction grating on its mode structure. A linear theory of electron bunching in the orotron is presented that allows computations of the threshold current of the device and its tuning characteristics. The effects of the finite electron beam temperature on the device characteristics are described within the context of the linear theory. A full nonlinear numerical calculation that predicts electron efficiencies is then presented.

II. PROPERTIES OF THE OPEN RESONATOR

The feature that distinguishes orotrons from other electron-beam driven devices is the use of an open resonator with a high quality factor (Q) as a feedback device. The open resonator allows the buildup of large electric-field oscillations near the electron beam, which cause the beam to become bunched. This bunching allows the orotron to operate at relatively high efficiency in spite of the fact that the basic radiation mechanism (the Smith-Purcell effect) is weak. In this section, we examine the properties of high-Q open resonators and determine the effect of a metallic diffraction grating imbedded in one of the mirrors on these properties.

A. Effect of the Reflecting Diffraction Grating

The problem of finding the mode structure and the electromagnetic field pattern in an open resonator containing a grating is complex and cannot be solved exactly. However, if the grating period is small compared with the transverse dimensions of the electromagnetic field pattern, the problem can be solved to a good degree of approximation. Consider the ideal-
ized situation depicted in Fig. 2. We consider a grating of grooves of rectangular cross section of period \( l \), groove depth \( h \), and groove width \( d \). A plane wave is incident on the grating, with the wave vector \( \mathbf{k} \) along \( -z \), the electric field vector \( \mathbf{E} \) along \( y \) (perpendicular to the grooves), and the magnetic induction vector \( \mathbf{H} \) along \( x \) (parallel to the grooves).

The solution of this electromagnetic boundary problem is given in detail by Mizuno et al. [18]. The electromagnetic field is a transverse magnetic (TM) field, and we may consider the \( x \)-component of the magnetic induction \( H_x \) alone. Above the grating surface, because of the periodic boundary condition, \( H_x \) assumes the form

\[
H_x = H_0 \left[ 2 \cos k(z - z_0) + \sum_{r=1}^{\infty} a_r \cos \left( \frac{2\pi r y}{l} \right) e^{-\Gamma_r z} \right] e^{-i\omega t} + c.c. \tag{1}
\]

where \( H_0 \) is the amplitude of the incident wave and

\[
\Gamma_r = \left( \frac{2\pi r}{l} \right)^2 - k^2 \right]^{1/2}. \tag{2}
\]

The unknown coefficients \( a_r \) are found by solving the appropriate boundary value problem on the grating surface.

Having solved the idealized problem of a plane wave incident on the grating, we can now write down the solution for the fields in the open resonator by multiplying the plane wave solution by the mode pattern [19] of the resonator. Thus, we obtain for the \( y \)-component of the electric field above the grating, for example,

\[
E_y(x, y, z) = E_0 \sqrt{w_{0x}w_{0y}} \left[ 2 \sin k(z - z_0) \right.
\]

\[
+ \sum_{r=1}^{\infty} a_r \frac{\Gamma_r}{k} \cos \left( \frac{2\pi r y}{l} \right) e^{-\Gamma_r z} \right] H_n \left[ \sqrt{\frac{2}{w_x(z)}} \right.
\]

\[
\times H_m \left[ \sqrt{\frac{2}{w_y(z)}} \right] \exp \left[ -\frac{x^2}{w_x(z)^2} \right.
\]

\[
- \frac{y^2}{w_y(z)^2} + ikz \frac{x^2}{2R_x(z)} + \frac{y^2}{2R_y(z)} \right]
\]

\[
- i \left[ n + \frac{1}{2} \right] n_x(z) - i \left[ m + \frac{1}{2} \right] n_y(z) \right] e^{-i\omega t} + c.c. \tag{3}
\]

where \( E_0 = -iH_0/c\varepsilon_0 \) and where \( H_n \) and \( H_m \) are Hermite polynomials of degree \( n \) and \( m \) [21]. The functions \( w_x(z) \), \( w_y(z) \), \( R_x(z) \), \( R_y(z) \), \( n_x(z) \), and \( n_y(z) \) depend on the resonator geometry [19], and \( w_{0x} \) and \( w_{0y} \) are the minimum values of \( w_x(z) \) and \( w_y(z) \). Near the grating, the sum on \( r \) adds a fine structure to the usual mode pattern of the resonator, which leads to electron bunching in the device. Far from the grating, each term in the \( r \) sum vanishes due to the exponential factor, and we recover the usual mode pattern.

### B. Quality Factor of the Resonator

The above expression for the field in the open resonator in the orotron can be used to determine the losses in the device. In practice, the mirror at \( z = 0 \) is only partially covered by the grating, as shown in Fig. 3. In the region above the grating, (3) is valid; we use the analog of (3) for the grating-free region. The transition between these two regions, in general, is a sharp discontinuity at the grating edges and, in general, cylindrical scattered waves propagate from the edges. The effect of the discontinuity can be minimized by raising the grating a distance \( z_0 \) above the rest of the mirror (so that there is phase matching) and by choosing the grating width in such a manner that a zero of the mode pattern falls on the grating edges. Diffraction losses can be made negligible by choosing sufficiently large mirrors for the open resonator [22]. At the wavelength of interest in this work, scattering losses also are negligible. Thus, outside of output coupling losses which presumably are chosen to optimize energy transfer from the resonator, the predominant loss mechanism is ohmic heating, which we consider here.

The quality factor of an open resonator is defined through

\[
P_{loss} = \frac{\omega U}{Q} \tag{4}
\]

where \( P_{loss} \) is the power dissipated in the resonator, \( U \) is the total energy stored in the resonator, and \( Q \) is the quality factor. The energy density is given by

\[
U = \frac{1}{2} \left\langle \int (e_0|\mathbf{E}|^2 + \mu_0 |\mathbf{H}|^2) \, dV \right\rangle \tag{5}
\]

where the integral is over the resonator volume and \( \mu_0 \) is the vacuum permeability. The power dissipated may be calculated by assuming that the resonator surfaces are good but not perfect conductors [23]:

\[
P_{loss} = \frac{1}{\sigma} \int \frac{|\mathbf{H}|^2}{\delta} \, dA \tag{6}
\]

where the integral is over the surfaces of both resonator mirrors, \( \sigma \) is the conductivity, and \( \delta \) is the skin depth. For a resonator composed of two smooth mirrors, we obtain, from (4),

\[
P_{loss} = \frac{\omega U}{Q}
\]
For a resonator with a grating imbedded in one mirror, we define an effective fraction of the mirror surface occupied by the grating as

$$Q_{\text{ideal}} = \frac{1}{2} \frac{L}{D}.$$  \hspace{1cm} (7)

where $D$ is the grating width. For the situation depicted in Fig. 3, with $w_x(0) = D$, we obtain $Q = 0.2$. If we define

$$\gamma = 1 + \frac{F}{2} \left\{ \frac{1}{4H_2^2} \int H_x^2 \, dl - 1 \right\}$$  \hspace{1cm} (9)

where the integral is over one period of the grating surface, then the quality factor of the resonator with the grating becomes

$$Q = Q_{\text{ideal}} / \gamma.$$  \hspace{1cm} (10)

where $u$ is the $y$-component of the velocity. The current density ($y$-component) at position $r$ at time $t$ is

$$j(r, t) = -e \int_{-\infty}^{\infty} \, du \, v \, f(r, v, t).$$  \hspace{1cm} (12)

In (11) and (12), $e$ is the magnitude of the electron charge. The dynamical equation of motion for the electron beam is the Boltzmann equation [27]

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial y} - \frac{e}{m_e} \frac{\partial f}{\partial u} = 0$$  \hspace{1cm} (13)

where $m_e$ is the electron mass and $E = E(r, t)$ is the longitudinal RF electric field. Note that $f$ depends parametrically on the electron coordinates $x$, $y$, and $z$. Within the constraints of assumptions 1)-3), (13) exactly represents the electron motion.

Assumption 4) is incorporated into our discussion by expanding (13) in a power series in the strength of the electric field. We introduce the strength parameter $r$ and let $E \rightarrow rE$ in (13). We then expand the distribution function in a power series in $r$:

$$f(r, v, t) = \sum_{n=0}^{\infty} r^n f_n(r, v, t).$$  \hspace{1cm} (14)

This expansion is then substituted into (13), and the coefficient of $r^n$ is equated to zero. The power radiated by the electron beam is given by

$$P_{\text{rad}} = -\left\langle \int \, j \, E \, dV \right\rangle$$  \hspace{1cm} (15)

where the integral is over the volume of the electron beam and the brackets indicate a time average. Substituting (14) into (13) and using (15), we obtain the result [28]
where the integral over the transverse coordinates $x$ and $z$ is over the cross-sectional area of the electron beam. In (16), $E_0$ is the Fourier transform

$$
\tilde{E}_0(p) = \int_{-\infty}^{\infty} e^{-ipy} E_0(r) dy
$$

and depends parametrically on $x$ and $z$.

Equation (16) is in terms of the zeroth-order distribution function, $f_0(v)$, which is chosen as a function of velocity only to correspond to the physical situation of a uniform dc electron beam. If all the electrons travel at the same velocity, $v_0$, then the power radiated is [26], [28]

$$
Prad = \frac{-e\omega I_0}{m_e v_0 A_0} \int_{\text{beam}} dx \frac{d}{d\omega} \left| \tilde{E}_0 \left( \frac{\omega}{v_0} \right) \right|^2.
$$

In applying (18) to radiation in an orotron, we must account for the detailed structure of the electromagnetic field distribution by means of (3). It is known that the radiation from the grating should peak at frequencies near those given by the Smith-Purcell condition for vertical radiation [11], [29]

$$
\omega = \frac{2\pi v_0}{l}.
$$

Therefore, referring to (17), we consider the Fourier transform near $p = 2\pi r/l$, where $r$ is an integer. (Thus, we are discussing an orotron operating on the $r$th harmonic.) We obtain the result

$$
Prad = -e\omega I_0 \frac{e_0 F H_0^2 g_0^2 \Gamma_r w_{ox} w_{oy} w_y^2}{m_e v_0 A_0 \omega^3} \times e^{-2\Gamma_r \alpha} \left[ 1 - e^{-2\Gamma_r \Delta} \right] H_m \left( \frac{u}{\sqrt{2}} \right)
$$

$$
\times \left\{ 2mH_{m-1} \left( \frac{u}{\sqrt{2}} \right) - H_{m+1} \left( \frac{u}{\sqrt{2}} \right) \right\} e^{-u^2/2}
$$

for a beam width $D$, thickness $\Delta$, and height $a$ above the grating where

$$
u = \frac{w_y}{l} \left( \frac{\omega l}{v_0} - 2\pi \right).
$$

In particular, the result for a TEM$_{20}$ mode peaks at a value $u = -1$; at this value of $u$, we obtain

$$
Prad \text{ (max)} = 0.0095 \epsilon_0 c U \frac{e_0^2 \Gamma_r^2 \epsilon^2 w_y^2}{m_e v_0 \omega D L \epsilon_e} \times \left\{ e^{-2\Gamma_r \alpha} \left[ 1 - e^{-2\Gamma_r \Delta} \right] \right\}
$$

where $U$ is the total energy stored in the open resonator, given by (5). The starting current of the orotron is determined by requiring that the power radiated, given by (22), is equal to the power loss, given by (4). Thus, the starting current is given by [26]

$$
I_s \text{ (min)} = \frac{m_e v_0^2 \omega^2 D L \epsilon_e}{0.0095 c_0^2 \epsilon^2 \epsilon_r \epsilon_L Q} \left\{ e^{-2\Gamma_r \alpha} \left[ 1 - e^{-2\Gamma_r \Delta} \right] \right\}^{-1}
$$

for the TEM$_{20}$ mode. The starting current for other modes may be derived in a similar manner.

B. Tuning Characteristics of the Orotron

The orotron may be represented by an equivalent circuit, as shown in Fig. 4. The current source $i(w)$ may represent a noise source due to fluctuations in the electron beam current (in the oscillator case) or the modulation in the electron beam due to the driver input (in the amplifier case). The passive open resonator is represented by a complex admittance [30] that can be approximated in the vicinity of a resonance at frequency $\omega_0$ by

$$
Y_0(w) = G_0 + iB_0 (\omega - \omega_0)
$$

where the quantities $G_0$ and $B_0$ are constants (independent of frequency). The output coupling is represented by an additional conductance $G_{out}$ in parallel with $Y_0$.

The important element in the circuit is the admittance $Y_{beam}$, which represents the effect of the RF oscillations in the resonator on the electron beam. The real part of the beam admittance is given by [28]

$$
\text{Re} Y_{beam} (\omega) = -\beta \frac{d}{d\omega} \left| \tilde{E}_0 \left( \frac{\omega}{v_0} \right) \right|^2
$$

and the imaginary part is

$$
\text{Im} Y_{beam} (\omega) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\text{Re} Y_{beam} (\omega')}{\omega' - \omega}
$$

where $\beta$ is a proportionality constant and $P$ is the principal value. Fig. 5 shows the real and imaginary parts of the beam admittance.

If we consider the orotron as an ideal oscillator [31] (that is, we neglect the noise source in Fig. 4), the condition for oscillation is that both the real and the imaginary parts of the circuit admittance must vanish. This condition gives

$$
I_s = \frac{I_s (\text{min})}{\epsilon \epsilon_r} e^{(\epsilon^2 - 1)/2}, \quad \epsilon < 0
$$

and
The thermal averages of the real and imaginary parts of the beam admittance, (25) and (26), can be obtained by expanding about \( v = v_0 \). The results can be used to obtain the corrected starting current as

\[
I_s^{\text{thermal}}(\text{min}) = \alpha^{-1} I_s(\text{min})
\]

where \( I_s(\text{min}) \) is given by (23). The real part of the beam admittance peaks at \( \tilde{u} = -1/\sqrt{\alpha} \) (instead of \( \tilde{u} = -1 \)), at which point the starting current is given by (33) and the electronic tuning transconductance is

\[
\left( \frac{d\omega}{dV} \right)_0^{\text{thermal}} = \alpha^{1/2} \left( \frac{d\omega}{dV} \right)_0
\]

where \( (d\omega/dV)_0 \) is given by (30). In (33) and (34), \( \alpha \) is given by

\[
\alpha = 1 \left[ 1 + \frac{k_B T}{m_e} \left( \frac{w_y \omega}{v_0} \right)^2 \right]^{-1/2}
\]

For realistic values of \( w_y, \omega, v_0, \) and \( T, \alpha \) is significantly less than unity, and the effects of finite beam temperature have substantial effects on \( I_s(\text{min}) \) and \( (d\omega/dV)_0 \).

### IV. NONLINEAR BUNCHING THEORY

The solution of the full nonlinear equations for the orotron is extremely complex, consisting of the coupled mechanical equations for the electron motion and Maxwell’s equations for the electromagnetic field. The mechanical equations couple to Maxwell’s equations through the source terms, and the solutions of Maxwell’s equations are driver terms in the mechanical equations. However, the facts that the open resonator of the orotron has a high \( Q \) and that the beam current density is low allow us to decouple the equations by assuming that the electromagnetic fields are given. The electron motion is then determined through the equation of motion, as a function of the electric field strength. This approach can be made self-consistent by requiring that the power radiated is equal to the power loss given by (4); this requirement determines the electric field strength. We will not do so here, however. We are concerned primarily with the behavior of an electron beam in a given electric field.

#### A. Numerical Computations

We consider the problem of electron motion in the RF electric field given by (3). We retain the assumptions 1)-3) of Section III so that the equation of motion for an electron is given by

\[
\frac{m_e y}{e v_0 A_0} \left( \frac{m_e}{2 \pi k_B T} \right)^{1/2} \exp \left[ -\frac{m_e (v - v_0)^2}{2 k_B T} \right] = -e E(y, t)
\]

where \( k_B \) is Boltzmann’s constant and \( T \) is the electron temperature. The rms spread in beam energy is calculated from (31) as

\[
\Delta E = \left[ \frac{m_e}{4} \left( v^4 - (v_0^4) \right) \right]^{1/2} = \sqrt{m_e v_0^2 k_B T + \frac{1}{2} k_B T^2}.
\]

For a typical electron beam temperature of 1200°C (1500 K) and electron beam velocity \( v_0 \approx 3 \times 10^7 \) m/s, we obtain \( \Delta E/E \approx 1 \) percent, which is sufficiently large so that the effects of finite beam temperature must be accounted for in computing starting currents.
Fig. 6. Electron efficiency versus electron velocity. Curve (a): $E_0 a_1 = 10^3$ V/m. Curve (b): $E_0 a_1 = 10^4$ V/m. Curve (c): $E_0 a_1 = 10^5$ V/m.

For different velocity, $v = v_f$. We define the electron efficiency as the fraction of electron kinetic energy lost by an electron

$$\eta = 1 - \left(\frac{v_f}{v_0}\right)^2.$$  \hspace{1cm} (38)

To represent the average effect of electrons in the beam, the electron efficiency should be averaged over the initial conditions. This amounts, in the present problem, to averaging over the initial velocity distribution and the $x$- and $z$-dependences of $E$ which we neglect here. Thus, we define the average electron efficiency as

$$\langle \eta \rangle = 1 - \left(\frac{\langle v_f \rangle}{v_0}\right)^2.$$  \hspace{1cm} (39)

The only loss mechanism in the problem as stated is the radiation of electromagnetic waves by the beam. Therefore, we may assume that the power radiated is given by the average efficiency times the input beam power, or

$$P_{\text{rad}} = \langle \eta \rangle V I_0$$  \hspace{1cm} (40)

where $V$ is the beam accelerating voltage and $I_0$ is the beam current.

We have computed the electron efficiency by numerically integrating differential equation (36) with the electric field given by (37). The frequency and the grating period, $f = 7.5$ GHz ($\omega = 4.719 \times 10^{11}$ s$^{-1}$) and $l = 0.4$ mm ($K = 1.5708 \times 10^4$ m$^{-1}$), were chosen to model the experiment described in the following paper [17]. The mode size parameter $w_x$ was chosen to be 10 mm. Operation on the first harmonic ($r = 1$) was considered, and thus electron velocities were chosen to be near $v_0 = \omega_p/K = 3 \times 10^7$ m/s.

It was found that the electron efficiency depends only on the combination $E_0 a_1$ and not on $E_0$ alone nor on $a_1$. The electron efficiencies are shown in Fig. 6 as a function of initial electron velocity for three different values of $E_0 a_1$. For $E_0 a_1 = 10^3$ V/m, the efficiency is as predicted by the linear theory. As the field becomes larger, the range in velocity with appreciable efficiency values becomes larger, and the orotron can be excited over a larger velocity spread. Saturation effects begin to set in at $E_0 a_1 \sim 10^4$ V/m; at $10^5$ V/m, there is a strong peak at $v_0 = 3.09 \times 10^7$ m/s.

The dependence of electron efficiency on $(E_0 a_1)^2$ for $v_0 = 3.08 \times 10^7$ m/s is shown in Fig. 7. For $(E_0 a_1)^2 < 0.2 \times 10^{10}$ (V/m)$^2$, the efficiency is quite low, but builds up quite rapidly beyond this point. The peak value, at $(E_0 a_1)^2 \sim 0.7 \times 10^{10}$ (V/m)$^2$, is about 7.5 percent. Not shown on this curve is a subsequent oscillation of the efficiency about a mean value of approximately 5 percent. This oscillation occurs also in an approximate model of the orotron [33].

V. SUMMARY AND CONCLUSIONS

By a proper consideration of the modes in an open resonator and the effect of the reflecting diffraction grating on these modes, we have derived many of the characteristics of the orotron from first principles. Quantities derived include the starting current of the device and its dependence on $u$, the frequency detuning variable; the electronic tuning transconductance, $d\sigma/dV$; and the efficiency of the device in a nonlinear integration of the equations of motion.

The calculations performed in this paper were instrumental in the design of the first operating orotron in the Western world [17], and the implementation of these calculations is described in Part II of this paper.

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REFERENCES


Fig. 7. Electron efficiency versus $(E_0 a_1)^2$ for $v_0 = 3.08 \times 10^7$ m/s.


[27] The concepts used in this discussion are similar to those used for lasers in [18, p. 300].
