Degree of Polarization in the Lyot Depolarizer

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Abstract—Analytic output equations for the degree of polarization of light exiting a Lyot depolarizer are derived from a coherency matrix representation. Design criteria are obtained for sources of different spectral shape.

I. INTRODUCTION

LYOT DEPOLARIZERS [1], [2] have recently attracted renewed attention due to the successful use of one in reducing polarization noise in a fiber-optic gyroscope [3]. Recently, such a device has been reported [4] which was constructed from birefringent single-mode fibers rather than the usual birefringent crystal plates.

The Lyot depolarizer classically consists of two birefringent crystal plates (X or Y cut), whose crystal axes are oriented at 45° with respect to each other, and whose thicknesses are in the ratio 1:2. The device is intended to be used with broadband light and may be viewed as performing a polarization conversion on the spectral components of polarized input light. At the output, each spectral component appears with a different polarization state, and upon averaging over bandwidth, the output is depolarized. The choice of a 45° angle between the plate axes is made so the depolarizer will be operative independent of the input polarization state. Depolarization is not achieved by the first plate, it will be achieved by the second.

Billings [2] analyzed the Lyot depolarizer by assuming sufficient birefringence to provide 2π of phase retardation over the source bandwidth in the first plate. His model for a rectangular-shaped source suggested that the plates should have an integer thickness ratio not equal to one. Loeber has treated the problem for a blackbody [5]. Recent approaches have focused on frequency decomposition of the output polarization states on the Poincaré sphere [6], attempting to achieve a design that provides a uniform distribution of such output states [7], [8]. However, these approaches are not capable of providing quantitative design rules for Lyot depolarizers.

Rashleigh and Ulrich [9] have pointed out that polarization-mode dispersion in single-mode fibers leads to a group delay difference which causes the depolarization of broad-band light. The role of polarization dispersion as opposed to birefringence in depolarization has also been emphasized theoretically by Sakai et al. [10]. In this paper, we follow this approach to calculate analytic output equations for the degree of polarization of a Lyot depolarizer, as a function of the input polarization state, source bandwidth and band shape, and fiber or crystal group delay difference. This result will aid in the de-

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sign of depolarization devices in applications. Our model has been presented qualitatively by Epworth [11]. We ignore mode coupling between polarization modes which has been treated in the paper by Böhm et al. [4].

II. Theory

We assume a configuration for the Lyot depolarizer as shown in Fig. 1. The birefringent sections may be crystal plates or sections of birefringent fiber, although we will use fiber terminology. We assume fiber-mode propagation constants $\beta_\chi$ and $\beta_\Psi$, which are functions of optical frequency $\omega$. The fiber birefringent axes are denoted by $X$ and $Y$ in the first section of length $L_1$, and $X'$ and $Y'$ in the second section of length $L_2 (L_2 > L_1)$. The second section is rotated relative to the first by $45^\circ$. We assume an optical source with spectral intensity $\nu(\omega)$ and center frequency $\omega_0$. The optical input to the depolarizer is assumed to be linearly polarized with input azimuth at an angle $\theta_p$ from the mode axis $X$. For unity power input, the time-dependent electric-field input to the fiber may then be represented

$$E_{0\chi}(t) = \begin{bmatrix} \cos \theta_p \\ \sin \theta_p \end{bmatrix} e^{i\omega_0 t}$$

and $(\cdot)$ signifies a time average. At the joint between the two sections, polarization-mode coupling will occur. The transfer matrix for such coupling is given by

$$
\begin{bmatrix}
E'_{\chi}(L_1) \\
E'_{\Psi}(L_1)
\end{bmatrix} = \frac{1}{\sqrt{2}}
\begin{bmatrix}
1 & 1 \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
E_{\chi}(L_1) \\
E_{\Psi}(L_1)
\end{bmatrix}
$$

Propagation in the fiber sections is governed by the frequency-dependent propagation constants $\beta_\chi(\omega)$ and $\beta_\Psi(\omega)$. Employing the transfer matrix in (2), we can write the frequency-dependent output fields at the end of section $L_2$ as

$$
\begin{bmatrix}
E'_{\chi}(\omega) \\
E'_{\Psi}(\omega)
\end{bmatrix} = \frac{1}{\sqrt{2}}
\begin{bmatrix}
e^{-i\beta_\chi(L_1+L_2)} e^{-i\beta_\Psi(L_1+L_2)} \\
e^{-i\beta_\chi L_1} e^{-i\beta_\Psi L_1 + L_2}
\end{bmatrix}
\begin{bmatrix}
E_{\chi}(\omega) \\
E_{\Psi}(\omega)
\end{bmatrix}
$$

where $E_{0\chi}(\omega)$ and $E_{0\Psi}(\omega)$ are the input fields at the frequency component $\omega$.

The degree of polarization is defined as the fraction of optical power that is polarized [10], [12]. It is calculated from the coherency matrix $J$

$$J=\langle E \cdot E^\dagger \rangle = \begin{bmatrix}
\langle E_{\chi} E_{\chi}^\dagger \rangle & \langle E_{\chi} E_{\Psi}^\dagger \rangle \\
\langle E_{\Psi} E_{\chi}^\dagger \rangle & \langle E_{\Psi} E_{\Psi}^\dagger \rangle
\end{bmatrix}$$

where $\dagger$ signifies the Hermitian transpose. To compute (4), we require time-varying electric-field outputs. We express time-varying electric fields in terms of a complex analytic signal in the usual way [10] by defining $e(t)$ in (1a) as

$$e(t) = 2 \int_0^\infty \nu(\omega) e^{i(\omega_0^\prime \omega_0^\prime t)} d\omega$$

where $\nu(\omega)$ is the (complex) amplitude spectrum of the source. The output time-varying fields from the depolarizer are then given by

$$
\begin{align}
E'_{\chi}(t) &= \frac{\cos \theta_p}{\sqrt{2}} \{ e^{i(\omega_0 t - \beta_\chi L_1 + L_2)} e^{i(\omega_0 t - \beta_\Psi L_1 + L_2)} + \sin \theta_p \} e^{i(\omega_0 t - \beta_\Psi L_1)} \\
&\hspace{1cm} e(t - \beta_\chi L_2 - \beta_\Psi L_1) \end{align}
$$

where $\beta_\chi(\omega) \equiv \beta_\chi(\omega)$, $\beta_\Psi(\omega)$ and $\beta'$ denotes differentiation of $\beta$ by $\omega$. To derive (6), we have used (3) and papers by Sakai et al. [10] and Burns et al. [13]. Using (6), we calculate the components of the coherency matrix by performing the necessary time averages

$$
\begin{align}
J'_{\chi}\chi &= \frac{1}{2} \{ S_0 + \sin 2\theta_p S_1(L_1) \cos \Delta \beta_0 L_1 \} \\
J'_{\chi}\Psi &= \frac{1}{2} \{ S_0 - \sin 2\theta_p S_1(L_1) \cos \Delta \beta_0 L_1 \} \\
J'_{\chi}\chi &= \frac{1}{2} e^{i\Delta \beta_0 L_2} \{ -\sin 2\theta_p S_1(L_2) + \frac{1}{2} \sin 2\theta_p e^{i\Delta \beta_0 L_1} [S_1(L_1 + L_2) \\
&\hspace{1cm} - e^{-i\Delta \beta_0 L_1} S_1(L_2 - L_1) \} \\
J'_{\chi}\Psi &= (J'_{\chi}\chi)^* \end{align}
$$

where $\Delta \beta_0 = \beta_\Psi(\omega_0) - \beta_\chi(\omega_0)$ and $S_0$ and $S_1(z)$ are defined by
\[ S_0 = 8\pi \int_{0}^{\infty} \lvert \nu(\omega) \rvert^2 \, d\omega \]  
\[ S_1(z) = 8\pi \int_{0}^{\infty} \lvert \nu(\omega) \rvert^2 \cos \left( (\omega - \omega_0) \delta \tau_{fg} z \right) \, d\omega. \]  

To derive (7) and (8), we have assumed the source spectrum is symmetrical with respect to \( \omega_0 \). In (8b), \( z \) is a length variable and \( \delta \tau_{fg} \) is the polarization-mode dispersion or group delay difference.

\[
\delta \tau_{fg} = \frac{d(\beta_f^2 - \beta_g^2)}{d\omega}.
\]

The degree of polarization is obtained from the coherency matrix as \( \gamma \).

\[
P(\theta_p) = \left[ 1 - \frac{4 \det \mathbf{F}}{(\tau_f)^2} \right]^{1/2}
\]

which yields \( P(\theta_p) \) as a function of the input polarization azimuth. The result is

\[
P(\theta_p) = (\cos^2 \theta_p \gamma^2(L_2) + \sin^2 \theta_p (\gamma(L_1) + [\gamma^2(L_1)]^2)
- \gamma(L_1 + L_2) \gamma(L_2 - L_1) \cos 2\Delta \theta_0 L_1
+ \frac{1}{2} [\gamma^2(L_1 + L_2) + \gamma^2(L_2 - L_1)])
- \sin 2\theta_p \cos 2\theta_0 \gamma(L_2) \gamma(L_1 + L_2)
- \gamma(L_2 - L_1) \cos \Delta \theta_0 L_1)^{1/2}
\]

where \( \gamma(z) \equiv S_1(z)/S_0 \) is the degree of coherence [12].

### III. Discussion

Equation (11) gives the output degree of polarization for the Lyot depolarizer. It depends on the degree of coherence \( \gamma(z) \)

\[
\gamma(z) = \frac{\int_{0}^{\infty} \lvert \nu(\omega) \rvert^2 \cos \left( (\omega - \omega_0) \delta \tau_{fg} z \right) \, d\omega}{\int_{0}^{\infty} \lvert \nu(\omega) \rvert^2 \, d\omega}
\]

which depends on the source parameters \( \omega_0 \) and \( \lvert \nu(\omega) \rvert^2 \) and the fiber parameter \( \delta \tau_{fg} \). We note that \( \gamma(z) \) is a real, even function of \( z \). For band shapes with spectral width \( 2\Delta \omega \), Sakai et al. [10] has calculated \( \gamma \) for the following spectral shapes.

**Rectangular:** \( \gamma = \frac{\sin \alpha}{\alpha} \)

**Gaussian:** \( \gamma = \exp \left( -\frac{(\alpha}{2\sqrt{\ln 2}} \right) \)

**Lorentzian:** \( \gamma = \exp (-\alpha) \)

where \( \alpha = \Delta \omega \delta \tau_{fg} z \). These functions are compared in Fig. 2.

For the Gaussian and Lorentzian cases, \( \gamma \) is a monotonically decreasing function of \( \alpha \). Experimental results have been given for rectangular- [9] and Gaussian- [14] shaped sources.

We first look at (11) for special cases of \( \theta_p \). For \( \theta_p = 0 \) the input is polarized parallel to the X-axis of the first section \( (L_1) \) and no depolarization occurs in this section. The input to the second section \( (L_2) \) is at 45° to its birefringent axes and we have \( P(0) = |\gamma(L_2)| \). For \( \theta_p = 45° \) depolarization will occur in the first section and the second section becomes unnecessary. We take \( \theta_p = 45° \) and \( L_2 = 0 \) and obtain \( P(45°) = |\gamma(L_1)| \).

These results agree with the calculation for a single fiber length in the paper by Sakai et al. [10]. In general, we want \( P(\theta_p) = 0 \) for any \( \theta_p \). We assume \( L_2 \geq L_1 \) and a monotonically decreasing \( \gamma(z) \). From inspection of (11), we require

\[
\gamma(L_1) = 0
\]

and

\[
\gamma(L_2) = 0
\]

which then implies \( \gamma(L_1 + L_2) = 0 \). We also require

\[
\gamma(L_2 - L_1) = 0
\]

which is satisfied if (14a) is satisfied and if \( L_2 - L_1 \geq 2 L_1 \). The entire condition can then be stated as

\[
L_2 \geq 2 L_1
\]

\[
L_2 = n L_1, \quad n \text{ integer} \neq 1
\]

It is clear from Fig. 2 that precision in \( L_1 \) and \( L_2 \) is only required with a rectangular-shaped source.

The condition for low output degree of polarization is that the quantity \( \Delta \omega \delta \tau_{fg} z \) be large (\( \geq \pi \)). We may express the group delay difference \( \delta \tau_{fg} \) in terms of the effective-index difference \( \Delta n_{eff} \) between the modes [9]. Since \( \Delta \beta = \Delta n_{eff} \omega/c \), where \( c \) is...
The theoretical significance of rotation of 45° insures that this condition is met in one of the resulting group delay is greater than the length of the coherent region. Using two fiber lengths with a relative axis of coherence 

The importance of source band shape and of group delay difference in the birefringent elements, and will allow quantitative device design. Our approach has been put in context with previous descriptions of the device. This analysis will be useful to device designers who will employ this important device in fiber gyroscopes.

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References


Abstract—A method for obtaining permanent low-loss coupling between arrays of single-mode fibers and Ti:LiNbO$_3$ waveguides is described. The technique, based on the use of silicon chip V-grooves, simplifies the coupling problem by simultaneously aligning the entire array and by providing a large surface area for a higher integrity adhesive bond. At $\lambda$ = 1.3 $\mu$m, we measure an average 1.5-dB coupling loss (exclusive of propagation loss) for the assembled array. The average excess loss due to the fiber array is 0.8 dB.

We present an analysis of the effect of various types of array misalignment on coupling efficiency. Angular alignment and array periodicity are found to be critical. If the fiber and waveguide periodicities are matched exactly, the fibers need only be placed within $\pm$1.3 $\mu$m of their optimum position to maintain coupling efficiencies greater than 90 percent.

The efficient coupling of light from a single-mode fiber into a single-mode waveguide (in glass or in an electrooptic material) is an important and formidable problem. Recent results [1], [2] demonstrate that through a careful choice of waveguide fabrication parameters, high efficiency can be attained. In this research, micropositioners were used to align one input and one output fiber to a single waveguide, in what amounts to a tedious and time-consuming manual operation. However, two important problems were not addressed. No attempt was made to simultaneously couple into more than one waveguide and no attempt was made to permanently attach the fibers. In this paper, we report our initial results on a multiple fiber coupling and calculate the influence of various misalignments on coupling efficiency.

There are several approaches to the solution of these problems that deserve consideration. Clearly, a technology which allows for a simple tongue-in-groove automatic alignment would be ideal. In principle, the tongue and groove could be formed by etching. Unfortunately, the submicrometer accuracy required here may preclude this possibility especially for chemically inert materials like lithium niobate. The other obvious and, in fact, more tractable approach involves the use of etched V-grooves in silicon. The etching of these grooves is well understood [3] and the grooves can be accurately placed by using standard photolithographic processes. Moreover, these chips are commonly used for other fiber alignment applications [4].

Several other authors have reported on permanent fiber-waveguide fixtureing. The earliest work involved the “flip-chip” approach [5]. Ramer et al. reported on a 2 x 2 switch at $\lambda$ = 0.83 $\mu$m with attached fiber pigtails [6]. The excess loss due to fiber coupling was approximately 1.5 dB per fiber per interface. In the work by Kondo et al., a pigtailed 1 x 4 switch array at $\lambda$ = 1.3 $\mu$m is reported. In this case, a four-element Si chip array was used at the switch output with an excess loss of at least 1.0 dB per fiber.

The approach we utilize here is to fabricate arrays of fibers in Si chips and to align them to appropriately spaced waveguides in LiNbO$_3$. If the periodicities of the fibers and waveguides are equal, alignment of any two fibers will result in the alignment of all of the fibers. The Si chips used here are standard Western Electric twelve-groove chips used for multi-mode-fiber ribbon connectors [4]. The nominal groove spacing is 228.5 $\mu$m. We use a standard fiber for $\lambda$ = 1.3 $\mu$m with a mode size (half width at 1/e amplitude) of 4.0 $\mu$m and a core eccentricity of 0.7 $\mu$m. The fiber array (see Fig. 1) is formed by stripping a small length of the fiber's plastic cladding and epoxying the fibers in a Si chip “sandwich.” The chip is then cut and polished to an optically smooth finish.

For this experiment, waveguides were fabricated on a z-cut