Integrated Navigation System Study Based On Multiscale Recursive Fusion Estimation

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Abstract—Multiscale recursive estimation is the algorithm of multisensor system based on the scale, which uses the multiscale error model correcting the state estimation on the coarse scale and gets the optimal estimation based on overall observations on the finest scale. The algorithm is used in SINS/GPS integrated navigation system. The estimates are more accurate than the results at the finest scale. The simulation results show that the accuracy is improved.

Keywords- Multiscale; estimation; navigation.

I. INTRODUCTION

The estimation technology maintaining easy to operate has a very good prospect of application in the integrated navigation system. But subsystems of the integrated navigation system usually stand in different scales; there are many restrictions in using that algorithm in the integrated navigation system. For purpose of above mentioned question, we put forward the multiscale Recursive Fusion Estimation algorithm based on the sensors at different scales. The algorithm combines the dynamic analysis based on the model and multiscale signal transform based on the statistic characteristic. The state model and the dynamic system are given at a certain scale. The new model and system are come into being at the new scale and establish new multiscale Recursive Fusion Estimation algorithm. Using the estimation algorithm in the integrated navigation system, we can obtain better estimation affect and break the restriction of sensor in the simple scale. This article has utilized the multisensor multiscale fusion estimate theory, has conducted the research to the GPS and the SINS(strapdown inertial navigation system) integrated system.

II. SYSTEM DESCRIBE

A multiscale multisensor dynamic system at scale N can be described by

\[ x(N,k+1) = A(N,k)x(N,k) + w(N,k) \quad k \geq 0 \]  
\[ z(i,k) = C(i,k)x(i,k) + v(i,k) \quad k \geq 0, \; i = N,N-1,...,2,1 \]  

\( x(N,k) \in \mathbb{R}^{n} \) is the quantity of interest at scale N and the matrix \( A(N,k) \in \mathbb{R}^{n \times n} \) is system matrix. The modeling error is specified by a stochastic process \( w(N,k) \in \mathbb{R}^{n} \).

\[ E\{w(N,k)\} = 0 \]  
\[ E\{w(N,k)w^{T}(N,l)\} = Q(N,k)\delta_{kl} \]  

At the different scale, the different sensor (2) sample in the different resolution. \( C(i,k) \in \mathbb{R}^{p \times n} \) is measurement matrix and the measurement error is given by \( v(i,k) \in \mathbb{R}^{p} \).

\[ E\{v(i,k)\} = 0 \]  
\[ E\{v(i,k)v^{T}(i,l)\} = R(i,k)\delta_{ij} \]  
\[ E\{v(i,k)v^{T}(N,l)\} = 0, k,l \geq 0 \]  

The state variable initial value is stochastic, and

\[ E\{x(N,0)\} = x_{0} \]  
\[ E\{[x(N,0) - x_{0}][x(N,0) - x_{0}]^{T}\} = P_{0} \]

III. MULTISCALE RECURSIVE FUSION ESTIMATION

Multiscale Recursive Fusion Estimation is the optimal estimation of linearity unbiased and variance least. Suppose all eigenvalues of \( A(i,k) \) are in the unit circle, the state model at scale \( i(1 \leq i \leq N) \) can be described by

\[ x(i,k+1) = A(i,k)x(i,k) + B(i,k)v(i,k), k \leq 0 \]  
\[ z(i,k) = C(i,k)x(i,k) + v(i,k), k \geq 0 \]  

The state \( x(i,k) \) can be defined as

\[ x(i,k) = \frac{[x(i+1,2k-1) + x(i+1,2k)]}{2} \]

\[ = \frac{[I + A(i+1,2k-1)]x(i+1,2k-1)}{2} \]
\[ z(i, k) \] is the measurement at scale \( i \).

\[ z(i, k) = [z(i + 1, 2k - 1) + z(i + 1, 2k)] / 2 \]  
(13)

where

\[ A(i, k) = A(i + 1, 2k)A(i + 1, 2k - 1) \]  
(14)

\[ C(i, k) = C(i + 1, 2k) \]  
(15)

\[ B(i, k)Q(i, k)B^T(i, k) = [I + A(i + 1, 2k + 1)]^* \]  
(16)

\[ [A(i + 1, 2k)B(i + 1, 2k - 1)]Q(i, k - 1) = [I + A^T(i + 1, 2k)]B(i, 2k)^* \]  
(17)

\[ Q(i, 2k)B^T(i, 2k)[I + A^T(i + 1, 2k + 1)] / 4 \]  
(18)

In the algorithm, the scale recursive fusion estimation is from the fine to the coarse scale. Namely the algorithm begins from the scale \( L_0 \). In the scale \( L_0 \), the dynamic equation is described by

\[ x(L, k + 1) = A(L, k)x(L, k) + B(L, k)v(L, k) \]  
(19)

\[ z(L, k) = C(L, k)x(L, k) + \nu(L, k) \]  
(20)

Using the standard kalman filtering

\[ \hat{x}(L, k + 1 | k + 1) = \hat{x}(L, k + 1 | k) + K(L, k)[z(L, k + 1) - C(L)\hat{x}(L, k | k)] \]  
(21)

where

\[ \hat{x}(L, k + 1 | k) = A(L, k)\hat{x}(L, k | k) \]  
(22)

\[ K(L, k) = P(L, k + 1 | k)C^T(L, k) \]  
(23)

\[ P(L, k + 1) = A(L, k)P(L, k)A^T(L, k) + B(L, k)Q(L, k)B^T(L, k) \]  
(24)

\[ P(L, k + 1 | k + 1) = [I - K(L, k + 1)^*]C(L, k)]P(L, k + 1 | k) \]  
(25)

The initial value \( x(L, 0) \) of the state vector is the stochastic variable.

\[ E\{x(L, 0)\} = x_0^* \]  
(26)

\[ E\{[x(L, 0) - x_0^*][x(L, 0) - x_0^*]^T\} = P_0^* \]  
(27)

Next we can use the multiscale modeling and observations at the different scale make the recursive estimation and get the optimal estimation based on the global at the most finest scale \( N \).

\[ Z(i, k) = [z(i, 1), z(i, 2), \ldots, z(i, k)]^T \]  
(28)

\[ \hat{x}(i, k) = E\{x(i, k) | Z(1, k), Z(2, k), \ldots, Z(i, k)\} \]  
(29)

We can obtain.

\[ \hat{x}(i - 1, k | k) = E\{x(i - 1, k) | Z(1, k), Z(2, k), \ldots, Z(i - 1, k)\} \]  
(30)

\[ P(i - 1, k | k) = E\{[x(i - 1, k) - \hat{x}(i - 1, k | k)]^T \} \]  
(31)

The unbiased estimation and estimation error variance of \( x(i, 2k - 1) \) at the scale \( i \) based on measurements at all scales are described respectively.

\[ \hat{x}(i, 2k - 1 | 2k - 1) = \hat{x}(i, 2k - 1) + K(i, 2k - 1)^* \]  
(32)

\[ [z(i, 2k - 1) - C(i)\hat{x}(i, 2k - 1)] \]  
(33)

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(34)

\[ [z(i, 2k | 2k - 1) - C(i)\hat{x}(i, 2k | 2k - 1)] \]  
(35)

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\[ \hat{x}(i, 2k - 1 | 2k - 1) = \hat{x}(i, 2k - 1) + K(i, 2k - 1)^* \]  
(36)

\[ [z(i, 2k - 1 | 2k - 1) - C(i)\hat{x}(i, 2k - 1 | 2k - 1)] \]  
(37)

\[ \hat{x}(i, 2k | 2k - 1) = [A(i, 2k - 1)]^* \hat{x}(i, 2k - 1 | 2k - 2) \]  
(38)

\[ \hat{x}(i, 2k - 1 | 2k - 2) = A(i, 2k - 1)^* \hat{x}(i, 2k - 2 | 2k - 2) \]  
(39)

\[ \hat{x}(i, 2k - 1 | 2k - 1) = 4[I + A(i, 2k - 1)]^* \hat{x}(i, 2k - 1 | 2k - 2) \]  
(40)

\[ \hat{x}(i, 2k - 1 | 2k - 2) = A(i, 2k - 1)^* \hat{x}(i, 2k - 2 | 2k - 2) \]  
(41)

\[ \hat{x}(i, 2k - 1 | 2k - 1) = 4[I + A^T(i, 2k - 2)]^* \]  
(42)

\[ \hat{x}(i, 2k - 1 | 2k - 2) = A^T(i, 2k - 2)^* \hat{x}(i, 2k - 2 | 2k - 2) \]  
(43)
\[ K(i, 2k - 1) = P(i, 2k - 1)C^T(i) \cdot \]
\[ (C(i)P(i, 2k - 1)C^T(i) + R(i, 2k - 1))^{-1} \]

\[ \alpha_i(i, 2k - 1) = \frac{1}{trP(i | i - 1, 2k - 1)} \cdot \]
\[ 1 \left( \frac{1}{trP(i | i - 1, 2k - 1)} + \frac{1}{trP(i, 2k - 1 | 2k - 2)} \right)^{-1} \]

\[ \alpha_2(i, 2k - 1) = \frac{1}{trP(i, 2k - 1 | 2k - 2)} \cdot \]
\[ 1 \left( \frac{1}{trP(i | i - 1, 2k - 1)} + \frac{1}{trP(i, 2k - 1 | 2k - 2)} \right)^{-1} \]

\[ \hat{x}(i, 2k | 2k - 1) = A(i, 2k)\hat{x}(i, 2k - 1 | 2k - 1) \]

\[ P(i, 2k | 2k - 1) = A(i, 2k)(i, 2k - 1 | 2k - 1) \cdot \]
\[ A^T(i, 2k) + B(i, 2k)Q(i, 2k)B^T(i, 2k) \]

\[ K(i, 2k) = P(i, 2k | 2k - 1)C^T(i) \cdot \]
\[ (C(i)P(i, 2k - 1)C^T(i) + R(i, 2k - 1))^{-1} \]

The multiscale recursive fusion estimation provides a new recursive algorithm, which derives the estimation in the coarse scale to the fine scale as the initial value in the fine scale. Finally we can obtain the state estimation in the fine scale.

### IV. Theory Simulation

In the simulation, there are three sensors at three scales respectively. They measure the object in the different resolution \( (N = 3, L = 1) \). The simulation gives that the true signal is the sin and its amplitude is five. The measurement signal at every scale is the true signal and noise signal, and noise error variance is in the Table I. The multiscale estimation results are also in it.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Scale 3</th>
<th>Scale 2</th>
<th>Scale 1</th>
<th>Kalman</th>
<th>Multiscale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.98</td>
<td>1.88</td>
<td>2.11</td>
<td>1.01</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>2.02</td>
<td>1.58</td>
<td>0.94</td>
<td>1.06</td>
<td>0.78</td>
</tr>
<tr>
<td>3</td>
<td>2.02</td>
<td>1.75</td>
<td>1.90</td>
<td>1.04</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>1.69</td>
<td>1.69</td>
<td>1.38</td>
<td>0.89</td>
<td>0.82</td>
</tr>
<tr>
<td>5</td>
<td>1.99</td>
<td>1.72</td>
<td>1.33</td>
<td>1.05</td>
<td>0.90</td>
</tr>
<tr>
<td>6</td>
<td>1.82</td>
<td>1.55</td>
<td>1.15</td>
<td>0.96</td>
<td>0.83</td>
</tr>
<tr>
<td>7</td>
<td>1.62</td>
<td>1.20</td>
<td>0.95</td>
<td>0.88</td>
<td>0.69</td>
</tr>
<tr>
<td>8</td>
<td>1.75</td>
<td>1.48</td>
<td>0.97</td>
<td>0.96</td>
<td>0.78</td>
</tr>
</tbody>
</table>

The multiscale estimation result in the situation 8, where the horizontal axis is sample and the vertical axis is the signal amplitude. From the result, we can make the conclusion that the multiscale estimation is better than kalman filter.

![Figure 1](image1.png)

Figure 1 gives the kalman filter and the multiscale estimation at scale 3.

### V. Integrated Navigation System Simulation

The navigation system is the north system without considering altitude. The state vector is the platform error \( \phi, \phi, U \), velocity error \( \delta v_x, \delta v_y, \delta v_z \), position error \( e_{x}, e_{y}, e_{z} \), gyro constant drift \( b_{x}, b_{y}, b_{z} \), accelerometer drift \( V_x, V_y \). The system state equation is described.

\[ X(N, t) = A(N, t)X(N, t) + B(N, t)W(N, t) \]

where

\[ X = \begin{bmatrix} \delta \phi & \delta \lambda & \delta v_x & \delta v_y & \delta v_z & e_x & e_y & e_z \end{bmatrix}^T \]

\[ W = \begin{bmatrix} \omega_x & \omega_y & \omega_z & \omega_x & \omega_y \end{bmatrix}^T \]

In the system, integrated matter of the position and velocity is used. The position and velocity measurement equation of SINS/GPS is described.

\[ Z(i, t) = \begin{bmatrix} q - q_x & \lambda - \lambda & v_x - v_y & v_y - v_x \end{bmatrix} \]

\[ = \begin{bmatrix} \delta q - \delta q_x & \delta \lambda - \delta \lambda & \delta v_x - \delta v_y & \delta v_y - \delta v_x \end{bmatrix} \]

\[ = C(i, t)X(i, t) + V(i, t) \]

where \( V(i, t) = [m_x, m_y, m_z, m_v] \) is the zero mean white noise of the GPS.

To start the algorithm, the initial condition is followed.

\[ \varphi = 126.2^\circ, \lambda = 46.7^\circ, \phi_x = \phi_y = \phi_z = 10^\circ \]

\[ e_{x} = e_{y} = e_{z} = 0.01^\circ / h \]

\[ \sigma_{v_x} = \sigma_{v_y} = \sigma_{v_z} = 0.005^\circ / h \]

\[ V_x = V_y = 5 \times 10^{-5} g, \sigma_{v_x} = \sigma_{v_y} = 2.5 \times 10^{-5} g \]
The system adopted two sensors of different accuracy, which noise is zero mean white. The variance vector is

\[
R = [(0.25 \text{ m/s})^2 \quad (0.25 \text{ m/s})^2 \quad (25 \text{ m})^2 \quad (25 \text{ m})^2] \\
R_i = [(0.50 \text{ m/s})^2 \quad (0.50 \text{ m/s})^2 \quad (50 \text{ m})^2 \quad (50 \text{ m})^2]
\]

The simulation is done in different situation for observing the system accuracy. The variance of position and velocity after filtering is in Figure 2, and the estimation error variance in different state in the Table II.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Error variance</th>
<th>Error variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>state</td>
<td>algorithm</td>
<td>(\delta \varphi /\text{m} )</td>
</tr>
<tr>
<td>1</td>
<td>Kalman</td>
<td>16.25</td>
</tr>
<tr>
<td></td>
<td>Multiscale</td>
<td>14.61</td>
</tr>
<tr>
<td>2</td>
<td>Kalman</td>
<td>16.08</td>
</tr>
<tr>
<td></td>
<td>Multiscale</td>
<td>14.08</td>
</tr>
<tr>
<td>3</td>
<td>Kalman</td>
<td>16.09</td>
</tr>
<tr>
<td></td>
<td>Multiscale</td>
<td>14.94</td>
</tr>
</tbody>
</table>

From the Figure 2 and Table II, we can find that the result of the multiscale in the integrated navigation system is better than that of the kalman filter.

VI. CONCLUSION

In this paper, the integrated SINS and GPS system has been studied, based on the scale of restrictions of traditional kalman estimation method, puts forward a new fusion estimation algorithm based on different scale sensors. The specific algorithm is derived, through algorithm analysis and computer simulation proves the validity of the new algorithm and superiority. Compared with traditional methods, the method with a broader range of applications, while maintaining a filtering method kalman simple and easy to operate, the actual navigation system will have a very good prospect of application.

ACKNOWLEDGMENT

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REFERENCES


Figure 2. Estimation errors of integrated system of Multiscale Recursive estimation