A novel naturally sampled space vector pulse width modulation algorithm

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Abstract—A novel algorithm for the naturally sampled SVPWM in overmodulation region is proposed in this paper, the comparison between naturally sampled SVPWM and conventional SVPWM in overmodulation region is carried out. It was proved that naturally sampled SVPWM could be used not only in under modulation but also in overmodulation region by the results of analysis and simulation. The work was done by this paper shows a promising use of the naturally sampled SVPWM. The obviously advantage is that it could be implemented by using DSP with only simplified computation and short time of on line calculation, or just by using analog circuits without the help of microprocessor at all. All the work did by this paper provide a platform for the implementation of naturally sampled SVPWM operating in under modulation and over modulation region.

Keywords- Overmodulation region; Unermodulation region; Naturally sampled SVPWM

I. INTRODUCTION

Space vector PWM has recently become a very popular pulse width modulation method for voltage-feed converter AC drives because of its superior harmonic quality and extended linear range of operation into undermodulation and overmodulation region. However, the implementation of conventional SVM is quite a hard work because it requires a very complex on line mathematical calculation [1].

Many papers have proved that the SVPWM is equivalent to the SPWM or carrier based PWM when a zero sequence component is added to the sinusoidal reference waveform [2-5]. However, all the above-mentioned references did not give their fully detailed analysis and discussion into the overmodulation region for naturally sampled SVPWM and the comparison with conventional SVPWM. While in this region, conventional SVPWM need even more complex calculation.

In this paper, the analysis and study into the overmodulation region for naturally sampled SVPWM are presented. Firstly, the principle of conventional SVPWM is reviewed shortly. Then, the principle of naturally sampled SVPWM in undermodulation and overmodulation region is discussed in detail, based on the expression in undermodulation, the three reference pole voltages of natural sampled SVPWM are expressed segmentally in overmodulation region, then the reference phase voltages are fabricated, two angles (α for mode-1 and α for mode-2) are derived from the Fourier Series Expansion of the reference phase voltage which corresponding to the modulation factor (m), the relationships between m and the magnitude of reference voltage $V^*$ is worked out based on the two angles, then, the relationship between $V^*$ and modulation factor (m) are built which covers the entire operating range from undermodulation up to six step. At the end of this paper, the performance of naturally sampled SVPWM based on the analysis discussed in this paper is further verified in an open loop V/Hz control of induction motor drives that cover all the undermodulation and overmodulation region.

II. REVIEW OF SVPWM IN UNDERMODULATION AND OVERMODULATION

Fig.1 explains SVPWM operation in the undermodulation and overmodulation regions, The operation in undermodulation or overmodulation is determined by the modulation factor, which is defined as:

$$m = \frac{V^*}{V_{1w}} \ (1)$$

where $V^*$ is the magnitude of command or reference voltage vector and $V_{1w}$ is the fundamental peak value ($V_d/\pi$) of the (square or six step) phase voltage wave [6].

In the undermodulation as shown in Fig.1(a), 0<m<0.907. In overmodulation Mode-1 as shown in Fig. 1(b), 0.907<m<0.9514. In overmodulation Mode-2 as explained in Fig.1(c), 0.9514<m<1.0.

III. PRINCIPLE OF NATURALLY SAMPLED SPACE VECTOR PWM

A. Principle of naturally sampled SVPWM

Fig.2 shows the three reference sinusoidal command voltages in naturally sampled SPWM. It has been proved that when a kind of zero sequence component is added to the sinusoidal reference waveform as listed in Table 1., The naturally sampled SPWM will become naturally sampled SVPWM with exactly the same output voltage as that of the conventional SVPWM [2][3][4]. Fig.3 shows the corresponding three pole command voltages $v_{ao}$, $v_{bo}$, $v_{co}$ in naturally sampled SVPWM.
When pole command voltage $v_{x}^{*}$ (x stands for a, b or c) reaches to $0.5V_d$ (limit of undermodulation), the peak value of line-to-neutral phase command voltage $v_{xn}^{*}$ will be:

$$V^{*} = \frac{V_d}{\sqrt{3}}$$

so, the maximum modulation factor that naturally sampled SVPWM in undermodulation can achieve will be:

$$m_{max} = \frac{V_d}{\sqrt{3}} \frac{\sqrt{3}}{2V_d} = 0.907$$  \hspace{1cm} (2)

If the magnitude of reference voltage $v_{x}^{*}$ increase further, peak value will exceed $0.5V_d$, then, inverter will be working in overmodulation region.

### Table 1  zero sequence components vs $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$0 \sim \frac{\pi}{6}$</th>
<th>$\frac{\pi}{6} \sim \frac{\pi}{2}$</th>
<th>$\frac{\pi}{2} \sim \frac{5\pi}{6}$</th>
<th>$\frac{5\pi}{6} \sim \frac{7\pi}{6}$</th>
<th>$\frac{7\pi}{6} \sim \frac{3\pi}{2}$</th>
<th>$\frac{3\pi}{2} \sim \frac{11\pi}{6}$</th>
<th>$\frac{11\pi}{6} \sim 2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>$v^*_a/2$</td>
<td>$v^*_b/2$</td>
<td>$v^*_c/2$</td>
<td>$v^*_d/2$</td>
<td>$v^*_b/2$</td>
<td>$v^*_c/2$</td>
<td>$v^*_d/2$</td>
</tr>
</tbody>
</table>

#### Fig 1. Voltage trajectories in undermodulation and overmodulation regions and the corresponding phase voltage

**B. Naturally sampled SVPWM in overmodulation mode-1**

Working in overmodulation region, no matter in mode-1 or mode-2, the peak value (absolute value) of the pole command voltage $v_{x}^{*}$ will be clamped to $0.5V_d$, as shown in fig.4. Where $\alpha_1$ is the clamping angle that is defined as the angle between $\pi/6$ (or $\pi/2$) and clamping points as shown. At the beginning of overmodulation mode-1, $\alpha_1 = \pi/6$, at the end of overmodulation mode-1, $\alpha_1 = 0$.

From fig.4, the description of three pole command voltages ($v^*_a$, $v^*_b$, $v^*_c$) in the first quarter-cycle can be obtained. Consider that the phase-a command voltage could be derived as:

$$v^*_a = \frac{2}{3}v^*_a - \frac{1}{3}v^*_b - \frac{1}{3}v^*_c$$  \hspace{1cm} (3)

So the description of phase-a command voltage in the first quarter-cycle can be obtained as following:
The simple and clear relationship between $V^*$ and $\alpha_1$ gives an indirect way for the control of $\alpha_1$ and modulation factor $m$ by following equation (5),

\[ m = \frac{V_1}{2V_d/\pi} \] (6)

From equation (5), the peak value of reference sinusoidal voltage can be derived as:

\[ V^* = \frac{V_d}{\sqrt{3\sin(\alpha_1 + \frac{\pi}{3})}} \] (7)

Mode-1 begins when $\alpha_1 = \frac{\pi}{6}$, with $m = 0.907$,

\[ m = 0.9566 \, , \quad V^* = \frac{2V_d}{3} \, . \]

C. Naturally sampled SVPWM in overmodulation mode-2

When $\alpha_1 = 0$ , overmodulation mode-1 ends and overmodulation mode-2 begins. Fig.6 gives the waveform of the three command pole voltage $v_{a0}$, $v_{b0}$, $v_{c0}$ in overmodulation mode-2, where $\alpha_2$ is the shifting angle which is defined as the angle between $\pi/6$ and crossover point as shown. At the beginning of overmodulation mode-2, $\alpha_2 = 0$ ,
the corresponding $v_{ao}^*$ is shown in dashed line. At the end of overmodulation mode-2, $\alpha_2 = \pi / 6$, three command pole voltage will become completely square wave.

In mode-2, as shown in fig.6, the three pole command voltages ($v_{ao}^*$, $v_{bo}^*$, $v_{co}^*$) in the first quarter-cycle can be obtained. Following the equation (3), we can get the description of command phase-a voltage in the first quarter-cycle as following:

$$v_{ao}^* = \begin{cases} v' \sin \alpha & 0 \leq \alpha \leq \frac{\pi}{6} - \alpha_2 \\ \frac{1}{3} V_d & \frac{\pi}{6} - \alpha_2 \leq \alpha \leq \frac{\pi}{6} + \alpha_2 \\ \frac{1}{2} V_d - \frac{1}{2} v' \sin \left( \alpha + \frac{2}{3} \pi \right) & \frac{\pi}{6} + \alpha_2 \leq \alpha \leq \frac{\pi}{2} - \alpha_2 \\ \frac{1}{3} V_d & \frac{\pi}{2} - \alpha_2 \leq \alpha \leq \frac{\pi}{2} \end{cases} \quad (8)$$

fig.7 (a) shows the simulation waveform of $v_{ao}^*$, fig.7 (b) gives the corresponding voltage trajectories. By means of the Fourier Series Expansion, the fundamental peak value ($V_1$) of phase-a command voltage can be derived.

Then, a computer program is written to solve clamping angle $\alpha_2$ as a function of modulation factor $m$. where:

$$\alpha_2 = \frac{\pi}{6} - \sin^{-1}\left( \frac{V_d}{3V^*} \right) \quad (9)$$

$$m = \frac{V_1}{2V_d / \pi} \quad (10)$$

Fig.6(b) shows the plot of this relation. From equation (9), the peak value of reference sinusoidal voltage can be derived as:

$$V^* = \frac{V_d}{3 \sin \left( \frac{\pi}{6} - \alpha_2 \right)} \quad (11)$$

Mode-2 begins when $\alpha_2 = 0$ at $m = 0.956$, $V^* = 2V_d / 3$. With the increase of $V^*$, $\alpha_2$ will increase. When $V^*$ goes to infinite, $\alpha_2$ will go to $\pi / 6$, and $m$ will go to 1. $V^*$ and $\alpha_2$ also have a simple and clear relationship, so $\alpha_2$ can also be controlled through the control of $V^*$ by following equation (9).

The above discussion is future verified in a open loop Volts/Hz control of ac motor drive with fan load covering undermodulation and overmodulation regions as shown in fig.8.

IV. CONCLUSIONS

Naturally sampled Space Vector PWM makes the implementation of SVPWM an easy work not only in undermodulation but also in overmodulation region. In overmodulation region, because of the approximate relationship between $\alpha_1$ (in mode-1) or $\alpha_2$ (in mode-2) and modulation factor $m$, output modulation factor $m$ can also be

![fig.6 three command pole voltages in overmodulation mode-2](image)

![fig.7 simulation results for phase command voltage and voltage trajectory in mode-2](image)

![fig.8 Drive acceleration with Volts/Hz control covering undermodulation and overmodulation regions](image)
controlled through controlling the magnitude of three sinusoidal reference wave \( V^* \) by following the equations and two lookup tables derived in this paper.

Another thing that need to do is to clamp the peak value of pole command voltage into 0.5\( V^d \). Then, all the other things will be the same as what done in undermodulation region. All the work did by this paper provide a platform for the implementation of naturally sampled SVPWM operating in undermodulation and overmodulation region.

REFERENCES


