SMALL OBSTACLE LOADING IN A TEM CELL

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Abstract
A typical transverse electromagnetic (TEM) cell measurement procedure involves calibrating an empty cell and introducing a test object. The loading effect due to the test object presence is normally assumed to be negligible. This paper examines the effect of the test object and the validity of the "non-perturbing" assumption. The analysis utilizes the small aperture theory, as applied to the dual problem of small obstacle scattering. The result is an equivalent T-network representation of the test loading which allows the overall transmission line circuit to be studied. In addition, evaluating the scattered modes gives the field perturbation due to the test object.

Introduction
The TEM cell has emerged as a very useful low frequency measurement device [1]. The basic TEM cell design, as shown in figure 1, consists of a section of rectangular coaxial transmission line (RCTL) coupled at each end to ordinary 50 ohm coaxial cable via a tapered section. The central RCTL test section supports a linearly polarized TEM mode which resembles a free-space plane wave and allows one to create an isolated, standard, broadband test field. TEM cell applications include the emission and susceptibility testing of electronic equipment for EMC purposes [2-3], the calibration of probes and antennas [4], the study of the biological effects of rf radiation [5-6], as well as the measurement of the shielding effectiveness of materials [7].

In each case, the presence of the test object perturbs the TEM cell environment from its well studied empty state. If the test object is kept small compared to the test chamber height, is it then reasonable to assume that the cell environment is not significantly perturbed? Small obstacle theory may be used to investigate the validity of this assumption from two points of view. First, if only the scattered TEM mode is considered, then the TEM cell may be viewed as a transmission line circuit. Analyzing the reflected and transmitted TEM mode enables us to represent the test object loading in terms of an equivalent T-network. The loaded TEM cell circuit may then be investigated. Second, the excitation of the initial higher-order modes may be modeled. This yields a theoretical assessment of the expected field perturbation due to test object scattering. If the sensitivity of the desired measurement is such that deviations in the cell characteristics are important, the present analysis should predict the limits to be placed on the test object size and/or measurement frequencies.

Our paper begins with a review of small obstacle scattering theory which also requires a knowledge of the normalized modes in a cell's RCTL section, the obstacle polarizabilities, and the incident field distribution. Combining these elements leads to a description of test object scattering. Numerical results for the impedance of a loaded cell will be compared to measured data. Also examined is the field perturbation from the empty cell values due to a test object. In each case, a spherical conducting scatterer is used. A more complete discussion of the above approach has appeared elsewhere [8].

Small Obstacle Theory
Small obstacle theory is the dual of the small aperture theory pioneered by Bethe [9] and others. The equivalence between the two follows from Babinet's principle. Both problems are of longstanding interest [10] and are outlined here as applicable to the TEM cell. The notation is that of Collin [11].

Given a discontinuity in a waveguide excited by an incident field, $E_0$, $H_0$, the scattered fields $E_s$, $H_s$ outside the discontinuity region may be expanded in terms of the waveguide's normalized modes $E_n^-, H_n^-$ according to [11]

\[
\begin{align*}
E_s^+ &= \sum a_n^+ E_n^+ , \\
E_s^- &= \sum a_n^- E_n^- , \\
H_s^+ &= \sum b_n^+ H_n^+ , \\
H_s^- &= \sum b_n^- H_n^- ,
\end{align*}
\]

where $\pm$ denotes propagation away from the discontinuity in either the forward (+) or backward (-) direction with the expansion coefficients given by $a_n$ and $b_n$, respectively. In general, the expansion coefficients must be determined by an integration over the source region. However, when the discontinuity consists of elementary electric (P) and magnetic (M) dipole moments, $a_n$ and $b_n$ are simply given by

\[
\begin{align*}
2a_n &= i\omega (\mu_0 \cdot \vec{M} - \vec{E} \cdot \vec{P}) , \\
2b_n &= i\omega (\mu_0 \cdot \vec{M} - \vec{E} \cdot \vec{P}) ,
\end{align*}
\]

where $\mu_0$ is the free space permeability, and an $\exp(i\omega t)$ time convention has been suppressed.

The basic premise of small obstacle theory is as follows: if both the incident fields and the scattered modes of interest vary only slightly over the obstacle, the scattered fields are equivalent to those produced by a pair of dipole moments located at the obstacle center. The observation point must be sufficiently removed from the scattering region to insure that the ignored evanescent modes do not contribute. This rather simple model has been proven to be quite useful and gives good accuracy in practical applications.

The dipole moments depend on the incident field exciting the obstacle, as well as the obstacle size, shape, and composition. Because the incident field is assumed to be essentially constant over the obstacle volume, its contribution may be summarized by its values $E_i(x_0), H_i(x_0)$ at the obstacle's center $x_0 = (x_0', y_0', z_0')$ which corresponds to the location of the equivalent dipole moments. The obstacle's geometric qualities are characterized by a pair of
dyadics termed the electric $\varepsilon_e$ and magnetic $\mu_m$ polarizabilities. In terms of this decomposition, the dipole moments $P$ and $M$ are given by

$$P = \varepsilon_0 \varepsilon_e \cdot \mathbf{E}_i (\hat{x}_0),$$

$$M = \mu_m \cdot \mathbf{H}_i (\hat{x}_0),$$

where $\varepsilon_0$ is the free-space permittivity. Combining (2) and (3) we arrive at the basic equation of interest

$$2a_n = i \omega \mu_0 \left[ \varepsilon_0 \varepsilon_e - \varepsilon_m \right] \cdot \mathbf{H}_i (\hat{x}_0) - \varepsilon_0 \varepsilon_e \cdot \mathbf{E}_i (\hat{x}_0),$$

$$2b_n = i \omega \mu_0 \left[ \varepsilon_0 \varepsilon_e + \varepsilon_m \right] \cdot \mathbf{H}_i (\hat{x}_0) - \varepsilon_0 \varepsilon_e \cdot \mathbf{E}_i (\hat{x}_0).$$

In order to apply this result, three quantities need to be specified: (1) the normalized modes, (2) the obstacle polarizabilities, and (3) the incident fields.

**The Normalized Modes in a RCTL**

Typically, a TEM cell is operated at frequencies where only the dominant TEM mode propagates. Thus, the higher-order RCTL modes have been considered only to the extent that their appearance limits a cell's usable bandwidth. A complete modal description for the RCTL is unavailable. Our description of obstacle scattering will account for the TEM mode and the first three higher-order modes. Modes beyond these first few should not contribute meaningfully. Several solutions for the TEM mode field distribution have appeared, based on either the conformal mapping approach [12] or the singular integral equation technique [13]. The latter solution will be used since it also generates the desired higher-order modes.

Figure 2 shows the RCTL cross-section and defines the relevant dimensions. The $x$ and $y$ axes are oriented as shown with the $z$-axis corresponding to the direction of propagation. The normalized waveguide modes may be written as follows:

$$\mathbf{E}_n = (E nt + \tilde{a}_n E nt) e^{ik_n z},$$

$$\tilde{\mathbf{H}}_n = (\tilde{a}_n H nt + a_n H nt) e^{ik_n z},$$

where $k$ is the propagation constant of the $n$th mode, and the transverse (subscript $t$) field components $E nt, H nt$ are related via the admittance dyadic $Y_n$ according to

$$\tilde{H}_n = Y_n \cdot \tilde{E}_n.$$  

For convenience the $z = 0$ plane will be located through the obstacle's center. The normalization condition takes the form

$$\int CS (\mathbf{E}_n \times \tilde{a}_n) \cdot \mathbf{E}_n = \delta_{mn},$$

where $CS$ denotes the RCTL cross-section, and $\delta_{mn}$ is the Kronecker delta function.

The first four RCTL modes are the TEM, TE01, TE10, and TE11 modes [14]. The TE01 mode will not interact with a test object located near the test zone center ($x = 0$) as this mode is largely confined to the gaps. Therefore the TE01 mode can be ignored. The remaining modes may be found via a residue calculation and a normalization consistent with (7). For the TEM mode, one finds that [13]:

$$E_{ox} = \frac{2}{\alpha z} \frac{1}{\cosh M (b - py)} \sum_{m=1}^{\infty} \frac{\sinh M b \sin M x \sin M a}{\sinh Mb} \sin Mx \sin Ma J_0 (Mg),$$

$$E_{oy} = \frac{2}{\alpha z} \frac{1}{\sinh M b} \sum_{m=1}^{\infty} \frac{\cosh M b \sin M x \sin M a}{\sinh Mb} \cos Mx \sin Ma J_0 (Mg),$$

where $Z_c$ is the characteristic impedance of the cell (typically designed to be nominal 50 $\Omega$, $\mu_0$ refers to a summation over odd $m$ only ($m = 1, 3, 5, \ldots$), $p$ is a sign function with $p = 1$ for $y > 0$ and $p = -1$ for $y < 0$, $M = \mu_0/(2a)$, and $J_0(x)$ is a Bessel function. The magnetic components may be found via (6), where for the TEM mode ($n = 0$) the admittance dyadic becomes

$$\tilde{Y}_0 = \frac{1}{\pi} (\tilde{a}_y \tilde{a}_x - \tilde{a}_x \tilde{a}_y), \quad \pi = 120_n.$$  

Similar expressions are available for the TE10, and TE11 modes [8].

**Obstacle Polarizabilities**

For a perfectly conducting obstacle, the electric dipole moment is related to the integral of the surface charge. The magnetic dipole moment is due to the integral of circulating currents. For a nonperfect conductor the surface integrals must be supplemented by a volume integral. These integrals are difficult to analyze and various approximations have evolved. The simplest is Bethe's static approach [9]. For a perfectly conducting sphere of radius $r$ this method yields [11]

$$\varepsilon_e = 4\pi r \frac{3}{\varepsilon},$$

$$\mu_m = -2\pi r \frac{3}{\mu},$$

where $\mathbb{I}$ is the unit dyadic. Results also exist for other shapes (circular and square disks, etc.) including dielectric bodies [15]. Our measured data are limited to scattering from a conducting sphere.

**Incident Field Distribution**

We will assume that the scattering obstacle is illuminated by a forward propagating TEM mode as would be the case in a typical TEM cell test procedure. Thus the incident field is determined by (8).
TEM Cell Transmission Line Circuit

An empty TEM cell constitutes a length of transmission line and, therefore, may be represented by an equivalent circuit. The test object constitutes a load which also has an equivalent circuit representation once we have specified the reflected and transmitted TEM mode. A knowledge of the TEM mode expansion coefficients \(a_0\) and \(b_0\) leads immediately to the reflection \((R)\) and transmission \((T)\) coefficients via

\[
R = b_0, \quad T = 1 + a_0. \tag{11}
\]

The dipole idealization of the scattering obstacle implicitly requires that the associated scattering matrix \(S\) be symmetric

\[
S = \begin{pmatrix} R & T \\ T & R \end{pmatrix}. \tag{12}
\]

The scattering matrix \(S\) is related to the normalized impedance matrix \(Z\) by \(Z = (I + S)(I - S)^{-1}\), where \(I\) is the identity matrix.

The impedance matrix \(Z\) is normalized by the transmission line characteristic impedance, in this case \(Z_C\). The equivalent T-network shown in figure 3a follows from \(Z\) where the element impedances \(Z_a\) and \(Z_b\) are related to \(a_0\) and \(b_0\) according to

\[
Z_a = -Z_c \left( \frac{a_0 - b_0}{a_0 + b_0} \right), \tag{13}
\]

\[
Z_b = -Z_c \left( \frac{1}{a_0 + b_0} + \frac{1}{2} \right) - \frac{1}{2} Z_a. \tag{14}
\]

If \(a_0\) and \(b_0\) are small, which is the case for a small scattering obstacle, then \(Z_a\) and \(Z_b\) are approximately given by

\[
Z_a = -Z_c \left( \frac{a_0 - b_0}{a_0 + b_0} \right), \tag{13'}
\]

\[
Z_b = -Z_c \left( \frac{1}{a_0 + b_0} \right). \tag{14'}
\]

As the obstacle size vanishes, both \(a_0\) and \(b_0\) will go to zero. Therefore \(Z_a \to 0\) (short circuit) and \(Z_b \to \infty\) (open circuit) and the T-network reduces to a simple transmission line section as expected.

As an example, consider a spherical conducting obstacle. In this case the polarizabilities are given by \(10\). A substitution of \(10\) into \(4\) along with the TEM mode results \(8\) and \(9\) will generate the following expressions for \(a_0\) and \(b_0\)

\[
a_0 = -i 4\pi \frac{Z_C}{\eta_0} k_0 r \left( \frac{r}{a} \right)^2 F_{xy}, \tag{15}
\]

\[
b_0 = -i 12\pi \frac{Z_C}{\eta_0} k_0 r \left( \frac{r}{a} \right)^2 F_{xy},
\]

where the factor,

\[
F_{xy} = \sum_{m=1}^{\infty} \frac{\sinh M (b - py)}{\sinh Mb} \sin M x \sin Ma J_0 (Mg)^2, \tag{16}
\]

\[
+ \sum_{m=1}^{\infty} \frac{\cosh M (b - py)}{\sinh Mb} \cos M x \sin Ma J_0 (Mg) \sin M y \cos My J_0 (Mg) \sin My J_0 (Mg),
\]

depends on the TEM mode field at the dipole location. Assuming that the sphere is electrically small \((k_0 r << 1)\) and small compared to the cell dimension \(a\), we have from \(14\),

\[
Z_a = -Z_c 4\pi \frac{Z_C}{\eta_0} k_0 r \left( \frac{r}{a} \right)^2 F_{xy}, \tag{17}
\]

\[
Z_b = -Z_c \left\{ 16\pi \frac{Z_C}{\eta_0} k_0 r \left( \frac{r}{a} \right)^2 F_{xy} \right\}^{-1}.
\]

Thus we see that loading due to a spherical conductor consists essentially of a small series capacitance \(Z_a\) and a large shunt capacitance \(Z_b\).

The T-network representing the obstacle loading may be included in the overall transmission line network as shown in figure 3b. The test object is assumed to be centered in the RCTL section which has a length \(l_{\text{TAPERT}}\) and a characteristic impedance \(Z_c\). Ideally, the taper sections, of length \(l_{\text{TAPERT}}\), should have the same impedance as the RCTL section, however, in practice some deviation is expected. Therefore, the taper sections are shown explicitly and their characteristic impedance is denoted by \(Z_{\text{TAPERT}}\).

Tests were performed in a NBS TEM cell to assess the validity of the above TEM cell equivalent circuit representation. The cell dimensions are \(a = 15\) cm, \(b/a = 1\), and \(g/a = .17\). The taper and RCTL impedances were measured using a time domain reflectometer (TDR) which yielded \(Z_c = 49.5\) \(\Omega\) and \(Z_{\text{TAPERT}} = 51\) \(\Omega\). The cell was designed to have an impedance of 50 \(\Omega\). Cell impedances within \(2\) \(\Omega\) of the ideal 50 \(\Omega\) value are considered acceptable. The RCTL section has a length of \(l_{\text{RCTL}} = 30\) cm. The taper length is somewhat ambiguous since it may be measured along the center of the inner conductor (15 cm), along the outside of the taper (25 cm), or somewhere in between. Because this is a small gap cell, the characteristic impedance in the tapers should be dominated by the gap size. Therefore it seems appropriate to use the outside taper dimension, that is \(l_{\text{TAPERT}} = 25\) cm.

The cell was terminated with a 50 \(\Omega\) load and a spectrum analyzer was used to measure the input impedance. Three spherical conductors were used to load the cell. Figure 4 shows the magnitude of the cell's input impedance versus frequency. Theoretical curves (solid lines) based on \(17\) and our transmission line circuit (fig. 3b) are shown along with measured data (stars) for the empty cell case (no loading). At the low frequency end (less than 100 MHz), the 50 \(\Omega\) termination load dominates. As the frequency is increased both \(Z_c\) and \(Z_{\text{TAPERT}}\) become important. The curves basically agree up to around 400 MHz whereupon they diverge significantly. An important factor affecting the accuracy of our model is the stability of the various impedances. Although measured values for \(Z_c\) and \(Z_{\text{TAPERT}}\) are being used, the TDR results represent time averaged data, in this case for a spectrum reaching the 12 GHz range. At specific frequencies, the actual RCTL and taper characteristic
impedances may well deviate 1% or more from the TDR values. In practice this means a VSWR of less than 1.05, which is acceptable for normal cell usage. Against this criterion, the qualitative agreement found in figure 4 is quite satisfactory.

Figure 5 gives results for the same cell with a 4.35 cm conducting sphere introduced. The sphere is centered in the test chamber (i.e., $x = 0$ and $y/b = 0.5$) and occupies slightly less than a third (29%) of the test chamber height. The curves vary little from the empty cell case, except above 400 MHz where the measured data decrease somewhat. If we increase the diameter of the spherical load to 6.5 cm (43% of the chamber height) as shown in figure 6, then the frequency dependence of the upper frequency range ($>400$ MHz) is accentuated. However, below this range the deviation from the unloaded cell again remains small. These figures suggest that a cell's characteristic impedance should not be greatly affected except near the top end of the cell's usable frequency range. For this particular cell, the first resonances associated with the TE$_{01}$ and TE$_{10}$ modes appear just beyond 800 MHz, and 600 MHz respectively. Therefore, the use of this cell would normally be below 400 MHz, the range where our model gives the best results. Finally, a 10 cm diameter conducting sphere was introduced and the results are shown in figure 7. This sphere occupies 67% of the test chamber height, and as may be seen the loading effect is significant when compared to the empty cell. This sphere did excite the TE$_{01}$ cavity resonance just above 400 MHz, beyond which our transmission line model no longer applies and the poor agreement is expected.

Taken together, these curves demonstrate that small obstacle theory gives a reasonable description for the loaded TEM cell. Below the first cell resonance, the model predicts well both the frequency dependence and the magnitude of the impedance variation due to loading. In addition, our theory supports the common practice of ignoring loading effects if the test object size is kept less than a third of the chamber height. These results should prove useful in predicting whether a particular measurement will be compromised by the impedance change due to loading.

Small Obstacle Field Scattering

A second concern to TEM cell users is whether the field distribution is altered by the test object presence. We will consider the RCTL section only, as if it were infinite, and again use a centrally located $(x = 0, \ y/b = 0.5)$ spherical conductor as the scattering obstacle. The RCTL modes will be excited according to (1) with the expansion coefficients determined by (4). The incident field is again assumed to be a forward traveling TEM mode, thus $a_0$ and $b_0$ are given by (15). The scattered TE$_{01}$ and TE$_{11}$ modes follow from (1) and (4) with the appropriate field expressions inserted [8]. The previous section indicates that the TE$_{01}$ mode will not be important. We will use a sphere of radius $r/b = 0.2$ for our numerical study. All other parameters will be chosen as in the previous examples.

Rather than examine the field distribution in any specific plane, we will investigate the perturbation to the primary TEM mode field components, $E_y$ and $H_x$, at the center of the test chamber cross section (i.e., the sphere location) as a function of $z$, the longitudinal distance away from the scattering sphere. The wave impedance $E_y/H_x$ will also be plotted. Each parameter will be normalized by its unperturbed (empty cell) value, thus unity represents a zero perturbation. Figure 8 shows numerical results for the above described analysis, with the frequency chosen such that $k_0a = 1$. At this frequency only the TEM mode propagates. As may be seen, there is no perturbation in the forward direction $(z > 0)$. However, some change is apparent for $z < 0$, caused by the greater backscatter coefficient of the TEM mode $(|b_0| > |a_0|)$ and the phase beating between the incident and reflected TEM modes. If we examine the relative phase of the evanescent higher-order modes, we find they do not contribute significantly in either direction. Thus we may conclude that the loading primarily affects the field distribution via the scattered TEM mode. Figure 8 also shows that for $z < 0$ the $E_x$ component is reduced while $H_x$ is enhanced and consequently the wave impedance $E_y/H_x$ shows the greatest deviation.

If we increase the frequency, and thereby the electrical cross section of the sphere, then we expect a greater scattering effect. Figure 9 shows the results for $k_0a = 1.4$ which lies in the TE$_{10}$ cutoff frequency $(k_0a = \pi/2)$. The same comments as for figure 8 apply in the backward $(z < 0)$ direction with the perturbations accentuated. In addition, we also see a definite change in the fields forward $(z > 0)$ of the scattering sphere. Both $E_y$ and $H_y$ are increased yielding essentially the same wave impedance. The interference between the incident and forward scattered TEM modes is less pronounced than in the backward direction. Again the variations are essentially due to the scattering of the TEM mode alone.

Conclusions

This paper has applied small obstacle theory to evaluating the effect of a test object on the TEM cell environment. In particular, we have examined the transmission line loading and the field scattering due to a conducting sphere. Results suggest that the field distribution inside the cell is more significantly affected by the presence of a test object than are the cell's transmission line characteristics. In each case, the present analysis may be used to assess whether loading should be of concern in a given measurement task involving a TEM cell.

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References


Figure 1. A TEM cell.

Figure 2. The cross section of an RCTL.

Figure 3. TEM cell equivalent circuit representations.

Figure 4. Input impedance vs. frequency for the empty cell.
Figure 5. Input impedance vs. frequency for the cell with a 4.35 cm diameter conducting sphere.

Figure 6. Input impedance vs. frequency for the cell with a 6.50 cm diameter conducting sphere.

Figure 7. Input impedance vs. frequency for the cell with a 10 cm diameter conducting sphere.

Figure 8. Field perturbation due to a conducting sphere of diameter 6.0 cm with $k_0a = 1.0$.

Figure 9. Field perturbation due to a conducting sphere of diameter 6.0 cm with $k_0a = 1.4$. 