A Review of Dipole Models for Correlating Emission Measurements made at Various EMC Test Facilities

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Abstract - In this paper we review dipole models for correlating measurements made at various electromagnetic compatibility (EMC) test facilities. We will present expressions for the maximum received voltage (at some measurement port) generated by a dipole radiator placed in free-space, an ideal half-space environment, a transmission line, and an over-modulated cavity. These different environments correspond to commonly used EMC test facilities, namely, a fully anechoic chamber (FAC), an open area test site (OATS) or a semi-anechoic chamber (SAC), a transverse electromagnetic (TEM) cell or stripline, and a reverberation chamber. These dipole models can then be used as the basis for correlating between emission measurements made at different facility types. Most of these expressions have appeared in previous publications. The purpose of this paper is to present these results in a review having consistent terminology and notation.

1. INTRODUCTION

Various types of EMC facilities have been developed over the years to meet a variety of test needs. EMC facilities range from indoor sites to outdoor sites, small volumes to large volumes, and from those possessing precise, well defined fields to those having complex, statistically defined fields. To date, emission standards have been based primarily on the use of the OATS and SAC (e.g., CISPR22, FCC Part 15, Subpart J). However, there is a trend toward additional independent emission and immunity standards that are based on other test facilities. IEC 61000-4-20, covering TEM cell testing, and IEC 61000-4-21, covering reverberation chamber testing, have recently been published. Work is underway in the IEC to develop an independent standard covering FAC testing. This variety of test methods gives product committees and others the flexibility to choose a test facility that best fits their needs.

In principal, emission results based on an independent standard do not need to be correlated to other methods. However, a reasonable goal is to set emission limits for each method that test products equivalently. Alternatively stated, a product that passes in one type of EMC test facility should also pass at another. Likewise, a product that fails in one type of EMC test facility should fail at another. Dipoles are the simplest emitters to model and thus dipole emissions are the simplest to correlate between different test facilities. Clearly, real test objects may have emission patterns that differ from that of a dipole. This is especially true for electrically large test objects. However, the dipole models reviewed in this paper are a good starting point for setting “equivalent” test limits.

Dipole-based correlation algorithms have already been developed and have appeared in the literature [1-4]. However, different terminologies and notations are used, which can make comparisons difficult. In this paper, using consistent terminology and notation, we will present the various expressions needed for correlating dipole emission measurements. We hope that this will serve as a convenient reference for the interested reader.

2. FREE SPACE (FS)

An ideal free space environment is the simplest case to analyze and represents that which a FAC seeks to replicate. We will consider only the case of an electric dipole. The results can be extended in a straightforward manner to a magnetic dipole or combined electric and magnetic dipoles, as will be discussed later. For a short electric dipole (length $d$, peak current $I_0$) located at the origin and aligned with the z-axis, the far-field radiation is given by

$$ E_\theta = \frac{j\mu_0 I_0 d \sin \theta \ e^{-ikr}}{4\pi r} \ (V/m) $$

$$ H_\phi = \frac{j\kappa I_0 d \sin \phi \ e^{-ikr}}{4\pi r} \ (A/m), $$

where $\omega$ is the angular frequency, $\mu$ is the permeability of the medium in which the dipole is located (e.g., air), $k = 2\pi / \lambda$, where $\lambda$ is the wavelength, $r = (\theta, \phi, r)$ represents the usual spherical coordinate system, and a $\phi$ time convention is suppressed. The total power $P_0$ radiated by the electric dipole is found by integrating the Poynting vector over a sphere enclosing the dipole, resulting in

$$ P_0 = \frac{2}{3} \eta I_0^2 \left( \frac{d}{\lambda} \right)^2 \ (W), $$

where $\eta$ is the intrinsic impedance of the medium ($\approx 120\pi \Omega$ for air). EMC measurements made in a FAC typically measure the electric field. For an emission measurement, the maximum electric field magnitude $E_{max}$ would be sought over some scan...
geometry. For the above electric dipole geometry, the maximum in (1) occurs when $\theta = \pi/2$, where

$$E_{\text{max}}^2 = \frac{\alpha^2 \mu_0^2 (U0d)^2}{(4\pi r)^2}. \quad (3)$$

Using (2), the above expression may be rewritten as

$$E_{\text{max}}^2 = \frac{3 \eta}{2 \cdot 4\pi^2} P_0. \quad (4)$$

This expression is the electric dipole case of the more general expression given in [1],

$$E_{\text{max}}^2 = D_{\text{max}} \frac{\eta}{4\pi^2} P_0. \quad (5)$$

for the maximum electric field from an emitter with maximum directivity $D_{\text{max}} = \pi/2$ for an electric or magnetic dipole) and total radiated power $P_0$. $E_{\text{max}}$ is actually measured as a voltage $V_{\text{max}}$ at the output of an antenna with an antenna factor AF, where $V_{\text{max}} = AF E_{\text{max}}$. Collecting results, we have

$$V_{\text{max}}^2 = AF^2 D_{\text{max}} \frac{\eta}{4\pi^2} P_0. \quad (6)$$

We will rewrite this further by defining a propagation-loss term $PL_{\text{FS}} = 1/(4\pi r^2)$ giving

$$V_{\text{max,FS}}^2 = \eta (AF_{\text{FS}}^2 D_{\text{max,FS}} PL_{\text{FS}}) P_0. \quad (7)$$

Subscripts (FS) have been added to differentiate between later cases. In particular, $AF_{\text{FS}}$ is not an inherent property of free space; rather, it simply denotes the AF of whatever antenna is used to make a free space measurement.

3. HALF SPACE (HS)

A half space is formed by introducing an ideal ground plane (a perfect conductor of infinite extent) into the free-space case and represents the test environment that an OATS or SAC seeks to replicate. An electric dipole is here located at a height $h$ above the ground plane and oriented either vertically or horizontally. Other orientations may be analyzed as the superposition of these two cases. The half-space case may be analyzed by introducing an image dipole. We introduce a rectangular coordinate system with the ground plane in the x-y plane, the dipole at $z = +h$, and the image dipole at $z = -h$. The distance from the source dipole to the measurement point is designated $r_1$, from the image dipole to the measurement point $r_2$ from the origin to the measurement point $r$, and the radial distance from the z-axis to the measurement point is designated $\rho$. In the far-field, the maximum electric field can be shown to be [2, 4]

$$E_{\text{max}}^2 = D_{\text{max}} \frac{\eta}{4\pi^2} g_{\text{max}}^2 P_0, \quad (8)$$

where $D_{\text{max}}$ is again $\pi/2$ for a single electric dipole. The geometry factor $g_{\text{max}}$ is defined by

$$g_{\text{max}} = \left[ \begin{array}{c} r_1 e^{-j\rho_1} - r_2 e^{-j\rho_2} \\ r_1 \\ r_2 \\ \rho_1 \\ \rho_2 \end{array} \right]_{\text{max}} \quad \text{for horiz.}$$

$$g_{\text{max}} = \left[ \begin{array}{c} \rho_1^2 r_1 e^{-j\rho_1} + \rho_2^2 r_2 e^{-j\rho_2} \\ r_1 \\ r_2 \end{array} \right]_{\text{max}} \quad \text{for vert.}$$

where the subscript max denotes the maximum value found over some scan geometry (e.g., a 1 to 4 m vertical height scan at some horizontal offset). We have introduced a normalizing factor $r$ into (9) (versus the expressions found in [2, 4]) to give (8) a form similar to that of (5). If the radial distance $\rho$ is significantly larger than the height above the ground plane $h$, such that $\rho/r_1 = 1$, $\rho/r_2 = 1$, $r/r_1 = 1$, and $r/r_2 = 1$, then $g_{\text{max}}$ reduces to

$$g_{\text{max}} = \left[ \begin{array}{c} 2 \sin k(r_1 - r_2)_{\text{max}} \\ 2 \cos k(r_1 - r_2)_{\text{max}} \end{array} \right] \text{horizontal and vertical} \quad (10)$$

Under these assumptions, $g_{\text{max}} = 2$ in both cases, assuming the measurement scan is such that, at some point, $k(r_1 - r_2)_{\text{max}} = \pi/2$ or $\pi$ respectively (or some suitable multiple). Physically, this simply means that the ground plane doubles the maximum electric field through constructive interference due to the reflected path. Figure 3 in [4] shows that $g_{\text{max}} = 2$ is a good approximation above 200 MHz for a typical 3 m EMC emission test (3 m horizontal separation, 1 m source height, 1 to 4 m measurement height scan) and is a good approximation above 30 MHz for a typical 10 m EMC emission test (10 m horizontal separation, 1 m source height, 1 to 4 m measurement height scan). Thus, a value of $g_{\text{max}}$ equal to 2 should be sufficient for setting equivalent EMC test limits and will be used here for the half-space case. Thus, the maximum measured voltage for an electric dipole above a perfect ground plane can be approximated as

$$V_{\text{max,HS}}^2 = \eta (AF_{\text{HS}}^2 D_{\text{max,HS}} PL_{\text{HS}}) P_0, \quad (11)$$

where $PL_{\text{HS}} = 4/(4\pi r^2)$ for the half-space case. However, if the ground-plane geometry needs to be accounted for more accurately, we can use $PL_{\text{HS}} = g_{\text{max}}^2/4(4\pi r^2)$.

4. TEM TRANSMISSION LINE (TL)

A TEM transmission line (stripline, TEM cell) seeks to approximate a linearly polarized plane-wave field over some test volume. A dipole placed in a TEM line will couple to the TEM mode and to higher-order modes, if present, and produce a voltage at the measurement port. This measured voltage
combined with suitable rotations of the dipole (similar in
case to the above receive antenna scans) can be used to
determine the dipole moment [1] or the total radiated power
from the dipole [1, 4]. For example, in IEC 61000-4-20 [5], \( P_0 \)
for an electric dipole is given as

\[
P_0 = \frac{2\pi}{\kappa} \frac{k^2}{\epsilon_0^\prime Z_0} S_{\nu}^2,
\]

where \( Z_0 \) is the characteristic impedance of the TL (typically
50 \( \Omega \)), \( \epsilon_0 \) is a normalized field factor, \( \epsilon_0^\prime \) is Z_0 / \( \kappa \), where
\( \kappa \) is the plate separation at the dipole location, and \( S_{\nu} \)
represents the sum of the measured output port voltage over a
set of dipole rotations. These expressions ignore contributions
from higher-order modes. If we simply orient the dipole for
maximum coupling, as in the previous sections, then the
voltage at the measurement port will be maximized:

\[
S_{\nu}^2 = V_{\text{max}}^2 \quad (\text{no rotations needed}),
\]

where we have substituted for \( \epsilon_0^\prime \). Replacing 3/2 with \( D_{\text{max}} \),
this may be rewritten as

\[
V_{\text{max}}^2 = \left( \frac{\epsilon_0}{\kappa} \right)^2 \frac{Z_0^2}{d^2} P_0.
\]

Comparing this result to (6), we see that (14) defines an
equivalent antenna factor for a TEM cell (the term in
parenthesis). The distance \( r \) is interpreted as the radial distance
to the test volume. In a uniform cross section TEM line (e.g., a
standard two-port TEM cell tapered at each end) \( r \) is the radial
distance from the measurement port along the cell taper to the
location of the dipole projected back to the plane beginning
the uniform section of the transmission line. In a constant flare
TEM line (e.g., GHz TEM cell) \( r \) is simply the radial distance
from the measurement port to the dipole location. The
equivalent antenna factor is then given by

\[
AF_{\text{TL}} = \frac{Z_0}{\kappa} \frac{r}{d}.
\]

This expression is consistent with a similar equivalent gain
factor for TEM lines derived using power expressions [6].
Using this definition, we again have

\[
V_{\text{max,TL}}^2 = \eta (AF_{\text{TL}}^2 D_{\text{max,TL}} P_{\text{TL}}) P_0,
\]

where \( PL_{\text{TL}} \) is defined the same as \( PL_{\text{FS}} \) but using \( r \) as defined
in this section.

5. REVERBERATION CHAMBER (RC)

A reverberation chamber is an over-moded cavity that seeks to
statistically approximate a uniform set of plane waves over
some test volume. The ideal set of plane waves would include
all directions and polarizations. A good reverberation chamber
approaches this ideal. We can show that the average power
\( \langle P_r \rangle \) (averaged over multiple modal distributions) received by
a matched, lossless reference antenna due to a source in the
cavity is given by [7]

\[
\langle P_r \rangle = \frac{\lambda^4 Q}{16\pi^2} \frac{V}{P_0},
\]

where \( Q \) is the quality factor of the chamber, \( V \) is the volume
of the chamber, and as before, \( P_0 \) is the total radiated power
from the source, which is an electric dipole in this case. The
difficulties with (17) are that \( Q \) is often not well characterized
and we need to correct for antenna-loss effects. To avoid these
difficulties, the more usual method of determining \( P_0 \) in a
reverberation chamber is to make a comparative measurement
with a reference source of known power \( P_{\text{ref}} \) while keeping
the chamber conditions the same: then

\[
P_0 = \frac{P_{\text{ref}}}{\langle P_r \rangle} \langle P_r \rangle.
\]

Solving for \( \langle P_r \rangle \), and rewriting this in terms of the average
received voltage yields

\[
\langle V_r \rangle = \frac{Z_c}{P_{\text{ref}}} \langle P_r \rangle.
\]

where \( P_t = V_r^2 / Z_c \) (\( Z_c \) is the impedance at the antenna
measurement port, typically 50 \( \Omega \)). In this case there is no \( D_{\text{max}} \),
subscript since there is no need to scan the receive antenna or
rotate the test object. There is also no directivity term as this is
negated by averaging over incidence angles and polarization.
For consistency with previous expressions, we define the
following for the reverberation chamber case: \( D_{\text{max,RC}} = 1 \) (no
directivity), \( V_{\text{max,RC}}^2 = \langle V_r \rangle^2 \) (the receive antenna scan in
previous methods is replaced by averaging over the modal
distribution variations), \( AF_{\text{RC}}^2 = sZ_c / \eta \) (\( s = 1 \) m is included
to give consistent units), and

\[
PL_{\text{RC}} = \frac{1}{s} \frac{\langle P_r \rangle}{P_{\text{ref}}}.
\]

We then have

\[
V_{\text{max,RC}}^2 = \eta (AF_{\text{RC}}^2 D_{\text{max,RC}} P_{\text{RC}}) P_0.
\]
Note that $V_{\text{max,RC}}$ does not denote the maximum voltage measured over the multiple modal distributions; it denotes the average, as defined in eq. (20) above.

6. CORRELATION

We now have expressions (7, 11, 16, 21) for the received voltage from an electric dipole located in four different test environments: free space (FS), half space (HS), a TEM line (TL), and a reverberation chamber (RC). Assuming the total radiated power from the dipole is the same in each case we can form the ratios of these expressions to correlate between EMC test facilities,

$$\frac{V_{\text{max,A}}^2}{V_{\text{max,B}}^2} = \frac{AF_A^2}{AF_B^2} \frac{D_{\text{max,A}}}{D_{\text{max,B}}} \frac{PL_A}{PL_B}$$

(22)

where A and B can be any combination of FS, HS, TL and RC.

The above expression (22) is based on consideration of an electric dipole. However, the identical form could be used for a magnetic dipole ($D_{\text{max}} = 3/2$), a combination of an electric and magnetic dipole ($D_{\text{max}} = 3$), or, in a general sense, for test objects having directivities other than these dipole values. If we are not correlating to a reverberation chamber, then the directivity ratio is unity regardless of the test object. For the correlation to a reverberation chamber where $D_{\text{max,RC}} = 1$, $D_{\text{max}}$ for a test object in the other facility must be either known (typically not the case) or must be estimated, as was done in [8], based on the electrical size of the test object. The correlation parameters are summarized in Table 1. The terms used in Table 1 may be found in the appropriate sections of the paper.

<table>
<thead>
<tr>
<th>EMC Test Facility</th>
<th>AF</th>
<th>$D_{\text{max}}$</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS (FAC)</td>
<td>receiving antenna</td>
<td>3/2 for dipole or see [8]</td>
<td>$1/(4\pi r^2)$</td>
</tr>
<tr>
<td>HS (SAC, OATS)</td>
<td>receiving antenna</td>
<td>3/2 for dipole or see [8]</td>
<td>$4/(4\pi r^2)$ or $R_{\text{max}}$ (9)</td>
</tr>
<tr>
<td>TL (TEM Cell, GTEM Cell, striplines)</td>
<td>$\frac{Z_0 r}{\eta d}$</td>
<td>3/2 for dipole or see [8]</td>
<td>$1/(4\pi r^2)$</td>
</tr>
<tr>
<td>RC (reverberation chamber)</td>
<td>$\frac{Z_c}{\eta}$</td>
<td>1 for all emitters</td>
<td>$\frac{1/P_{\text{ref}}}{1/P_{\text{ref}}}$</td>
</tr>
</tbody>
</table>

Table 1

REFERENCES