Reverberation Chamber Relationships: Corrections and Improvements

or

Three Wrongs Can (almost) Make a Right

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Abstract: We correct a number of errors that have crept into the reverberation-chamber literature over the past several years. These include errors in equations and interpretation of data. Wherever possible, we identify the causes and implications of these errors. We pay particular attention to the so-called 4π/8π controversy where measurements appear to disagree with theory by approximately 3 dB. We also compare the power received by an antenna in an anechoic chamber and a reverberation chamber and give approximate equivalence relationships for comparison of such facilities.

INTRODUCTION

We use some recent advances in electromagnetic and statistical theory of reverberation chambers to identify and correct some particularly persistent errors that have been around for over 13 years. In our research, we have found that one of the challenges of working with reverberation chambers is that it is easy to obtain incorrect results that appear plausible. Compound this problem with the fact that, until recently, the theory of reverberation chambers has been significantly lacking, and the potential for error is staggering. Fortunately, the theory of reverberation chambers has advanced to the point that common errors can be detected and corrected.

One error, related to the relationship between measurements of received power and total electric field, has been especially difficult to eliminate. This is because it is usually accompanied by other errors, and the combination gives a result that is surprisingly close to the correct answer. Thus it is not much of a problem in typical experiments (what's the problem as long as my answer is close?) but some of these errors can have subtle implications. For these reasons, we will step through the incorrect derivation and the correct one in the hopes of laying this problem to rest once and for all.

We also show how the variability of reverberation chamber data can cause problems, and even plotting data on a decibel scale can cause surprising results.

We also address some problems and errors in comparing measurements in reverberation chambers with measurements in anechoic chambers. Additionally, we discuss some of the problems caused by the fact that the term electric field has two distinct meanings: total electric field and a rectangular component of the electric field. This distinction is generally unimportant in plane-wave facilities such as anechoic chambers but is very important in reverberation chambers. We give our opinion as to the best way to compare these facilities.

NOTATION

First, we must introduce some notation. We use the symbols 〈 〉 to indicate an ensemble average. Thus, 〈X〉 is the ensemble average or expected value of X. If we average a finite number N of samples of X, we will write this as 〈X〉N. Using this notation, 〈X〉 is shorthand notation for 〈X〉N. Similarly, we use the notation 〈X〉N to indicate the maximum observed value out of a set of N samples of X. The ratio of the maximum observed value out of a set of N samples of X to the average of N samples of X (the maximum-to-average ratio) is useful enough to warrant its own shorthand notation. We use the symbol 〈X〉N to indicate the maximum-to-average ratio of N samples of some parameter X. For example, the maximum to average ratio of 225 samples of the magnitude of the total electric field E_T (see why we need a shorthand notation?) can be written as 〈225 〈E_T〉〉.

HOW MANY 7 TS WOULD YOU LIKE WITH THAT?

We begin with one of the most fundamental relationships of reverberation chambers: the relationship between the average power 〈P_R〉 received by an ideal antenna in a reverberation chamber and the average squared magnitude of the total electric field 〈|E_T|^2〉.

The relationship given by Hill [1] is

\[ 〈|E_T|^2〉 = 〈P_R〉 \cdot \frac{8\pi\eta}{\lambda^2}, \]

(1)

where \( \lambda \) is the wavelength of the driving cw signal and \( \eta \) is the wave impedance of the medium (usually free space). There has been some controversy over whether the numerator of (1) should be 4\pi\eta or 8\pi\eta (the 4\pi / 8\pi controversy). Electromagnetic theory [1] clearly supports the use of 8\pi\eta (a factor of 4\pi\eta for an isotropic environment, and a factor of 2 to account for polarization mismatch). We can confirm this experimentally by comparing measurements of...
Figure 1. Comparison of measurement of average squared total electric field and two possible predictions based on measurements of average received power. 

\[ \langle |E_T|^2 \rangle \text{ with predictions based on equation (1) and measurements of } \left( \langle P_R \rangle \right). \text{ The results are shown in Figure 1.} \]

The data clearly support equation (1) as is, including the \( 8\pi \eta \) term. Given the uncertainty associated with the measurements and indicated by the error bars, it is unlikely that the \( 4\pi \eta \) term is correct. Great! The problem is solved. But where did the controversy originate in the first place?

The source of the problem is in the manipulation of equation (1) to give an estimate of the maximum total electric field \( |E_T|_N \). Using simple (and incorrect) arguments, the relationships were developed as follows. First, since we are interested in the magnitude rather than the squared magnitude of the total electric field, we should take the square root of both sides of equation (1):

\[ \langle |E_T| \rangle = \sqrt{\langle P_R \rangle \cdot \frac{8\pi \eta}{\lambda^2}}. \quad ** \text{WRONG} ** (2) \]

Second, to estimate \( |E_T|_N \), we can simply replace the average received power \( \langle P_R \rangle \) in equation (2) with the maximum received power \( \langle P_R \rangle_N \):

\[ |E_T|_N = \sqrt{P_R_N \cdot \frac{8\pi \eta}{\lambda^2}}. \quad ** \text{WRONG} ** (3) \]

We have written "**WRONG " to indicate that the associated equation should not be used, and the reasons will be discussed below. Equation (3) is similar to equation (10) given by Crawford and Kopec in NBS Technical Note 1092 [2] with the exception that they chose to use \( 4\pi \eta \) rather than \( 8\pi \eta \) in the numerator because it appeared to show better agreement with measurements. If we now plot measurements of \( |E_T|_N \) along with estimates based on equation (3) and measurements of \( P_R_N \), as shown in Figure 2, we are in for a surprise. Now the data clearly agree better with the \( 4\pi \eta \) assumption than the \( 8\pi \eta \) assumption.

So what went wrong? To find out, we need to go back to equations (2) and (3) and carefully examine each step. For equation (2), since both sides of equation (1) are nonnegative, it should be valid to take the square root of each side of the equation. However, there is one step in deriving equation (2) that is not specifically spelled out, and that is the source of one error: the assumption that the order of computing the averaging and taking the square root can be interchanged, that \( \sqrt{\langle |E_T|^2 \rangle} \) is the same as \( \langle |E_T| \rangle \). We can experimentally verify that these are not the same. Taking our measurements of the squared total electric field, we can process the measurements both ways and see that indeed these two processes give different results. For our measurements, the difference is consistent but small (approximately 4 percent or 0.36 dB). Such an error is obviously too small to cause our discrepancy of 3 dB and is probably small enough to ignore. However, for completeness, we need to correct equation (2). To do so requires some information about the statistical distribution of the squared magnitude of the total electric field. Fortunately, this distribution is known to be a \( \chi^2 \) (chi-square) distribution with six degrees of freedom (dof). This can also be written as a \( \chi^2_9 \) distribution. Based on work in reference [3], we can write the correct form of equation (2) as

\[ \langle |E_T| \rangle_N = \sqrt{\left( \frac{P_R}{\lambda^2} \right) \cdot \frac{8\pi \eta}{\lambda^2}}. \quad (4) \]

where \( \Gamma(X) \) is the gamma or factorial function evaluated at \( X \). The fact that equation (4) depends on the number of samples \( N \) might be surprising. However, we have already seen that for our data \( (N = 225) \) we need a correction factor of 4 percent. For \( N = 1 \), though, no correction is needed because \( \sqrt{\langle |E_T|^2 \rangle} \) is the same as \( \langle |E_T| \rangle \). Thus, the correction factor must be a function of \( N \).
Regardless, simply assuming that $\sqrt{\langle |E_T|^2 \rangle}_N$ and $\langle |E_T| \rangle_N$ are equal for all $N$ will result in an error of less than 0.36 dB for any value of $N$. Also, the gamma function correction is also always less than 0.36 dB, less than 0.1 dB for $N \geq 4$, and less than 0.01 dB for $N \geq 37$.

Since we have established that there is an error in equation (2), and have also established that the error is not large enough to account for the 3 dB error shown in Figure 2, then the remainder of the error must be related to equation (3). The primary assumption made in equation (3) is that if equation (2) gives a relationship that is approximately accurate for the average total electric field and the average received power, then it should also be reasonably accurate if we replace the averages with maximums. This is equivalent to assuming that the maximum-to-average ratio of the total electric field $\hat{N} \langle |E_T| \rangle$ is approximately the same as the square root of the maximum-to-average ratio of the received power $\sqrt{\hat{\gamma}_N} (P_R)$. From reference [3], however, we know that these ratios are different because the distributions are different. To demonstrate that these values are indeed different, we computed these ratios based on measurements of $|E_T|$ and $P_R$ taken at 124 different frequencies and 225 paddle positions ($N = 225$). The results are plotted in Figure 3.

On average, $\hat{\gamma}_{225} (P_R) = 6$, so $\sqrt{\hat{\gamma}_{225}} (P_R) = \sqrt{6} \approx 2.45$, and $\hat{\gamma}_{225} (E_T) = 1.9$. Thus, any attempt to estimate the maximum total electric field based on equation (3) and the maximum received power will result in an overestimate of approximately 30 percent or 2.2 dB.

If we examine the combined effect of each error: using $4\pi \eta$ instead of $8\pi \eta$ in equation (1), ignoring the correction factor in equation (2), and replacing averages with maximums to give equation (3), the combined error factor is $\frac{1}{\sqrt{2}} \approx 1.04 \cdot 13 = 0.956$. The combination of these three errors gives a result that is within 5 percent or 0.4 dB of the correct result. A few more errors and the result might have been exact!

**The Other Electric Field**

![Graph showing the maximum-to-average ratios of $|E_T|$ and $\sqrt{\hat{\gamma}_{225}} (P_R)$ vs. frequency.](image)

**Figure 3.** $\hat{\gamma}_{225} (|E_T|)$ and $\sqrt{\hat{\gamma}_{225}} (P_R)$.

So far, we have examined three parameters: $|E_T|^2$, $|E_T|$ and $P_R$. Notice we mention $|E_T|^2$ and $|E_T|$ separately since they have unique characteristics that cannot be converted easily from one to the other. Another useful parameter is the magnitude of a rectangular component of the electric field $|E_R|$. This is the component measured by a dipole probe and is often referred to as $|E_X|$, $|E_Y|$, or $|E_Z|$. These are related to the total electric field by the equation $|E_T| = \sqrt{|E_X|^2 + |E_Y|^2 + |E_Z|^2}$. We use $|E_R|$ rather than $|E_X|$, $|E_Y|$, or $|E_Z|$ because all rectangular components have been shown to have the same distribution [3].

**Maximum-to-Average Ratios**

The maximum-to-average ratios of $|E_T|$ and $P_R$ have proven to be valuable quantities. We tabulate approximate values for these and other key parameters in Table 1. The derivation of these values is too complicated to give in the space provided, but the details can be found in reference [3].

In Table 1, we take advantage of the fact that $P_R$ and $|E_R|^2$ are from the same family of distributions, as are $\sqrt{P_R}$ and $|E_R|$ and therefore these have the same maximum-to-average ratios. The values presented here are only approximations. The ratio of the maximum of a set of data to the average of that set is typically noisy. We present the expected value of the maximum of $N$ samples divided by the expected value of the $N$ samples, assuming that the maximum and average are independent. This is not quite the same thing as the expected value of the ratio of the sample maximum to the sample

**Table 1. Approximate maximum-to-average ratios for typical measurement parameters.**

| $N$ | $|E_T|^2$ | $|E_T|$ | $P_R$ or $|E_R|^2$ | $\sqrt{P_R}$ or $|E_R|$ |
|-----|----------|--------|----------------|------------------|
| 1   | 1.000    | 1.000  | 1.000          | 1.000            |
| 2   | 1.313    | 1.166  | 1.500          | 1.293            |
| 3   | 1.499    | 1.253  | 1.833          | 1.456            |
| 4   | 1.630    | 1.311  | 2.083          | 1.567            |
| 5   | 1.732    | 1.354  | 2.283          | 1.650            |
| 6   | 1.815    | 1.388  | 2.450          | 1.715            |
| 7   | 1.885    | 1.416  | 2.593          | 1.770            |
| 8   | 1.945    | 1.439  | 2.718          | 1.816            |
| 9   | 1.998    | 1.460  | 2.829          | 1.856            |
| 10  | 2.046    | 1.478  | 2.929          | 1.891            |
| 20  | 2.352    | 1.588  | 3.598          | 2.110            |
| 50  | 2.746    | 1.719  | 4.499          | 2.371            |
| 100 | 3.036    | 1.810  | 5.187          | 2.552            |
| 200 | 3.321    | 1.894  | 5.878          | 2.721            |
| 225 | 3.369    | 1.908  | 5.996          | 2.748            |
| 400 | 3.602    | 1.974  | 6.570          | 2.879            |
| 500 | 3.691    | 1.998  | 6.793          | 2.929            |
| 800 | 3.878    | 2.049  | 7.262          | 3.030            |
| 1000| 3.967    | 2.072  | 7.485          | 3.077            |
average because the sample average and sample maximum are not independent (if you know the average, you know the maximum must be greater than or equal to the average). Even so, the approximations are quite good. The values in Table 1 are calculated this way because it is much easier than calculating the expected value of the true maximum-to-average ratio. Because of the complexity of calculating the true values, we do not know the error introduced by the approximations, but numerous Monte Carlo simulations indicate that the error is small relative to the uncertainty in the experiment. Notice that means listed in Figure 3 were predicted quite accurately by Table 1. Specifically, \( \mathbb{E}[|E_T|] \) had an average value of 1.9 and Table 1 predicts 1.908. Similarly, \( \sqrt{\mathbb{E}[P_R]} \) had an average value of 2.45, and Table 1 predicts \( \sqrt{5.996} = 2.45 \).

The data given in Table 1 were calculated based on the assumption that the parameters can be described by ideal distributions given in reference [3], and that all samples are independent. These assumptions are known to be invalid in certain situations:

1. The distribution for the received power is incorrect if average chamber losses are small (less than 10 dB). This typically occurs only at low frequencies, and therefore these techniques should not be used at low frequencies.
2. The samples might not be independent if only small changes are made to the system, such as small changes in paddle position, frequency, or location.

For details on these limitations, as well as additional limitations, see reference [3].

A BETTER WAY

We have shown that the "wrong" way to estimate the maximum total electric field gives reasonably good results, but we have not yet given a "right" way. Now, armed with the information in Table 1, we are able to recommend some improved methods for estimating field parameters in a reverberation chamber. We can easily derive expressions for \( \left[ |E_T|^2 \right]_N \) from equation (1) and \( \left[ |E_T| \right]_N \) from equation (4):

\[
\left[ |E_T|^2 \right]_N = \mathbb{E}[|E_T|^2] \cdot \Phi_N(|E_T|^2)
\]

(5)

and

\[
\left[ |E_T| \right]_N = \mathbb{E}[|E_T|] \cdot \Phi_N(|E_T|)
\]

(6)

The improved estimate given in equation (6) is plotted in Figure 4. If we compare this estimate with the "4\( \pi \)" estimate given in Figure 2, we see that the new estimate not only agrees better with

![Figure 4. Comparison of measurement of maximum total electric field and improved predictions based on measurements of average received power and number of samples.](image)

measurements on average, but the variability of the measurement as a function of frequency is also reduced.

Although it is possible to derive these equations as a function of \( \mathbb{E}[P_R] \) rather than \( \mathbb{E}[P_R] \), such a derivation will generally be noisier (have a greater uncertainty) than that presented here. In fact, if you want to estimate \( \left[ |E_T|^2 \right]_N \) from measurements of \( \mathbb{E}[P_R] \), you first have to estimate \( \mathbb{E}[P_R] \) by dividing \( \mathbb{E}[P_R] \) by \( \Phi_N(P_R) \), so you are much better off simply computing \( \mathbb{E}[P_R] \) directly.

We can also generate equations for \( \left[ |E_R|^2 \right]_N \) and \( \left[ |E_R| \right]_N \).

Since \( |E_T|^2 = |E_X|^2 + |E_y|^2 + |E_z|^2 \), and since each rectangular component is assumed to be drawn from the same distribution, we can equate the averages \( \mathbb{E}[|E_T|^2] = 3\mathbb{E}[|E_R|^2] \). From equation (1) we can write

\[
\mathbb{E}[|E_R|^2] = \frac{\mathbb{E}[|E_T|^2]}{3} = \mathbb{E}[P_R] \cdot \frac{8\pi n}{3\lambda^2},
\]

(7)

and from this, we can write

\[
\left[ |E_R|^2 \right]_N = \mathbb{E}[|E_R|^2] \cdot \Phi_N(|E_R|^2)
\]

(8)

\[
\left[ |E_R| \right]_N = \mathbb{E}[|E_R|] \cdot \Phi_N(|E_R|)
\]

(9)

As before, we cannot simply estimate \( \mathbb{E}[|E_R|] \) as \( \sqrt{\mathbb{E}[|E_R|^2]} \).

Instead, we need to apply a correction factor similar to that given in equation (4):

\[
\mathbb{E}[|E_R|] = \sqrt{\mathbb{E}[P_R] \cdot \frac{8\pi n}{3\lambda^2} \cdot \sqrt{3} \cdot \frac{\Gamma(N)}{\Gamma(N+1/2)}},
\]

From equation (9) we can write
Another problem we encountered when processing our data is related to the conversion of data between linear units and decibels. The problem is that many specifications give amplitude and/or uncertainty requirements in decibels. In addition, some equipment gives readings in decibels, others give readings in linear units, and still others can give readings in either format. While this would not seem like a problem (we can always convert from one unit to another), there is still a question of when to convert. This, too, might not seem like a problem, so we offer the following example. We took 1024 samples of received power, measured at 1 GHz in a reverberation chamber. The test was set up to give an average received power of exactly 1 W (by normalizing the data). We then converted the data to decibels relative to 1 W and computed the average. Instead of an average received power of 0 dB relative to 1 W, the average was -2.5 dB relative to 1 W. This is a significant difference. The cause of this difference is similar to the cause of the difference between $\log(P_R)$ and $\log(\langle P_R \rangle)$. Knowing that the distribution of $P_R$ is a $\chi^2$ distribution, we can show [3] that the average of the decibel data should have been -2.507 dB relative to 1 W, which agrees well with the observed difference. For now, we recommend that all calculations be done using linear units, and any conversion to decibels should be performed as a final step.

**COMPARISON WITH ANECHOIC CHAMBERS**

As with any alternative EMC test facility, there is an inevitable push to compare results obtained in a reverberation chamber with those obtained in an anechoic chamber. Although these facilities are completely different, some attempts must be made if reverberation chambers are ever to be accepted as viable supplements or alternatives to anechoic chambers. Before a comparison can be made, however, we must decide on what parameters will be matched in the two facilities. Will we attempt to generate the same power density in each facility, or the same magnitude of the electric field? And if we choose to match the magnitude of the electric field, do we want to use the total electric field or a rectangular component?

At this point, we are forced to admit that a plane wave transmitted into an ideal anechoic chamber has certain advantages, and foremost among these is a lack of ambiguity. If we generate a linearly-polarized 10 V/m plane wave, we know that a rectangular component of the electric field aligned the polarization of the plane wave will be 10 V/m, and the total electric field is 10 V/m. The squared magnitude of the electric field (total or rectangular) is 100 V²/m², and the minimum, average, and maximum electric field (total or rectangular) are all 10 V/m. Because of this, conversion to and from decibels is unambiguous.

The same cannot be said for reverberation chambers. Although we have the tools to convert from one framework to another, there is still significant confusion. If a device is to be tested for immunity at 10 V/m, we need additional information. Most likely we can assume that this is a maximum electric field of 10 V/m, not an average field. However, we do not know if the test is to be based on a total electric field or a rectangular component. The difference can be significant, as much as 4.77 dB for $N = 1$, 3.25 dB for $N = 10$, 2.5 dB for $N = 100$, and 2 dB for $N = 1000$. If a device is to be tested to 20 dB relative to 1 V/m (10 V/m for a plane wave), things are even more complicated because of the possible 2.5 dB offset mentioned above.

To demonstrate some of the complications, we will compare the power received by an ideal (lossless and impedance matched) antenna in an anechoic chamber with the power received by the same antenna placed in a reverberation chamber, assuming that some test parameter is held constant in both facilities. We will show that the selection of different test parameters can result in significantly different relationships. For simplicity, we will compare the power received in an anechoic chamber $P_{\text{anech}}$ (pronounced panic?) with the average power received in a reverberation chamber $\langle P_{\text{reverb}} \rangle$. If a comparison with the maximum power received in a reverberation chamber is desired, the result for $\langle P_{\text{reverb}} \rangle$ can be multiplied by the appropriate maximum-to-average ratio.

In an anechoic chamber, assuming a linearly polarized antenna that is copolarized with the field and aligned in the direction of maximum coupling, the power received by an ideal antenna can be written as [4]

$$P_{\text{anech}} = |\mathbf{S}_c| \cdot A_e = \frac{|\mathbf{E}_T|^2}{\eta} \cdot \frac{A_e^2 D}{4\pi} = \frac{|E_{Rm}|^2 D}{4\pi},$$  

where $|\mathbf{S}_c|$ is the magnitude of the power density of the transmitted plane wave, $A_e$ is the effective area of the antenna, and $D$ is the directivity of the antenna. In a reverberation chamber, the average power received by a lossless impedance-matched antenna is [1]

$$\langle P_{\text{reverb}} \rangle = \langle |\mathbf{S}_r|^2 \rangle \cdot \langle A_e \rangle = \frac{\langle |\mathbf{E}_T|^2 \rangle}{\eta} \cdot \frac{\lambda^2}{8\pi} = \frac{\langle |E_{Rm}|^2 \rangle}{\eta} \cdot \frac{\lambda^2}{8\pi},$$  

where $\langle |\mathbf{S}_r|^2 \rangle = \frac{\langle |\mathbf{E}_T|^2 \rangle}{\eta}$ is the average scalar power density in the reverberation chamber and $\langle A_e \rangle = \frac{\lambda^2}{8\pi}$ is the average effective area of an ideal antenna in a reverberation chamber. One possible approach, and the approach used by Crawford [2], is to equate the power densities in both facilities. This is equivalent to equating the squared magnitude of the total electric field. In this case, it is easy to combine equations (11) and (12) to show that

$$\frac{P_{\text{anech}}}{\langle P_{\text{reverb}} \rangle} = 2D, \quad |\mathbf{S}_c| = |\mathbf{S}_r| or |E_{Rm}|^2 = \langle |E_T|^2 \rangle.$$  

Crawford claimed that this value should be equal to $D$ instead of $2D$. This is because he also assumed that the numerator of equation (1) should have been $4\pi\eta$ rather than $8\pi\eta$. 

\[
\left[|E_R| \right]_N = \langle |E_R| \rangle \cdot \frac{\Gamma(N)}{\Gamma(N+1/2)} \cdot \frac{\sqrt{\pi}}{\Gamma(N)\sqrt{2}} \cdot \frac{\nabla N}{\nabla |E_R|}.
\]
Another reasonable alternative is to equate the squared magnitudes of a rectangular component of the electric field, which gives us:

\[
\frac{P_{\text{Anch}}}{P_{\text{Reverb}}} = \frac{2D}{3}, \quad |E_x|^2 = \langle |E_x|^2 \rangle, \quad \text{(14)}
\]

Here, for a nondirective antenna, it is actually possible to receive less power in an anechoic chamber than a reverberation chamber.

Two other possibilities are to equate the magnitudes of the total electric fields by combining equations (4) and (11), which gives us:

\[
\frac{P_{\text{Anch}}}{P_{\text{Reverb}}} = 2D \left( \frac{15}{16} \right)^2 \cdot \frac{\pi}{3} \cdot 3N \cdot \left( \frac{\Gamma(3N)}{\Gamma(3N + 1/2)} \right)^2, \quad |E_F| = \langle |E_F| \rangle, \quad \text{(15)}
\]

or equating the magnitudes of a rectangular component of the electric field by combining equations (9) and (11), which gives us:

\[
\frac{P_{\text{Anch}}}{P_{\text{Reverb}}} = \frac{\pi D}{6} \cdot N \cdot \left( \frac{\Gamma(N)}{\Gamma(N + 1/2)} \right)^2, \quad |E_R| = \langle |E_R| \rangle, \quad \text{(16)}
\]

Both equations (15) and (16) depend on \( N \), which is somewhat surprising. If we evaluate these functions at small \( N \) (\( N = 1 \)) and large \( N \) (\( N > 100 \)) we can find the expected range of these relationships. For equation (15), equating the average magnitude of the total electric field, the received power ratio begins at \( 2D \) for \( (N = 1) \) and decreases down to \( 2D \left( \frac{15}{16} \right)^2 \cdot \frac{\pi}{3} \approx 1.84D \) for large \( N \).

Similarly, for equation (16) equating the average rectangular components of the electric field, the received power ratio begins at \( \frac{2D}{3} \) for \( (N = 1) \) and decreases down to \( \frac{\pi D}{6} = 0.52D \) for large \( N \).

These differences must be remembered in any facilities comparison.

**Will the Real Electric Field Please Stand Up?**

The question of which definition of electric field to use for immunity measurements in a reverberation chamber is still an open question. Scalar power density (and therefore the squared magnitude of the total electric field), the magnitude of the total electric field, and the magnitude of a rectangular component of the electric field have all been suggested as measures of the electric field. Of these, any measure based on the total electric field would be preferable because the amplitude is higher and the uncertainty is lower than any measure based on a rectangular component. Unfortunately, we have no sound technical argument for using any measure based on the total electric field. Hill [1] claims that the power received by any impedance-matched antenna will have a \( \chi^2 \) distribution, not a \( \chi^2 \) distribution as would be required if the antenna responded to the total electric field rather than a rectangular component. If the antenna is neither perfectly efficient nor impedance-matched, it is unlikely that the distribution of the power received by the antenna will change substantially. Thus, assuming that a susceptible device can be modeled as an antenna (possibly a poor one) connected to some arbitrary electronic system, the distribution of the power delivered to the electronics should be approximately described by the \( \chi^2 \) distribution, or alternatively, the distribution of the voltage delivered to the electronics should be approximately described by the \( \chi^2 \) distribution. One of the subtle implications of these statements is that it is *impossible* to build a linear two-terminal antenna with an output voltage or current that is proportional to the total electric field in the vicinity of the antenna. In fact, the only suggestions we have heard for devices that respond to the total electric field rather than a rectangular component are either artificial (constructing an isotropic probe by mathematically combining the responses of three orthogonal dipoles) or thermal (the temperature of a drop of water could be a function of the total electric field). Since neither of these cases are adequate for modeling typical electronic systems, it is difficult to justify any tests based on the total electric field. We are open to alternatives, and, in fact, would appreciate any suggestions. For now, however, we are forced to recommend using a rectangular component of the electric field as the basis of any immunity tests in a reverberation chamber.

**Conclusion**

Processing of reverberation-chamber data must be done with great care, since many errors can be disguised by the potentially large variability of the data and erroneous derivations. Many of these errors can be avoided if each step involved in the processing of the data is compared with values predicted by electromagnetic and statistical theory that is now available for reverberation chamber tests.

We have shown that the theoretical relationship between the average power received by an ideal antenna and the magnitude of the total electric field in a reverberation chamber agrees well with theory. We have also shown that previous claims that the theoretical relationship was in error by approximately 3 dB were in fact caused by erroneous derivations and interpretations. Once the data are analyzed step-by-step, these errors can be detected and corrected.

We have also shown that converting data to decibels must be done with care, and differences of 2.5 dB are possible simply by changing the order in which the data are processed.

Finally, we compared measurements of received power in an anechoic chamber and a reverberation chamber and demonstrated the ratio of these received powers is proportional to the directivity of the antenna, but the proportionality constant depends on which parameters are equated in the two facilities. Different choices can affect the comparison by almost a factor of four. We give our recommendation that the magnitude of a rectangular component of the electric field should be equated in these facilities.


