LOCALLY WEIGHTED TOTAL VARIATION DENOISING FOR PSF MODELING ARTIFACT SUPPRESSION IN PET RECONSTRUCTION

Arthur Mikhno¹, Elsa D. Angelini¹,², and Andrew F. Laine¹

1. Department of Biomedical Engineering, Columbia University, New York
2. Institut Mines-Telecom, Telecom ParisTech, CNRS LTCI, Paris, France

Abstract

Incorporating the point spread function (PSF) into the iterative MLEM reconstruction of PET images introduces contrast and size dependent ringing and over enhancement artifacts. We previously developed a new method, called TV-PSF-MLEM, to suppress these artifacts based on the introduction of a locally-weighted total variation regularization within the MLEM reconstruction algorithm. On non-noisy PET measures, we proposed to compute the TV spatial weights based on the point-wise convergence rate of a preliminary MLEM reconstruction, for each voxel. In this work we extend the TV-PSF-MLEM weighting scheme to noisy measures introducing a noise-independent weighting scheme. We compare its performance to a state of the art PET denoising method. Results on numerical phantoms show that the TV-PSF-MLEM offers substantial advantages in the recovery of small cylinders and gains in contrast recovery of larger cylinders.

Index Terms— PET, image reconstruction, point spread function, MLEM, Total Variation

1. INTRODUCTION

Iterative reconstruction techniques, such as the maximum likelihood expectation maximization (MLEM), provide a flexible framework in positron emission tomography (PET) imaging for modeling the physics and the scanner geometry, yielding greater image contrast, visual quality and noise robustness than analytical reconstruction methods. Incorporating the point spread function (PSF) of the scanner into the iterative MLEM reconstruction process (called PSF-MLEM) improves spatial resolution but introduces significant contrast and size dependent over-enhancement and ringing artifacts [1, 2]. The over-enhancement artifacts might be explained by the mismatch between the true and the measured PSF, while the ringing artifacts are related to object sizes [3]. Most approaches previously proposed to compensate ringing artifacts tend to blur the PET data which undermines the benefits of including the PSF in the reconstruction [3].

A promising approach was recently proposed by Rapisarda et al. for artifact suppression by incorporating a new regularization prior into the reconstruction process that locally modifies the image estimate at each iteration in an attempt to locally control edge enhancement [4]. This method provides excellent ringing suppression in the image, especially for large structures, but also tends to suppress the benefits of PSF modeling in smaller objects.

We previously developed a new method (called TV-PSF-MLEM) to suppress PSF-MLEM artifacts based on a locally weighted total variation regularization applied at each iteration [5]. Amplitude of the weights specify the amount of TV regularization at each voxel. These local weights can be pre-calculated based on the convergence rate of MLEM at each voxel since we established that the convergence rate is directly related to the object size and contrast and that artifact magnitudes are directly proportional to this convergence rate. Using noiseless simulations, we showed that TV-PSF-MLEM suppresses ringing artifact while providing edge and contrast recovery, beyond that of standard MLEM, especially in small cylinders. In this paper we extend the TV-PSF-MLEM formalism to simulations corrupted with Poisson noise. A new weighting scheme is introduced robust to noise and results are presented, comparing to the Rapisarda method that is also based on a regularization prior.

2. METHODOLOGY

2.1. Iterative image reconstruction: MLEM algorithm

The ML estimate of the PET image is computed using the MLEM algorithm, and following the notation from [4], leads to the following iterative update equation:

$$
\lambda_{b}^{k+1} = \frac{\lambda_{b}^{k}}{BP_{b}^{\lambda_{b}^{k}}} BP_{b} y_{d} - \frac{\lambda_{b}^{k}}{P_{d}^{\lambda_{b}^{k}}} y_{d} = \lambda_{b}^{k} u_{b}^{k}
$$

(1)

where $\lambda_{b}^{k}$ represents the intensity of the voxel $b$ within the image $\lambda$ at iteration $k$, $y_{d}$ is the sinogram measure, equal to the number of counts along the line of response (LOR) $d$, and $1$ is a unit matrix of the same size as $y_{d}$. Following the methodology introduced in [4], incorporating the PSF model into Eq. (1) is achieved by modifying the projector $P$ and backprojector $BP$ as follows:

$$
P_{d}^{\lambda_{b}^{k}}(\cdot)_b = \sum ((\cdot) * PSF)_b P_{bd}
$$

$$
BP_{d}^{\lambda_{b}^{k}}(\cdot)_d = PSF^{T} \sum ((\cdot) d P_{bd}
$$

(2)
where PSF represents the PSF kernel and $\ast$ is a discretized convolution operator as defined in Appendix of [6]. For a symmetric kernel, $PSF = PSF^T$. Hereafter PSF-MLEM shall refer to the situation when $PSF$ is a non-zero matrix, and MLEM when $PSF = 1$.

2.2. Simulation framework: software and phantom

All methodological developments were performed using simulated sinograms. PET image reconstructions were performed using the STIR open source C++ software (v2.2) [6]. The modified projectors in Eq. (2) were implemented in C++ within the STIR source code. A 200mm diameter cylindrical phantom object was simulated, with a background intensity equal to 10, and containing three hot spots of diameter 25mm, 16mm, 12mm with a 1.5:1 contrast ratio (CR), as well as three 8mm diameter hot spots with CR of 1.25:1 and 2:1, respectively. The phantom was blurred with a symmetrical 4.5mm FWHM Gaussian kernel [5] to simulate the resolution loss inherent to the ECAT HR+ scanner (Siemens/CTI) at use in our facility. Figure 1 illustrates the phantom image with resolution and contrast loss due to blur.

2.3. TV regularization prior (Rapisarda method)

A Bayesian formulation of the iterative reconstruction algorithm can be used, leading to a maximum a posteriori (MAP) estimate that includes an a priori probability term. Rapisarda proposed the following iterative MAP estimate [4]:

$$\lambda_b^{k+1} = \frac{\lambda_b^k}{BP_b[1 - \alpha D_b \lambda_b^k]}BP_b \frac{y_d}{P_d \lambda_b^k}$$

(3)

where $\alpha > 0$ weights the contribution of the value $D_b$ at pixel $b$ of a regularization prior $D$ defined as:

$$D[\lambda] = \nabla \cdot (\phi'(\|\nabla \lambda\|) \frac{\nabla \lambda}{\|\nabla \lambda\|})$$

(4)

The term $\phi'(x)$ is called the variational prior and is based on a modification of the generalized p-Gaussian kernel:

$$\phi'(x) = \begin{cases} x^{(p-1)} & x < \delta \\ 1 - \frac{d(d+\delta)}{d+x} & x \geq \delta \end{cases}$$

(5)

where $d = (1 - \delta^{p-1})$, with $\delta$ being a gradient threshold.

2.4. TV-PSF-MLEM: algorithm

We introduced in [5] a locally-weighted total variation regularization scheme for PSF-MLEM reconstruction. We outline its components briefly here. Using previous notations, at each iteration of Eq. (1) the total variation problem amounts to finding an estimate image $\hat{\lambda}$ that satisfies:

$$\hat{\lambda} = \min_{\theta} \frac{1}{2} \|\lambda^k - \theta\|^2 + \beta |\nabla \theta|$$

(6)

where $\|\cdot\|$ and $|\cdot|$ are the $l_2$ and $l_1$ norms, respectively, and $\beta$ is a positive scalar regularization weight. The classical framework given by Eq. (6) minimizes TV over the whole image. Instead, local weights $w_b$ were introduced for each individual pixels $b$ as follows:

We define $\lambda_b^k$ as the locally weighted TV estimate:

$$\hat{\lambda}_b^k = \lambda_b^k + w_b \Delta \hat{\lambda}_b^k$$

(7)

where $w_b$ is a spatially-varying weight imposed on the net change of each voxel and $\Delta \hat{\lambda}_b^k$ is defined as:

$$\hat{\lambda}_b^k = \lambda_b^k + (\hat{\lambda}_b^k - \lambda_b^k) = \lambda_b^k + \Delta \hat{\lambda}_b^k$$

(8)

Note that if $w_b = 1$ then $\lambda_b = \lambda_b$, corresponding to the classical solution to Eq. (6).

We proposed to use the convergence rate, derived from the value of $u_b$ in Eq. (1), for the definition of the spatial weights $w_b$, as detailed in [5], using the following values:

$$c_b = \min_k |1 - u_b^k| \leq \epsilon$$

$$w_b = 1 - \frac{c_b - \min(c)}{\max(c) - \min(c)}$$

(9)

where $c_b$ is defined as the earliest MLEM iteration $k$ at which voxel $b$ converges (in practice when $u_b^k \approx 1$), $w_b$ is derived by normalizing $c_b$ such that $0 \leq w_b \leq 1$, and $\epsilon$ is a convergence threshold ($\epsilon = 1 \times 10^{-4}$ in [5]). Figure 2 illustrates the spatial TV-weights $w_b$ obtained with this approach for noiseless and noisy measures. PSF-MLEM with TV denoising method (TV-PSF-MLEM) is summarized in Algorithm 1.
2.5. TV-PSF-MLEM: TV spatial weights for noisy data

From Eq. (9) it can be seen that weights depend on the convergence rate of individual voxels $b$ and will therefore be highly affected by the presence of noise. To adapt our approach to noisy data, we first studied the oracle approach where ideal weights (derived from the noiseless case) were applied to TV-PSF-MLEM reconstruction of the noisy sinograms, corrupted with Poisson noise. Results, illustrated in Figure 2, show that reconstructions after N=500 iterations were similar with and without noise, with good suppression of noise and ringing artifact, but with some over enhancement of the $8mm$ cylinders in both cases. To approximate these oracle weights using the iterative MLEM reconstructions from the noisy data, we propose to use a local spatial aggregation of the convergence rates estimates $\lambda^b_k$ from Eq. (1), using the average kernel of size $n \times n$ with $n = 3$, along with a modified value for $\epsilon$: $\epsilon = \epsilon = 5 \times 10^{-4}$. Results illustrated in Figure 2, show that such approach leads to a profile of the local weights that mimics well the general shape of the oracle weights although local differences in magnitude exist. Thus, the calculation of the parameter $c$ used in Eq. (9), on noisy data becomes:

$$\hat{\lambda}^b_k = \sum_{b \in N_b} u^b_k$$

(10)

where $N_b$ is the $3x3$ neighborhood around voxel $b$, and $\hat{\lambda}^b_k$ replaces $\lambda^b_k$ in Eq. (9) for noisy data.

3. RESULTS

3.1. Experimental setup using synthetic phantom data

Performance of MLEM, PSF-MLEM, TV-PSF-MLEM, and the Rapisarda method was quantitatively assessed using the recovery coefficient defined in Eq. (11). For TV-PSF–MLEM the regularization weight $\beta = 0.02$ was used as in [5], and for the Rapisarda method parameters were set as recommended by the authors in [4]: $p = 1.33$, $\delta = 0.3$, and $\alpha = 0.002$. In an attempt to suppress over-enhancement in the $8mm$ cylinders, as observed in Figure 3, we set a minimum cutoff value for the $w_b$ values (stemming from the observation that $w_b$ in those cylinders were lower than anywhere else in the image). By increasing $w_b$ values we enforce more TV regularization to prevent overshooting. Experiments were run until convergence, corresponding to: N=200 for MLEM and N=500 for TV-PSF-MLEM and Rapisarda.

$$RC = \frac{\sum_{b \in \Omega_{ROI}} \lambda^b_{\text{recon}}}{\sum_{b \in \Omega_{ROI}} \lambda^b_{\text{true}}}$$

(11)

3.2. Evaluation of artifact suppression

Visually TV-PSF-MLEM and Rapisarda reconstructions provide very comparable results in large cylinders ($\geq 12mm$), as seen in Figure 4. In smaller cylinders, the Rapisarda method underestimates while the TV-PSF-MLEM overestimates the activity. TV-PSF-MLEM appears more homogenous in the background and in larger cylinders. At the outer edges of the
phantoms artifacts in the TV-PSF-MLEM reconstruction are due to the non regularization with local weights $w_b$ there. The overshoot in small cylinders and background edges is largely remedied by applying a simple cutoff, as shown in Figure 5.

The performance characteristic of the two methods is better understood in terms of RC values, reported in Figure 6 for the different cylinders. The Rapisarda method yields RCs for all cylinders that closely track the noiseless MLEM estimate, with slightly higher RCs for cylinder diameters $\geq 12mm$, and outperforming standard MLEM on noisy data. Hence this method provides excellent noise suppression but greatly limits the contrast enhancement gained from PSF modeling as it does not recover contrast loss due to blurring effects. In contrast, TV-PSF-MLEM (with or without cutoff on the $w_b$) yields higher RC values for all cylinders. For the 24mm and 8mm cylinders the recovery level on noisy data approaches the recovery level of PSF-MLEM without noise. When a cutoff value is applied on the $w_b$, all RC values are higher than with the Rapisarda method, especially in the highest contrast 8mm cylinder.

4. CONCLUSIONS

We compared two ML iterative PET reconstruction algorithms that both exploit local regularization of the image data: our proposed TV-PSF-MLEM method and the Rapisarda method described in [4]. Our simulation results demonstrate that in the presence of noise, our proposed method is robust and provides advantages in terms of visual quality and contrast recovery, especially in small structures. Further testing with varying levels of noise, higher contrast ratios, real phantom objects and clinical data is planned.

5. REFERENCES


