ABSTRACT

We present a compressed sensing based approach to remove gain field from magnetic resonance (MR) images of the human brain. During image acquisition, the inhomogeneity present in the radio-frequency (RF) coil appears as shading artifact in the intensity image. The inhomogeneity poses problem in any automatic algorithm that uses intensity as a feature. It has been shown that at low field strength, the shading can be assumed to be a smooth field that is composed of low frequency components. Thus most inhomogeneity correction algorithms assume some kind of explicit smoothness criteria on the field. This sometimes limits the performance of the algorithms if the actual inhomogeneity is not smooth, which is the case at higher field strength. We describe a model-free, non-parametric patch-based approach that uses compressed sensing for the correction. We show that these features enable our algorithm to perform comparably with a current state of the art method N3 on images acquired at low field, while outperforming N3 when the image has non-smooth inhomogeneity, such as 7T images.

Index Terms— MRI, intensity non-uniformity, intensity inhomogeneity, 7T, bias field, bias correction

1. INTRODUCTION

Several core MR image processing algorithms such as registration and segmentation, use image intensity as a primary feature. Any artifact in the intensities affects the performance of the algorithms severely. Intensity inhomogeneity (IIH) or intensity non-uniformity (INU) is an example of one such artifact. At low magnetic field (1T-3T), this is primarily caused by the presence of non-linear characteristics present in the RF receiver coil. At high field (e.g. 7T), this effect is accentuated by the interactions between RF waves and electromagnetic properties of the tissues [1].

Inhomogeneity correction algorithms can be categorized into two primary classes, prospective and retrospective. Prospective methods [2] correct the inhomogeneity by including the imaging equations in the correction methodology, usually by combining multiple images [3, 1] acquired under different parameters. These methods are not applicable to many studies where it is not always possible to acquire multiple images with pre-defined parameters or the acquisition protocols are simply unknown. In contrast, retrospective methods are essentially post-processing methods. They usually assume that the inhomogeneity field is a smoothly varying non-anatomic multiplicative field, and is usually written as a linear combination of low order smooth polynomials. Then entropy minimization [4, 5] or deconvolution [6] is used to estimate the smooth IIH field. Often, the correction step is combined with a segmentation algorithm, where simultaneous estimation of tissue classes and inhomogeneity correction can be achieved by a maximum likelihood estimator [7, 8].

The smoothness property of the IIH field has been well studied for low field strength. But in high field, the smoothness assumption is often violated. Fig. 1 shows T1-weighted images from a 1.5T, 3T, 4T and 7T scanner, where at 7T, the inhomogeneity is quite different. In this case, a small window of intensity measurements should provide information about the inhomogeneity. This idea is exploited in [10], where a joint entropy minimization framework is described to remove bias from many images simultaneously. In this work, however, we propose a non-parametric compressed sensing based intensity non-uniformity correction (CSI-NC) approach that does not have any explicit smoothness model on the estimated field and does not require many images, thus being more versatile and applicable to situations where the actual IIH is not smooth, e.g. in 7T images.
2. METHOD

2.1. Compressed Sensing

We use compressed sensing for our IIH approach. Compressed sensing recovers sparse vectors from their projections onto a set of random vectors [11, 12].

Suppose we want to reconstruct a signal \( x \in \mathbb{R}^d \) which is \( s \)-sparse, i.e. has at most \( s \) non-zero elements. We want to observe another vector \( y \in \mathbb{R}^n \), \( n < d \), such that each element of \( y \) can be obtained by an inner product of \( x \) and another vector from \( \mathbb{R}^d \). In short, compressed sensing deals with reconstructing \( x \in \mathbb{R}^d \) from \( y \in \mathbb{R}^n \), with \( y = \Phi x \), \( n < d \), \( x \) being \( s \)-sparse, \( \Phi \in \mathbb{R}^{n \times d} \).

One approach of finding \( x \) is,

\[
\hat{x} = \min ||x||_0 \text{ such that } ||y - \Phi x||_2^2 < \epsilon_1,
\]

where \( \epsilon_1 \) is the noise in the measurement and \( ||\cdot||_0 \) indicates the number of non-zero elements in the vector. Although this approach provides simple conditions on \( \Phi \) [13], it is an NP-hard problem. A second approach comes from,

\[
\hat{x} = \min ||x||_1 \text{ such that } ||y - \Phi x||_2^2 < \epsilon_2,
\]

where \( ||x||_1 \) is the \( L_1 \) norm of a vector. This is a convex problem and can be transformed into a linear program that can easily be solved. If \( \epsilon_2 \) is unknown, Eqn. 2 can be written in a more compact form,

\[
\hat{x} = \arg\min_x \{||y - \Phi x||_2^2 + \lambda ||x||_1\},
\]

where \( \lambda \) is a weighing factor. The sparsity on \( \hat{x} \) increases as \( \lambda \) increases.

It has been shown that if \( \Phi \) follows the global restricted isometry property (RIP) [12], then the solutions to Eqn. 1 and Eqn. 2 are identical and the optimal solution can be obtained by such an \( L_1 \) minimization problem. This result is interesting because most random matrices satisfy the RIP [14]. Thus, to reconstruct \( x \), we have to observe its projections onto a set of random vectors.

2.2. Patch Based Correction

Assume the MR image is partitioned into \( p \times q \times r \) patches. If the bias field is not globally smooth, then we can assume that it is at least uniform over a small image patch. Let \( d = pqr \), thus each patch can be thought of as a \( d \times 1 \) vector. Assuming that the gain field is multiplicative, each image patch \( y_k \in \mathbb{R}^d \), \( k \in \Omega \), can be written as,

\[
y_k = g_k y_k^* + \eta_k, \quad g_k > 0
\]

(4)

Here, \( \Omega \) is the image domain, \( y_k^* \) is the inhomogeneity free image patch, \( g_k \) is the bias field for \( k^{th} \) location, and \( \eta \) is the image noise. For further analysis, for simplicity we assume that \( \eta_k = 0, \forall k \).

The idea behind a patch-matching process is that we want to match \( y_k \) to a set of vectors, called a dictionary, which is given by \( \Phi \in \mathbb{R}^{d \times N} \), such that \( y_k = \Phi x_k \), \( x_k \in \mathbb{R}^N \), and the high-dimensional sparse vector \( x_k \) carries the information about the multiplicative field \( g_k \).

Intuitively, for a particular \( y_k \), its sparsest representation \( x_k \) from Eqn. 2 is such a vector that \( x_k \) has exactly one non-negative element. That implies \( y_k \) is matched to exactly one vector \( \phi \) from \( \Phi \) by a scaling factor \( f \), \( \phi \) being a column of \( \Phi \). Thus any multiplicative effect on \( y_k \) is reflected on the scaling factor \( f \), which gives a measure of inhomogeneity. In the same context, we follow the idea of the non-negative garrote [15] so that the sparse representations are constrained by \( x_k \geq 0 \), to have a meaningful positive weighting of the dictionary \( \Phi \).

Specifically, consider the sparse representations of \( y_k \) and \( y_k^* \) as \( x_k \) and \( x_k^* \), respectively. Then Eqn. 3 gives,

\[
\hat{x}_k = \arg\min_x \{||y_k - \Phi x||_2^2 + \lambda ||x||_1\}, \quad x \geq 0
\]

(5)

\[
\hat{x}_k^* = \arg\min_x \{||y_k^* - \Phi x||_2^2 + \lambda^* ||x||_1\}, \quad x \geq 0
\]

(6)

Now if \( y_k = g_k y_k^* \), Eqn. 5 gives,

\[
\hat{x}_k = \arg\min_x \left\{||g_k y_k^* - \Phi x||_2^2 + \lambda ||x||_1\right\},
\]

(7)

\[
\Rightarrow \hat{x}_k = \arg\min_x \left\{||y_k^* - \Phi \frac{x}{g_k}||_2^2 + \frac{\lambda}{g_k} ||x||_1\right\}.
\]

By appropriate conditions on \( \Phi \) as described earlier, and choosing \( \lambda = g_k \lambda^* \), Eqn. 7 and Eqn. 6 give \( \hat{x}_k = x_k^* g_k \). Thus, an appropriate estimator of \( g_k \) is given by,

\[
g_k = \frac{||\hat{x}_k||_1}{||\hat{x}_k^*||_1}.
\]

(8)

2.3. Choice of \( \hat{x}_k^* \) and \( \Phi \)

Eqn. 8 suggests that in an ideal situation, if two inhomogeneity free patches \( y_i \) and \( y_j \) are of same the tissue and have the same intensity, then \( \hat{x}_k^* = \hat{x}_i^* \) and \( g_i \propto ||x_i||_1 \), i.e., the \( L_1 \) norm of the sparse representation gives the relative amount of gain field for the same tissue.

For a particular \( k \), \( \hat{x}_k^* \) can be found using a prior information about the tissue classes, which can be obtained from a segmentation of the image. The image being already corrupted with IIH, we only need an approximate segmentation of the image. In our experiments, we have used an atlas based segmentation called TOADS [16] that uses a probability atlas to do the segmentation. We use an approximate segmentation from TOADS, which gives a reasonable segmentation even in the presence of heavy inhomogeneity (c.f. Fig. 3(e)). We use 1 iteration of the segmentation algorithm, which provides a hard segmentation image with 4 classes, cerebro-spinal fluid (CSF), gray matter (GM), white matter (WM) and ventricles. For each of the classes, a mean \( y(l) \), \( l = 1, \ldots, 4 \) is computed, and their corresponding \( \hat{x}_k^* (l) \) are found from Eqn. 5.
The compressed sensing literature suggests that a good choice of $\Phi$ is a random matrix. For our experiments, we generate $\Phi$'s from a uniform distribution once.

The algorithm is described as follows,

1. Partition the image into patches $y_k, k \in \Omega$ of size $p \times q \times r$. In our experiments, $p = q = r = 3$. Generate a random $d \times N$ matrix $\Phi$. In our experiments, we arbitrarily select $N = 1000$.

2. Find an approximate segmentation of the image using TOADS and compute the mean $y^{(l)}$'s that represents the $l^{th}$ class, $l = 1, \ldots, 4$. Find the corresponding $\hat{x}^{*}(l)$'s using $\Phi$ in Eqn. 5.

3. For each $k$, find the type of the tissue $l_0 \in \{1, \ldots, 4\}$ that $y_k$ belongs to. If there is more than one tissue type in $y_k$, we choose the dominant one for simplicity. Then $\hat{x}_k^* = \hat{x}^*(l_0)$

4. Find the sparse representation of $y_k$ as $\hat{x}_k$ from Eqn. 5. We use $\lambda = 0.5$.

5. Find the gain field $g_k$ using $\hat{x}_k$ and $\hat{x}_k^*$ from Eqn. 8.

Here we note that all images are normalized, so that their WM peaks have the same value. The final inhomogeneity field becomes blocky because there is no explicit smoothness criterion on the image model. As a post-processing step, we smooth the field by a Gaussian filter with size $\sigma$, that we estimate based on phantom validation, described in the next section. The filter also smooths out bad estimates of $\hat{x}_k$'s near the tissue boundaries.

3. RESULTS

3.1. Phantom Validation

We compare CSI-NC with a current state of the art method called non-parametric non-uniform intensity normalization (N3) [6], which tries to restore the corrupt image by a histogram deconvolution assuming the field is smooth. We use a phantom image from Brainweb [17] database, for which

the inhomogeneity field is known (referred to as “field C” on [17]). The final inhomogeneity field is smoothed by a 3D Gaussian filter of size $\sigma$, chosen such that the coefficient of variation (CV) of the estimated smooth field and true field is minimum. CV is defined as $C = \sigma(B_c/B_e)$, where $B_c$ and $B_e$ are the estimated and the applied field and it measures how much the estimated field differs from the true field in a normalized way. Fig. 2 shows the plot of $\sigma$ vs the CV. The minimum CV = 0.01 is attained at $\sigma = 10$mm. The magnitude of the CV is comparable to that reported earlier [4]. The original and corrected images and the corresponding fields are shown in Fig. 3.

3.2. Experiment on 7T images

As Brainweb phantoms have very smooth inhomogeneity, it is very difficult to see the improvement of CSI-NC over other methods. It is explicitly seen when the inhomogeneity is not smooth. In those cases, the polynomial based algorithms will not perform better simply by increasing the degree of polynomials because the coefficients can not be estimated in a robust fashion in a high-dimensional parameter space. Instead, we employ a patch based method. We use both CSI-NC and N3 on Philips 7T MPRAGE (magnetization prepared rapid gradient echo) and FLAIR (fluid attenuated inversion recovery) images, $256 \times 320 \times 320$ volumes, 0.70mm isotropic.

As we don’t have any ground truth to compare the inhomogeneity fields, we visually compare the results from the two algorithms. Fig. 4 shows a 7T FLAIR image and its correction by both N3 and CSI-NC. As N3 assumes some underlying smoothness criterion on the inhomogeneity field, it can not remove the localized IIH present in the CSF at the
superior part of the brain (Fig. 4(g)), while CSI-NC, being a patch-based method, corrects the bias in that area (Fig. 4(h)). We also use one 7T MPRAGE image to test the effectiveness of our algorithm. In the absence of a ground truth, we visualize the inner cortical surfaces, generated using a geometric deformable model based surface reconstruction algorithm [18]. Fig. 5(a) shows the original MPRAGE image, showing large inhomogeneity near the anterior and posterior parts of the brain. Its correction by N3 and CSI-NC are shown in Figs. 5(a)-(b). A zoomed view of the anterior part shows that inner surface is pushed outwards deep inside GM on N3 corrected image, while it respects the WM-GM contrast in the CSI-NC corrected image.

4. SUMMARY AND CONCLUSION

We have presented a patch based intensity inhomogeneity correction method that uses the concept of compressed sensing. Our method is non-parametric, model-free and does not impose any smoothness constraint on the field. We have shown that our method works better on 7T images, at least visually, compared to N3, although there is some over-correction visible near the peri-ventricular region, where the method needs improvement. Future work includes estimation of optimal size \( \lambda \), \( N \) and size of patches. We would also validate the method on 7T images.

5. REFERENCES


