Neural Inverse Optimal Control for Discrete-Time Uncertain Nonlinear Systems Stabilization

Fernando Ornelas-Tellez*, Edgar N. Sanchez**, Ramon Garcia-Hernandez*, Jose A. Ruz-Hernandez* and Jose L. Rullan-Lara*

*Universidad Autonoma del Carmen, Cd. del Carmen, Campeche 24180, Mexico
Email: {fornelas, rghernandez, jruz, jruellan}@pampano.unacar.mx

**CINVESTAV, Unidad Guadalajara, Jalisco 45015, Mexico
Email: sanchez@gdl.cinvestav.mx

Abstract—This paper discusses neural inverse optimal control to achieve stabilization for discrete-time uncertain nonlinear systems, and minimizing a meaningful cost functional. A neural identifier scheme is used to model the uncertain nonlinear system, and based on this neural model and the knowledge of a control Lyapunov function, then the inverse optimal controller is synthesized in order to achieve exponential stability. An example illustrates the applicability of the proposed control technique.

I. INTRODUCTION

For optimal nonlinear control, we deal with determining a control law for a given system such that a predefined optimality criterion is minimized. This criterion is usually formulated as a meaningful cost functional, which is a function of state and control variables. For nonlinear optimal control, it is required to solve the Hamilton-Jacobi-Bellman (HJB) equation, which is a challenging task, then the inverse optimal control is proposed in this paper.

The aim of the inverse optimal control is to avoid the solution of the HJB equation [1], [2]. In this approach, a stabilizing feedback control law is designed first, and then it is established that this control law optimizes a meaningful cost functional. The main characteristic of the inverse approach is that the meaningful cost function is a posteriori determined from the stabilizing feedback control law. We refer the reader to [3], [4], [5] for inverse optimal control of continuous-time linear systems and to [2], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15] and references therein for the nonlinear continuous-time case. The discrete-time inverse optimal control has been treated in the frequency domain for linear systems in [3], [16] based on the return difference function fulfillment. Although, there already exist many important results on inverse optimal control for continuous-time nonlinear systems, the discrete-time case is seldom analyzed [17], [18], in spite of its advantages for real-time implementation.

On the other hand, for realistic situations, a control based on a plant model could not perform as desired, due to internal and external disturbances, uncertain parameters, or unmodelled dynamics [19]. This fact motivates the need to derive a model based on recurrent high order neural network (RHONN) in order to identify the dynamics of the plant to be controlled. Also, the RHONN model advantages are: easy implementation, relative simple structure, robustness, capacity to on-line adjust its parameters [20], [21], and incorporation of a priory information about the system structure.

In this paper, we propose a neural inverse optimal controller for discrete-time uncertain nonlinear systems in order to achieve exponential stability, minimizing a meaningful cost function, and avoiding the need to solve the associated Hamilton-Jacobi-Bellman equation. For this neural scheme, an assumed uncertain discrete-time nonlinear system is identified by a RHONN model, which is used to synthesize the inverse optimal controller from an appropriate control Lyapunov function. The neural learning is performed on-line through an extended Kalman filter (EKF) as proposed in [21].

The paper is organized as follows. Section II gives a brief review of optimal control and Lyapunov stability. Section III establishes inverse optimal control via control Lyapunov function. In Section IV the neural identification scheme for uncertain nonlinear systems is described. In Section V, the combination of the inverse optimal control and the neural identification is presented to achieve stabilization for uncertain nonlinear systems. Simulation results illustrate the applicability of the proposed control methodology. Finally, the conclusions are stated in Section VI.

II. MATHEMATICAL PRELIMINARIES

A. Optimal Control

This section briefly discusses the optimal control methodology and their limitations. Consider the nonlinear discrete-time affine system

\[ x_{k+1} = f(x_k) + g(x_k) u_k, \quad x_0 = x(0) \] (1)

with the associated meaningful cost functional

\[ V(x_k) = \sum_{n=k}^{\infty} l(x_n) + u_n^T R u_n \] (2)

where \( x_k \in \mathbb{R}^n \) is the state of the system at time \( k \in \mathbb{Z}_+ \cup \{ 0 \} = \{ 0, 1, 2, \ldots \} \), \( u \in \mathbb{R}^m \) is the control input, \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) and \( g : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m} \) are smooth mappings; \( f(0) = 0 \) and...
if for all vectors $z \neq 0$; $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$; $l : \mathbb{R}^n \rightarrow \mathbb{R}_+$ is a positive semidefinite\(^1\) function and $R : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times m}$ is a real symmetric positive definite\(^2\) weighting matrix. Meaningful cost functional (2) is a performance measure\(^[22]\). The entries of $R$ could be functions of the state system in order to vary the weighting on control effort according to the state value\([22]\).

Equation (2) can be rewritten as

$$V(x_k) = l(x_k) + u_k^T R u_k + \sum_{n=k+1}^{\infty} l(x_n) + u_n^T R u_n$$

$$= l(x_k) + u_k^T R u_k + V(x_{k+1})$$

(3)

where we require the boundary condition $V(0) = 0$ so that $V(x_k)$ becomes a Lyapunov function.

From Bellman’s optimality principle\((23, 24)\), it is known that, for the infinite horizon optimization case, the value function $V(x_k)$ becomes time invariant and satisfies the discrete-time (DT) Bellman equation\([25, 26]\)

$$V(x_k) = \min_{u_k} \{ l(x_k) + u_k^T R u_k + V(x_{k+1}) \}$$

(4)

where $V(x_{k+1})$ depends on both $x_k$ and $u_k$ by means of $x_{k+1}$ in (1). Note that the DT Bellman equation is solved backward in time\([25]\).

Minimization of (4) through $u_k$ is achieved by calculating the gradient of (4) right-hand side with respect to $u_k$, then

$$0 = 2R u_k - \frac{\partial V(x_{k+1})}{\partial u_k}$$

$$= 2R u_k + g^T(x_k) \frac{\partial V(x_{k+1})}{\partial x_{k+1}}$$

Therefore, the optimal control law is formulated as

$$u_k = -\frac{1}{2} R^{-1} g^T(x_k) \frac{\partial V(x_{k+1})}{\partial x_{k+1}}$$

(5)

with the boundary condition $V(0) = 0$. Hence, $V(x_k)$ can be used as a candidate Lyapunov function.

Moreover, $H(x_k, u_k)$ has a quadratic form in $u_k$ and $R > 0$, then

$$\frac{\partial^2 H(x_k, u_k)}{\partial u_k^2} > 0$$

holds as a sufficient condition such that optimal control law (5) globally\([22]\) minimizes $H(x_k, u_k)$ and the performance index (2)\([23]\).

Substituting (5) in (4), it results in the following DT Hamilton-Jacobi-Bellman (HJB) equation:

$$l(x_k) + V(x_{k+1}) - V(x_k) + \frac{1}{4} \frac{\partial V^T(x_{k+1})}{\partial x_{k+1}} g(x_k) \times R^{-1} g^T(x_k) \frac{\partial V(x_{k+1})}{\partial x_{k+1}} = 0.$$ 

Nevertheless, solving the partial differential equation (6) for $V(x_k)$ is not simple. Thus, to solve the above HJB equation constitutes the main disadvantage in discrete-time optimal control for nonlinear systems.

B. Stability Analysis

**Definition 1 (Radially Unbounded Function [27]):** A positive definite function $V(x_k)$ satisfying the condition $V(x_k) \to \infty$ as $\|x_k\| \to \infty$ is said to be radially unbounded.

**Definition 2 (Control Lyapunov Function [28]):** Let $V(x_k)$ be a radially unbounded, positive definite function, with $V(x_k) > 0$, $\forall x_k \neq 0$ and $V(0) = 0$. If for any $x_k \in \mathbb{R}^n$, there exist real values $u_k$ such that

$$\Delta V(x_k, u_k) < 0$$

where the Lyapunov difference $\Delta V(x_k, u_k)$ is defined as $V(x_{k+1}) - V(x_k) = V(f(x_k) + g(x_k) u_k) - V(x_k)$. Then $V(\cdot)$ is said to be a “discrete-time control Lyapunov function” (CLF) for system (1).

**Theorem 1 (Exponential Stability [29]):** Suppose that there exists a positive definite function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ and constants $c_1$, $c_2$, $c_3 > 0$ and $p > 1$ such that

$$c_1 \|x\|^p \leq V(x_k) \leq c_2 \|x\|^p$$

$$\Delta V(x_k, u_k) \leq -c_3 \|x\|^p, \quad \forall k \geq 0, \quad \forall x \in \mathbb{R}^n.$$ 

Then $x_k = 0$ is an exponentially stable equilibrium of system (1).

III. INVERSE OPTIMAL CONTROL

For the inverse approach, a stabilizing feedback control law is first developed, and then it is established that this control law optimizes a meaningful cost functional. When we want to emphasize that $u_k$ is optimal, we use $u^*_k$. We establish the following assumptions and definitions which allow the inverse optimal control solution.

In the next definition, we establish the discrete-time \{inverse optimal\} control.

**Definition 3 (Inverse Optimal Control Law):** The control law

$$u_k^* = -\frac{1}{2} R^{-1}(x_k) g^T(x_k) \frac{\partial V(x_{k+1})}{\partial x_{k+1}}$$ 

(8)

is inverse optimal (globally) stabilizing if

(i) it achieves (global) asymptotic stability of $x = 0$ for system (1);

(ii) $V(x_k)$ is (radially unbounded) positive definite function such that inequality

$$\overline{V} := V(x_{k+1}) - V(x_k) + u_k^T R u_k \leq 0$$

(9)

is fulfilled. When we select $l(x_k) := -\overline{V}$, then $V(x_k)$ is a solution for (6).

As established in Definition 3, inverse optimal control is based on the knowledge of $V(x_k)$; thus, we propose a CLF $V(x_k)$ such that (i) and (ii) can be guaranteed.
For the control law (8), let us consider a twice differentiable positive ($C^2$) definite function

$$V(x_k) = \frac{1}{2} x_k^T P x_k$$

(10)
as a CLF, where $P \in \mathbb{R}^{n \times n}$ is assumed to be positive definite ($P > 0$) and symmetric ($P = P^T$) matrix. Considering one step ahead for (10) and evaluating (8), we obtain

$$u_k^* = -\frac{1}{2} R^{-1} g^T(x_k) \partial V(x_{k+1})$$

$$= -\frac{1}{2} R^{-1} g^T(x_k) (P f(x_k) + P g(x_k) u_k^*).$$

(11)

which solving for $u_k^*$ it results in the following state feedback control law defined as:

$$\alpha(x_k) := u_k^* = -\frac{1}{2} (R + P_2(x_k))^{-1} P_1(x_k)$$

(12)

where $P_1(x_k) = g^T(x_k) P f(x_k)$ and $P_2(x_k) = \frac{1}{2} g^T(x_k) P g(x_k)$.

Once we have proposed a CLF for solving the inverse optimal control in accordance with Definition 3, the following theorem is established.

Theorem 2 (Inverse Optimal Control [30]): Let us consider the affine discrete-time nonlinear system (1). If there exists a matrix $P = P^T > 0$ such that the following inequality holds:

$$V_f(x_k) - \frac{1}{4} P_1^T(x_k) (R(x_k) + P_2(x_k))^{-1} P_1(x_k) \leq -\zeta_Q \|x_k\|^2$$

(13)

where $V_f(x_k) = |V(f(x_k)) - V(x_k)|$, with $V(f(x_k)) = \frac{1}{2} f^T(x_k) P f(x_k)$ and $\zeta_Q > 0$; $P_1(x_k)$ and $P_2(x_k)$ as defined in (12); then, the equilibrium point $x_k = 0$ of system (1) is globally exponentially stabilized by the control law (12), with the CLF (10).

Moreover, with (10) as a CLF, this control law is inverse optimal in the sense that it minimizes the meaningful functional given by

$$V(x_k) = \sum_{k=0}^{\infty} (l(x_k) + u_k^T R(x_k) u_k)$$

(14)

with

$$l(x_k) = -\nabla V|_{u_k^* = \alpha(x_k)}.$$

(15)

Proof: For proof details see [30].

IV. IDENTIFICATION

For realistic situations, a control law derived from a plant model can not perform as desired due to internal and external disturbances, uncertain parameters, or unmodeled dynamics [19]. This fact motivates to derive a model based on recurrent high order neural network (RHONN) [20] to identify the dynamics of the plant.

We analyze a general class of systems which are affine in the control with disturbance term as in [31].

A. Nonlinear Systems

Consider a class of discrete-time disturbed nonlinear system

$$\chi_{k+1} = \bar{f}(\chi_k) + \bar{g}(\chi_k) u_k + \Gamma_k$$

(16)

where $\chi_k \in \mathbb{R}^n$ is the system state at time $k$, $\Gamma_k \in \mathbb{R}^n$ is an unknown and bounded disturbance term representing modeling errors, uncertain parameters and unmodelled dynamics; $\bar{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $\bar{g} : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ are smooth mappings. Without loss of generality, $\chi_k = 0$ is an equilibrium point for (16). We assume $\bar{f}(0) = 0$ and $\text{rank}\{\bar{g}(\chi_k)\} = m$ $\forall \chi_k \neq 0$.

B. Discrete-Time Recurrent High Order Neural Network

To identify system (16), let consider the following discrete-time RHONN proposed in [21]:

$$x_{i,k+1} = w_{i,k}^T \rho_i(x_k, u_k)$$

(17)

where $x_k = [x_{1,k}, x_{2,k}, \ldots, x_{n,k}]^T$, $x_i$ is the state of the $i$-th neuron which identifies the $i$-th component of state vector $\chi_k$ in (16), $i = 1, \ldots, n$; $w_i$ is the respective on-line adapted weight vector and $u_k = [u_{1,k}, u_{2,k}, \ldots, u_{m,k}]^T$ is the input vector to the neural network; $\rho_i$ is a $L_p$ dimensional vector defined as

$$\rho_i(x_k, u_k) = \begin{bmatrix} \rho_{i_1} \\ \rho_{i_2} \\ \vdots \\ \rho_{i_L} \end{bmatrix} = \prod_{\ell \in I_1} Z_{i_{\ell_1}}^{d_{i_{\ell_1}}(1)} \prod_{\ell \in I_2} Z_{i_{\ell_2}}^{d_{i_{\ell_2}}(2)} \cdots \prod_{\ell \in I_{L_p}} Z_{i_{\ell_{L_p}}}^{d_{i_{\ell_{L_p}}}(L_p)}$$

(18)

where $d_{i_{\ell}}$ are nonnegative integers, $L_p$ is the respective number $p$ of high-order connections, $\{I_1, I_2, \ldots, I_{L_p}\}$ is a collection of non-ordered subsets of $\{1, 2, \ldots, n + m\}$, $Z_i$ is a vector defined as

$$Z_i = \begin{bmatrix} Z_{i_1} \\ \vdots \\ Z_{i_n} \\ Z_{i_{n+m}} \end{bmatrix} = \begin{bmatrix} S(x_{1,k}) \\ \vdots \\ S(x_{n,k}) \\ u_{1,k} \\ \vdots \\ u_{m,k} \end{bmatrix}$$

where the sigmoid function $S(\cdot)$ is defined by

$$S(x) = \frac{\alpha_i}{1 + e^{-\beta_i x}} - \gamma_i$$

with $S(\cdot) \in [-\gamma_i, \alpha_i - \gamma_i]$; $\alpha_i$, $\beta_i$ and $\gamma_i$ are positive constants.

We propose the following modification of the discrete-time RHONN (17) for the system described by (16):

- Neural weights associated to the control inputs could be fixed ($w_i^f$) to ensure controllability of the identifier.

Based on this modification and using the structure of the system (16), we propose the following neural network model:

$$x_{i,k+1} = w_{i,k}^T \rho_i(x_k) + w_i^T \psi_i(x_k, u_k)$$

(19)
in order to identify (16), where $x_i$ is the $i$-th neuron state; $w_{i,k}$ is the on-line adjustable weight vector and $w_i^*$ is the fixed
weight vector; $\psi$ denotes a function of $x$ or $u$ corresponding to the plant structure (16) or external inputs to the network, respectively. Vector $\rho_i$ in (19) is like (18), however $Z_i$ is redefined as

$$Z_i = \begin{bmatrix} Z_{i1} \\ \vdots \\ Z_{in} \end{bmatrix} = \begin{bmatrix} S(x_{1,k}) \\ \vdots \\ S(x_{n,k}) \end{bmatrix}$$

The on-line adjustable weight vector $w_{i,k}$ is defined as

$$w_{i,k} = \begin{bmatrix} w_{i1,k} & \cdots & w_{iL_p,k} \end{bmatrix}^T.$$

**Remark 1:** It is worth to notice that (19) does not consider the disturbance term $(\Gamma_k)$ due the RHONN weights are on-line adjusted, and hence the RHONN identifies the dynamics of the nonlinear system, including the disturbance effects.

**C. RHONN Models**

From results presented in [20], we can assume that there exists a RHONN which models (16); thereby, plant model (16) can be described by

$$x_{i,k+1} = W_k^* \rho_i(x_k) + W^{*\prime} \psi(\chi_k, u_k) + v_k$$ (20)

where $W_k^* = [w_{1,k}^T \ w_{2,k}^T \ \cdots \ w_{n,k}^T]^T$ and $W^{*\prime} = [w_{1,k}^{*\prime\prime\prime\prime} \ w_{2,k}^{*\prime\prime\prime\prime} \ \cdots \ w_{n,k}^{*\prime\prime\prime\prime}]^T$. These are the optimal unknown weight matrices, and $v_k$ is the modelling error, which can be rendered arbitrarily small by appropriately selecting the number of $L_p$ high-order connections [20]. The ideal weight matrices $W_k^*$ and $W^{*\prime}$ are artificial quantities required for analytical purposes. In general, it is assumed that they exist and are constants. Optimal unknown weight vectors $w_{i,k}^*$ will be approximate by the on-line adjustable weight vectors $w_{i,k}$ [21]. For identification of (16), two possible models for (19) can be implemented

- **Parallel model**
  $$x_{i,k+1} = w_{i,k}^T \rho_i(x_k) + w_{i,k}^T \psi(x_k, u_k)$$ (21)

- **Series–Parallel model**
  $$x_{i,k+1} = w_{i,k}^T \rho_i(x_k) + \rho_i^T \psi(x_k, u_k).$$ (22)

In this paper, (22) is used.

**D. On-line Learning Law**

For the RHONN weights on-line learning, we use an EKF as proposed in [21]. The weights become the states to be estimated; the main objective of the EKF is to determine the optimal values for the weight vector $w_{i,k}$ such that the identification error $e_{i,k} = \chi_{i,k} - x_{i,k}$ is minimized, where $\chi_i$ is the plant state and $x_i$ is the RHONN state. The EKF solution to the training is given by the following recursion:

$$
\begin{align*}
M_{i,k} &= [R_{i,k} + H_{i,k}^T P_{i,k} H_{i,k}]^{-1} \\
K_{i,k} &= P_{i,k} H_{i,k} M_{i,k} \\
w_{i,k+1} &= w_{i,k} + \eta_i K_{i,k} e_{i,k} \\
P_{i,k+1} &= P_{i,k} - K_{i,k} H_{i,k}^T P_{i,k} + Q_{i,k}
\end{align*}
$$ (23)

where vector $w_{i,k}$ represents the estimate of the weight (state) of the $i$-th neuron at update step $k$. This estimate is a function of the Kalman gain $K_i$ and of the identification error $e_{i,k}$. The Kalman gain is a function of the approximate error covariance matrix $P_i$, a matrix of derivatives of the network’s outputs with respect to all trainable weight parameters $H_i$ as follow

$$H_{i,k} = \frac{\partial e_{i,k}}{\partial w_{i,k}}^T$$ (24)

and a global scaling matrix $M_i$. Here, $Q_i$ is the covariance matrix of the process noise and $R_i$ is the measurement noise covariance matrix. As an additional parameter we introduce the rate learning $\eta_i$ such that $0 \leq \eta_i \leq 1$. Usually $P_i$, $Q_i$ and $R_i$ are initialized as diagonal matrices, with entries $P_i(0)$, $Q_i(0)$ and $R_i(0)$ respectively. We set to $Q_i$ and $R_i$ fixed. During training, the values of $H_i$, $K_i$ and $P_i$ are ensured to be bounded [21].

**Theorem 3 (Identification [21]):** The RHONN (17) trained with the EKF based algorithm to identify the nonlinear plant (16), ensures that the identification error $e_{i,k} = \chi_{i,k} - x_{i,k}$, is semiglobally uniformly ultimately bounded; moreover, the RHONN weights remain bounded.

**Proof:** For proof details see [21].

Stabilization for (16), defined in terms of the plant state $\chi_i$, can be established as the following inequality:

$$\|\chi_i\| \leq \|\chi_i - x_i\| + \|x_i\|$$ (25)

where $\|\cdot\|$ stands for the Euclidean norm. It is possible to establish inequality (25) based on the separation principle for discrete-time nonlinear systems [32]. From (25), we establish the following requirements for the neural network tracking and control solution:

**Requirement 1:**

$$\lim_{k \to \infty} \|\chi_i - x_i\| \leq \zeta_i$$ (26)

with $\zeta_i$ a small positive constant.

**Requirement 2:**

$$\lim_{k \to \infty} \|x_i\| = 0.$$ (27)

An on-line neural identifier based on (19) ensures (26) [21], while (27) is guaranteed by a discrete-time controller developed using the inverse optimal control technique [30]. A general control scheme is shown in Fig. 1.
V. NEURAL INVERSE OPTIMAL CONTROL APPLICATION

In this section, the applicability of the inverse optimal control for the neural identifier is illustrated by stabilizing an assumed uncertain nonlinear system as follows.

Model (22) can be represented as a system of the form (1), and in accordance with Theorem 2, if there exists $P = PT > 0$ satisfying condition (13), this system can be exponentially stabilized by the inverse optimal control law described by (12).

A. Example

We synthesize a neural inverse optimal control law for a discrete-time second order nonlinear system (unstable for $u_k = 0$) of the form (1) with:

$$f(\chi_k) = \begin{bmatrix} 0.5 \chi_{1,k} \sin(0.5 \chi_{1,k}) + 0.2 \chi_{2,k}^2 \\ 0.1 \chi_{1,k}^2 + 1.8 \chi_{2,k} \end{bmatrix}$$

(28)

and

$$g(\chi_k) = \begin{bmatrix} 0 \\ 2 + 0.1 \cos(\chi_{2,k}) \end{bmatrix}.$$  
(29)

The phase portrait for this unstable open-loop ($u_k = 0$) system with initial conditions $\chi_0 = [2 - 2]^T$ is displayed in Fig. 2.

1) Neural Identifier for (28)-(29): Assume system (28)-(29) to be unknown. In order to identify this uncertain system, from (19) and (22), we propose the following series–parallel neural network

$$\begin{align*}
x_{1,k+1} &= w_{11,k} S(\chi_1) + w_{12,k} S(\chi_2)^2 \\
x_{1,k+1} &= w_{21,k} S(\chi_1)^2 + w_{22,k} S(\chi_2) + w'_2 u_k
\end{align*}$$

(30)

which can be rewritten as $x_{k+1} = f(x_k) + g(x_k) u_k$, where

$$f(x_k) = \begin{bmatrix} w_{11,k} S(\chi_1) + w_{12,k} S(\chi_2)^2 \\ w_{21,k} S(\chi_1)^2 + w_{22,k} S(\chi_2) \end{bmatrix}$$

(31)

and

$$g(x_k) = \begin{bmatrix} 0 \\ w'_2 \end{bmatrix}.$$  
(32)

with $w'_2 = 0.8$. Initial adjustable weights are given in a random way.

2) Control Synthesis: Consider the control law (12) for which we select $P = \begin{bmatrix} 2 & 1.5 \\ 1.5 & 2 \end{bmatrix}$ and $R = 0.5$.

3) Simulation Results: Fig. 3 presents the stabilization time response for $x_k$ this system with initial conditions $\chi_0 = [2 - 2]^T$. Initial conditions for RHONN are $x_0 = [0.5 \ 2]^T$. Fig. 4 displays the applied inverse optimal control law $\alpha(x_k)$, which achieves exponential stability; this figure also includes the meaningful cost functional evaluation.

VI. CONCLUSION

A neural inverse optimal control scheme to achieve stabilization for discrete-time uncertain nonlinear systems is presented. We use discrete-time recurrent neural networks to model the uncertain system and based on this neural model, then the inverse optimal controller is synthesized, which uses a quadratic control Lyapunov function to achieve exponential stability. An example illustrates the applicability of the proposed control technique.

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