Optoelectronic Fractal Scanning Technique for Wavelet Transform and Neural Net Pattern Classifiers

Sonlinh Phuvan, Tae Kwan Oh, Nick Caviris, Yao Li*, Harold Suzuki**
NAVSWC, Code G43, **Code R44, Silver Spring, MD 20993
*City College of New York, Dept. of EE**, New York, NY

ABSTRACT
A 1-D scan which follows Peano's curve to a desired resolution is demonstrated to preserve 2-D proximity relationship and is furthermore shown to be efficient for Wavelet Transform (WT) processing and artificial neural network pattern recognition. This deterministic fractal sampling method can be implemented in real time using optoelectronic scanning. For example, 2-D texture patterns are analyzed by using 1-D Wavelet Transformation (WT). Those WT coefficients can be fed into a standard backpropagation neural network for pattern recognition. To speed up training time, a top down design which generalize Hopfield's energy landscape approach is given in terms of Min-Max pattern classifiers.

1 Introduction
A space-filling curve with a lower dimensionality that preserves a neighborhood relationship has been an important recurrent theme throughout the last century. It began with Peano's mathematical proof and Hilbert's verification in the last century. Recently, numerous applications have been developed, e.g. the meal on wheel program, computer map storage, optical neural network storage of 4-D interconnects within a 3-D photorefractive crystal (with 3/2 fractal dimensionality), and neural networks. We will demonstrate a fractal scanning technique which can be implemented using optoelectronic methods and is useful for real time neural net pattern recognitions.

A critical element for a solution in pattern detection is how to represent the data in such a way as to facilitate subsequent processing. Some signal processing techniques are more easily processed if the data is in a one dimensional form than in a two dimensional form. Some techniques such as raster scanning and its derivatives do not preserve the adjacency of information (i.e. two points on adjacent rows in a 2D image may be farther apart in the unidirectional or bidirectional raster scan representation then two points which are farther apart in the 2D image). A complete basis of Peano's curve is introduced for the first time.

Texture analysis is important in image processing for detection of patterns of interest in two dimensional data representations such as overhead imagery, infraograms and VLSI wafer inspection.

Several approaches to this problem that have been used, include standard digital image processing techniques (i.e. image transforms), fractal dimensions and fractal signatures. Those techniques have been successful for the special cases for which they were developed but not in the general case because they are found to provide nonunique measures in some test cases. A new nonlinear signal processing technique based on artificial neural networks and fractal measures has been used with some success and seems promising.

2 Fractal Scanning
In order to preserve adjacency in the mapping of a two dimensional tensor image to a one dimensional vector image, the scanning must fill space in a neighborhood before moving to another neighborhood. As can be seen in Fig. 1 the proximity relationship of images is better preserved by a fractal scan then a bidirectional raster scan. The deterministic fractal scan, as scanning density increases, shows self similarity with respect to scales.

To preserve orientation the scan output must produce identical results independent of the scan direction (i.e. top-down, bottom-up, right-left and left-right). This restricts the scan to symmetric orderly scans. The Peano space filling scan is composed of 4 3x3 scan cell primitives (Fig. 2).

![Figure 1. Scan Proximity Relationship](image)

![Figure 2. Peano Scan Cell Primitives](image)

In order for the sampling to be self similar for all scales and to preserve orientation at all scales, the scan must be an orderly, symmetric and antichiral fractal scan (Fig. 3).

Given an \(N \times N\) scan the angular resolution is \(N^2/2 - 3(N - 1)\) and scale dynamic range is \(N\). The scan sampling is at the correct sampling density when the fractal dimension of the fractal scan is equal or greater than the feature of the image with the largest fractal dimension.
The fractal scanning can be implemented by programming a photocathode tube to scan in the desired sequence (Fig. 4). A computer controls a driver which modulates an electron beam to scan in the desired sequence. The output of the TV tube is then the fractal scanning of the 2D image. By selecting a moderately fast TV tube this fractal scan can be implemented in real time (video rates).

3 Wavelet Transform Processing

A Wavelet transform and its inverse are defined by

\[ W(a, b) = \int f(x) g_{a,b}(x) dx, \quad f(x) = \int \int W(a, b) g_{a,b}(x) dx db /C \]

where \( g(x) \) is the mother wavelet and \( \{ g_{a,b} = g((x - a)/b) \sqrt{a} \} \) is the set of daughter wavelets and \( C = \int \int \left| \mathcal{F}(g(x)) \right| dx < \infty, \mathcal{F}(g(x)) \geq 0 \).

The Wavelet function must satisfy certain conditions; the area of the Wavelet function must be equal to zero, it must be of finite extent and decrease at least geometrically fast for increasing \( |x| \) (i.e. \( C < \infty \)). The Wavelet function must be selected in such a way as to enhance the desired features of the image. In this case the images are binary and thus a bipolar Wavelet function is selected (i.e. the Haar function, see Fig. 5).

The Haar function applied to a bipolar image through a Wavelet transform will extract edge features from a given image (Fig. 6). For each Wavelet transformed image a measure of the number of edges it detects is obtained by summing all correlation points for a given scale and adjusting the sum with respect to the scale,

\[ E(a) = \left( \frac{1}{2} \sum_{x} \left( \sum_{y} f(x, y) g((x - y)/a) \right) \right)^{-1} \]
A test example (Fig. 6) is used to test the algorithm. The top figure is a 1-D object comprised of two bars with four edges. The bottom left figure is the representation of a WT using the Haar function with 11 daughters (WT over 12 different scales). The bottom right figure is the graph of the measure of the number of edges versus the scale (i.e., sampling density or resolution).

It can be seen from Fig. 6 that as the scale increases the measure of the number of edges initially stays constant then monotonically decreases. This is because the sampling resolution becomes inadequate to measure the desired features. This implies that the sampling resolution can be adjusted in such a way as to discriminate against undesirable features.

![Figure 7. Thin Diagonal Lines](image)

This monotonic decrease of the measure of edges with increasing scale, and the asymptotic approach to a limit as the scale becomes finer is similar to the way in which fractal dimensions are measured. The fractal dimension typically found in the limit of the ratio of the log of the number of features measured with the log of the scale used to measure those features as the scale becomes very small. This implies that the fractal dimension can also be measured as follows.

$$D_f = \lim_{a \to 0} \left( \frac{\log \sum_{b=1}^{2^k} f(x_b) g((x_b - b)/a)}{\log \sum_{b=1}^{2^k} g((x_b - b)/a)} \right)^{-1}$$

Thus the fractal dimension of an image is contained within its wavelet transform.

![Figure 8. Diagonal Lines](image)
A fractal scan for six different simple geometric patterns is performed and the output is processed via a Wavelet transform. The six geometric figures can be subdivided into two classes; periodic array of diagonal lines and stacks of circles. Within each class the thickness and number of features are varied.

In Figs. 9-12, the upper left graph is the scan pattern and the 2D input image, the lower left graph is the 1D result of the fractal scan, the upper right graph is the WT of the 1D output of the fractal scan using the Haar function over 12 scales (i.e. one mother and 11 daughters), and the lower right graph is the plot of the measure of the edges with respect to the scale used during the WT processing.

![Scan Pattern and 2D Image](image1)

![Wavelet Transform](image2)

Figure 9. Thicker Diagonal Lines

![Scan Pattern and 2D Image](image3)

![Wavelet Transform](image4)

Figure 10. Thickest Diagonal Lines

It can be seen that as the thickness of the diagonal lines increases, certain periodicity in the Wavelet transform becomes apparent (Figs. 7-12). The measure of the number of edges decreases monotonically as the scale increases (Figs. 7-12).

It can be seen from Fig. 7-12 that the Wavelet transform using the Haar wavelet function produces distinctly different output for both the intraclass and interclass images. This implies that the Wavelet transform of a modified Peano fractal scan using the Haar wavelet function can be used for highly discriminatory 2D pattern classification. A candidate for classification is to feed those wavelet coefficients into a multiple layer artificial neural network with a standard backward error backpropagation training algorithm. However, in order to speed up training time, we considered in Sect. 4 other approaches.
4 Artificial Neural Net Classification

4.1 Designs of Energy Cost Functions in a Neuronic Vectorial Representation

An important question for practical applications is how to speed up the training process and to ensure a fast convergence of weight adjustment? We have suggested a general procedure of Taylor series expansion of the clustering-declustering mini-max energy to estimate the synaptic weights. Here, we extend the procedure by a self-consistently variational technique to make the truncated higher order terms of the Taylor series negligible.

A top-down design of a hard-wired neural network algorithm has been initiated by Hopfield et al for constrained optimizations. We consider a supervised top-down design goes beyond Hopfield's attempt. The minimum clustering of the alike and the maximum de-clustering of the dis-alike seems to be two contradicting goals. A tradeoff can be mathematically constructed by the linear combination of those pairs alike in the numerator and the pairs of dis-alikes in the denominator of the mini-max energy formalism (schematically shown in the cost energy expression, as follows)
4.2 Top Down Design of Hard Wired Neural Network Mini-Max Energy Principle

To speed up training time, a top down design which generalizes Hopfield's energy landscape approach is given in terms of mini-max pattern classifiers. The mini-max energy function is defined to be

\[ E(f) = a \sum_{c} |f_c| + b \sum_{c=1,2} |f_c - f_c^0| + d |f_c - f_c^0| \]

where \( I \) is a template of the c-class = 1,2 together with feature \( f \), and the coefficient of the direction cosine via inner product \( \langle \phi \rangle \) may be heavily weighted, eg by setting \( a = 10 \) (relative to \( b = 1 \), and \( d = 10 \)). Because the template image is fractal scanned to preserve the neighborhood proximity relationships, a Taylor series expansion along the Peano curve can be used to derive multiple layer interconnects. While the first order derivative is reserved for the neurodynamics equations, the second order derivative is evaluated at the equilibrium state: \( V_c = V_c^{eq} \), \( V_c = V_c^{eq} \) becomes the Taylor's coefficient, identified as a hard-wired Hopfield interconnect weights,

\[ \frac{\partial^2 E}{\partial V_c \partial V_j} \]

Then, the Hopfield-like hard-wired interconnect \( T_{ij}^{eq} \) becomes softwired \( T_{ij} \) by means of the Hebbian learning that makes the cubic order negligible.

\[ T_{ij} = T_{ij}^{eq} + \sum V_c \Delta V_{ij} \]

Similarly, the procedure can be extended to three layers.

5 Results

A fractal scan has been introduced which preserves orientation and proximity relationships. Once texture images are fractal scanned, the scanned binary template are fed to determine feature vectors by using Cauchy simulated annealing technique. Fig. 13 shows two extracted features from two texture inputs, thin diagonal lines and thicker circles. The diagonal feature output does not have pixels at the center, which makes it orthogonal to the other feature. In general, when an object is sampled above the Nyquist rate (the size of the Peano scan cell is compatible with the size of the object), the feature output would be more distinguishable. A hetero-associative memory, constructed between input and its feature, is used to classify these two classes as follow.

\[ f_c = \sum_{i,j} T_{ij} f_i f_j \]

The Wavelet transform of the fractal scan produces distinct output for both intraclass and interclass images implying possible uses as input for subsequent classification processes such as artificial neural networks.

A relationship between the edge measures obtained for the Wavelet transform using the Haar function and a real time optoelectronic implementation of the fractal using computer controlled TV tubes has been proposed.

6 Conclusions

A fractal scanning method has developed which preserves 2D proximity relationships and orientation. The complete basis for making Peano curves is introduced.

This technique is very well matched for use in Wavelet transform processing because of its self similarity with respect to scale and because its 1D vector data representation simplifies Wavelet transform analysis. The Wavelet transform of a 1D vector produces a 2D output, but that of a 2D tensor produces a 4D output, this makes it difficult to analyze. The Wavelet transform is essentially a correlation of a reference function with an arbitrary waveform in which the size of the reference function is changed with respect to scale (i.e. a correlation at various scales). The self similarity with respect to scale of the fractal scan match the variation of scale of the Wavelet transformation.
Using the nonlinear classification and feature extraction properties of artificial neural networks, operating on the output of the Wavelet transform analysis of the fractal scanned image, may produce a high discrimination and high sensitivity pattern detection and classification technique.

One potential classification technique is to use the Mini-Max type artificial neural networks to find orthogonal Wavelet basis functions to classify image textures. The Mini-Max algorithm is ideally suited for this purpose since it tries to maximize interclass differences and minimize intraclass differences (i.e. orthogonalize classification features). This may subsequently allow us to produce an alphabet of texture primitives for texture classification.

7 References