AOA GEOLOCATION FOR FAST-MOVERS USING NONLINEAR OPTIMIZATION

Stephen Hartzell, Marshall Haker, Richard Martin, Clark Taylor, Andrew Terzuoli

Institute of Electrical & Electronic Engineers (IEEE), Dayton, Ohio USA

1. INTRODUCTION

When observing an object of interest (herein simply called “object”), determining the geo-location of the object as accurately as possible, together with an accurate estimate of the geo-location uncertainty, is a key step in many remote sensing applications. To compute the geo-location of an object, Angle of Arrival (AoA) measurements can be used from multiple different sensor platforms such as aerostats, aircraft, and satellites. In a previous paper, nonlinear optimization (NLO) was used to provide an optimal geolocation estimate which converges to the minimum mean squared error (MMSE) for the problem of a stationary object and simultaneous measurements. This paper describes a generalization of the same (NLO method applied to fast-moving objects and non-simultaneous measurements. For some sensor platforms, the object may be so far from the sensor that it moves significantly by the time the sensor measures the AoA. This is especially true for satellites in GEO or HEO. The time-delay from the Earth to such satellites may be as much as 200 ms, meaning the sensors produce measurements of where the object was rather than where it is at the time of measurement. In addition, sensors rarely produce measurements at the same time. This temporal incongruence of measurements can significantly complicate the geo-location problem for a rapidly moving object. Thus a scheme which can produce an optimal geolocation estimate and a confidence estimate for this scenario is desirable. Nonlinear optimization is proposed and examined as a means of satisfying this objective.

2. DEVELOPMENT OF GEOLOCATION SCHEME

In the previous paper on NLO for geolocation, a single location estimate was computed from the different AoA estimates. With fast-moving object, however, the velocity of the object must be computed as well, so the quantity to be estimated is

\[
\mathbf{X} = \begin{bmatrix} X_x & X_y & X_z & V_x & V_y & V_z \end{bmatrix}^T,
\]

where \( x, y, \) and \( z \) are the axes for the location coordinate system (Earth-Centered Earth-Fixed – ECEF is used in our system) in which the object is being located, and \( v_{\text{?}} \) is the corresponding velocity for the subscript axis (\( x, y, \) or \( z \)).

To perform nonlinear optimization, Gauss-Newton iterations are performed by linearizing about the most recent solution, producing a weighted over-
determined system, and solving this system to find the incremental update to the solution. Given an initial guess of the object’s location, this system is solved iteratively to converge to the optimal geolocation estimate.

The first requirement to perform Gauss-Newton iterations is to have an initial estimate. To compute an initial estimate for this system, a simple triangulation system is used assuming that all measurements were collected at the same time and that the object is not moving. This step is repeated for two different time steps and a velocity estimate is computed as the difference between the two triangulation estimations.

The second requirement for NLO is to have a linear model of the geolocation problem at the current estimate. To do this the measurements must be described as a function of the object’s state. The AoA measurements produce by sensors each of which measures the line-of-sight to the object at a given time. These line-of-sight measurements are described by two angles: an azimuthal angle and an elevation angle \( \theta \) and \( \phi \), respectively. The noise in the in these angular measurements is assumed to be Gaussian with a mean of the true line-of-sight to the object. If the object were stationary, the measurements could be described as

\[
\Omega_{\text{stat}}(P) = \left[ \begin{array}{c}
\theta(P) \\
\phi(P)
\end{array} \right]^T
\]

\[
= \left[ \begin{array}{c}
\tan^{-1}\left( \frac{P_y}{P_x} \right) \\
\tan^{-1}\left( \frac{P_y}{\sqrt{P_x^2 + P_y^2}} \right)
\end{array} \right]^T
\]

where \( P \) is the object’s position.

Since the object is moving and the measurements undergo time-delay, this model is no longer sufficient. Rather, the measurements are observations of where the object was previously located. Therefore, the time-delay must be added to this model. Furthermore, it may be desirable to localize the object at times other than the time of measurement. Thus, the difference between the time of measurement and the desired geo-location time must be taken into account. The measurements at the time of measurement as a function of the object at the time of estimation is then given by

\[
\Omega(X(t_e); t_m) = \Omega_{\text{stat}} \left( P(t_e) - \int_{t_m - \tau}^{t_e} V(t) \, dt \right).
\]

In the above equation, \( \tau \) is the time-delay, \( t_e \) is the time at which the object is localized, \( t_m \) is the time of measurement, and \( V \) is the velocity of the object. \( X = [P \ V]^T \) A simplifying assumption is that the object’s motion is approximately linear between \( t_m \) and \( t_e \). Thus \( V \) is an unknown constant.

The equation above then becomes

\[
\Omega(X(t_e); t_m) = \Omega_{\text{stat}} \left( P(t_e) - \tau V - (t_e - t_m)V \right).
\]

A remaining difficulty is \( \tau \). However, \( \tau \) is also expressible in terms of known quantities. It is a function of the sensor’s position and the object’s position and velocity. The sensor’s position at the time of measurement \( S(t_m) \) is known therefore,
the equation may be express solely in terms of \( P(t_e) \) as

\[
\Omega(P(t_e); t_m) = \Omega_{\text{real}}(P(t_e) - \tau(P(t_e)S(t_m))V - (t_e - t_m)V).
\]

This equation may be linearized by making the assumption

\[
\Omega(X) \approx \Omega(X_0) + J(X - X_0),
\]

where \( J \) is the Jacobian of \( \Omega \). It is assumed that this model is accurate in the region about \( X_0 \).

Several measurements of the object may be combined from multiple sensors and/or the same sensors over time to form an over-determined system of equations may be produced. Furthermore, the accuracy of each sensors’ measurements may be included via a covariance matrix \( \Sigma_{\Omega} \). The initial estimate of the object’s state is \( X_0 \). This initial estimate may be used to iteratively solve this weighted system of nonlinear equations for a more likely state estimate.

3. EVALUATION METHODOLOGY

This nonlinear optimization scheme is evaluated using simulated data. The simulator is capable of generating measurements from multiple different sensors, sensor altitudes (satellites from low to high-earth orbits), frequency of sensor observations, and different velocities for the objects being geo-located. For this paper, the simulation consists of a system of pseudo-randomly generated satellites. These are generated with the constraint that they must exist within real orbits, and their line of sight must not be occluded by the Earth. Measurements are assumed to be normally distributed with a mean of the true line-of-sight to the object. Measurements are synthesized for two different objects and were generated for two purposes. The first object moves with constant velocity. This object is generated to verify that the NLO converges to an optimal unbiased estimate. The static NLO is also used on this object to compare the performance of the robust NLO that includes time-delay and velocity.

The second object is a realistic worst-case object. The worst-case object is the fastest realistic object that might be observed. This provides a lower bound for the performance of the NLO. The accuracy and confidence improvement provided by this algorithm is excellent compared to the previous nonlinear optimization algorithm that accounts for neither velocity nor time-delay (static NLO). On average, the NLO accuracy of the position estimate from approximately 10km to 100 m compared to the static NLO. The velocity estimates accuracy was improved from 500 m/s to 50 m/s. The NLO’s confidence also matched the MMSE.

4. CONCLUSIONS

The results demonstrate that for fast-moving objects, the NLO which includes the object’s velocity and the time-delay in its optimization is vastly superior to the static NLO for both objects. The NLO is an optimal unbiased estimator of the ideal constant velocity object. The confidence in its estimate also appears to be very accurate. The NLO’s confidence matches the MMSE, and so it neither overestimates nor
underestimates its confidence in our results. In the case of the realistic worst-case object, the NLO is no longer optimal since the realistic object’s complex motion is not accounted for in the model. However, the NLO drastically outperforms the static NLO and the initial estimates provided by triangulation. Its confidence surface is also very accurate although it is slightly overconfident.

5. BIBLIOGRAPHY


