Abstract—The paper debates a novel solution for the long-term monitoring of the system polarimetric quality based on radiometrically and polarimetrically stable targets, herewith named Polarimetric Permanent Scatterers (PPS). The technique is completely scene-based and thus cost effective. It allows, aided by the integration with the available Distributed Target (DT) information, for the relative calibration of the channel imbalance and cross-talk parameters. Their complex time-series information is indeed extracted with respect to the absolute unknown values of an arbitrary image of the stack. The performance achieved on a Radarsat-2 stack show that the accuracy is consistent with that returned by DT techniques. The attention shall be however focused on the fact that the more information can actually be extracted thanks to the proposed monitoring method.

Index Terms—Synthetic Aperture Radars, Polarimetric calibration, Permanent Scatterers, Multi-temporal analysis.

I. INTRODUCTION

Full-polarization acquisitions in spaceborne systems are operationally attained by alternating in transmission (Tx) the horizontal (H) and vertical (V) polarizations and by recording them simultaneously at reception (Rx). In such architecture, the signal can be affected by amplitude and phase perturbations and by channel cross-talks (CTs). The former lead to fluctuations in

- \( A \), the overall radiometric gain of the system, referred to the HH channel
- \( f_1 \), the complex imbalance ratio between the vertical and the horizontal polarizations in the Rx phase (ideally 1)
- \( f_2 \), the complex imbalance in the Tx phase (ideally 1)

whereas the latter contribute to

- \( \delta_1 \), the fraction of H signal leaking into the V receiver
- \( \delta_2 \), the fraction of V signal leaking into the H receiver
- \( \delta_3 \), the fraction of H field transmitted alongside the V pulses
- \( \delta_4 \), the fraction of V field transmitted alongside the H pulses.

The overall system distortion is mathematically accounted into the observation \( \mathbf{Y} \) of the target backscatter matrix \( \mathbf{S} \) through the model:

\[
\mathbf{Y} = \mathbf{A} \cdot e^{j\phi} \cdot \mathbf{R}^T \cdot \mathbf{S} \cdot \mathbf{T} + \mathbf{N},
\]

with:

\[
\mathbf{Y} = \begin{bmatrix} y_{hh} & y_{hv} \\ y_{hv} & y_{vv} \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} s_{hh} & s_{vh} \\ s_{hv} & s_{vv} \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} n_{hh} & n_{vh} \\ n_{hv} & n_{vv} \end{bmatrix}, \quad \mathbf{R}^T = \begin{bmatrix} 1 & \delta_2 \\ \delta_1 & f_1 \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{bmatrix}
\]

where the \( hh, hv, vh, vv \) subscripts refer to the 4 different polarization modes, \( \phi \) is the overall target phase, \( \mathbf{N} \) is the signal noise (both thermal noise and deviations from the linear distortion model).

The accuracy requirements for the each distortion parameter are in general agreed by the scientific community as a result of assessment analyses such as [2], [7] to name a few. Some common requirements for a polarimetrically calibrated system are reported in Table I. Two types of information are commonly adopted for calibrating the system: 1) Calibrated Point Targets (herewith referred to as PTs) which can be either active, such as the PARC, or cheaper passive targets such as trihedral and dihedral reflectors. 2) Distributed Targets (DTs) which take advantage of the reciprocity constraint, \( s_{hv} = s_{vh} \), and of the supposed reflection symmetry, i.e. \( \langle s_{hh} s_{vh}^* \rangle = \langle s_{vv} s_{hv}^* \rangle = 0 \), where \( \langle \cdot \rangle \) stands for the ensemble average.

A single PT as well as a DT alone cannot provide full polarimetric calibration, but only estimate some unambiguous subset of the distortion parameters. In particular, the DT-based approach provides unambiguous estimates of the cross-talks of the imbalance ratio [8], though it cannot guarantee radiometric and phase consistency of the channels throughout the acquisitions. It is then necessary to use three or more PTs (the reader is referred to [3] for an overview) or to make a joint use of the DT and PT [4], [9]. It must be then remarked that the calibration process should not be regarded just as a once only operation but as part of a timely mission monitoring framework. This implies that, besides the deployment of a large number of PTs to assess the performance of all the beam modes, such sites should undergo frequent mainaitance in order to guarantee the required calibration accuracy.
Herewith a novel scene-based technique is proposed to deal in a cost-effective way with the long-term monitoring of spaceborne polarimetric systems. The idea is that of exploiting the natural stable targets in the scene, hereon referred to as Polarimetric Permanent Scatters (PPSs), as calibrated PTs. The complexity introduced, related to the fact that the PPSs need to be found and “calibrated” (their polarimetric signature must be retrieved) before being employed, required the design of an appropriate PS framework, outlined in section II. The algorithm is not constrained to a specific PDM model, thus its implementation is practically feasible for any SAR sensor. The PPS-derived PDMs contain though ambiguous distortion information, which can find an absolute determination only by means of two different kinds of external information: 1) by using at least one calibrated image out of the whole stack for a full disambiguation [5] and 2) by assimilation with DT information for a partial resolution of the ambiguity. The second methodology is discussed by the paper, specifically in section III. It will be shown that an unambiguous temporal monitoring of all the distortion parameters, up to an absolute radiometric scale factor, can be achieved. Section IV will be finally dedicated to the technique results obtained on a Fine Quad-Pol Radarsat-2 dataset comprising 26 images over the Barcelona area.

II. THE POLARIMETRIC PS FRAMEWORK

A. The PPS Model

Let convert the generic observation in (1) in its vectorized form, i.e.,

\[ y_4 = A \cdot e^{j\phi} \cdot G \cdot P_{4-3} \cdot s + n_4 \]  

(3)

with

\[ y_4 = \begin{bmatrix} y_{hh} \\ y_{hv} \\ y_{vh} \\ y_{vv} \end{bmatrix}, s = \begin{bmatrix} s_{hh} \\ s_{hv} \\ s_{vh} \\ s_{vv} \end{bmatrix}, n_4 = \begin{bmatrix} y_{hh} \\ y_{hv} \\ y_{vh} \\ y_{vv} \end{bmatrix}, \]

\[ P_{4-3} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = (T^T \otimes R^T) \]  

(4)

Polarimetric Distortion Matrix (PDM) of the system and \( \otimes \) standing for the kronecker product. A convenient functional and statistical model in both time and space (target index) shall now be defined for the PPSs. To this aim, the model employed in [1] for single-pol radiometric calibration has been borrowed and extended to the full-pol scenario. Let consider an uncalibrated stack comprising \( N_p \) Permanent Scatters observed for \( N_f \) acquisitions. The total backscatter \( s_{i,p} \) returned by the \( p \)-th cell into the \( i \)-th image of the stack can be further expressed as

\[ s_{i,p} = x_p + w_{i,p} \]

(5)

where \( x_p \) is the stable PPS component, thus only dependent on the target number, and \( w_{i,p} \) is the backscatter fluctuation, hereon referred to as clutter, statistically modeled with:

\[ w_{i,p} \sim \mathcal{CN}(0, C_p) \]

(6)

i.e., a generic stationary circular complex gaussian process. The system (3) can then be rephrased as

\[ y_{4,i,p} = H_i \left(e^{j\phi_{i,p}}x_p + w_{i,p}\right) \]

(7)

where \( \phi_{i,p} \) is the target phase, the 4 by 3 PDM \( H_i = A_i G_i P_{4-3} \) contains now the radiometric gain, and where it was further supposed that the clutter fluctuations \( \hat{w}_{i,p} \) represent the most significant disturbance, even in the weak HV channel, thus making the residual noise \( n_4 \) negligible. The dependence of each element on the image or target number has been then explicited in the subscripts. Notice in particular that in (7) the distortion \( H \) is only image-dependent. This is indeed the second fundamental model assumption after (5) and relies on the fact that the distortion parameters can be safely assumed as slow-varying across the image, usually with a preferred range variation trend for the antenna distortion. The processing can be performed upon limited portions of the image, that will be called ‘imagettes’, where the parameters are supposed to be uniform.

B. The PolPSCal Inversion

In the PS calibration problem the \( N_f \) \{H\}, the \( N_p \) \{s, C\} and the \( N_f \times N_p \) phases \( \phi \) must all be considered unknowns. It is nonetheless readily demonstrated that their retrieval is feasible for \( N_f \geq 3 \) unless the intrinsic \( 3 \times 3 \) ambiguity matrix \( K \) common to the whole stack and the ambiguous phase terms \( \psi_i, \xi_p \) for each image and target:

\[ H_i \left(e^{j\phi_{i,p}}x_p + w_{i,p}\right) = H'_i \left(e^{j\phi'_{i,p}}x'_p + w'_i\right) \]

(8)

\[ H'_i = e^{j\psi_i} H_i \cdot K, \quad x'_p = K^{-1}e^{j\xi_p}x_p \]

(9)

\[ w' = K^{-1}w \rightarrow C_p = K^{-1}C_pK^{-H} \]

Whereas the determination of the phase terms is arbitrary, the retrieval of \( K \) will be of utmost importance once the ambiguous estimates are computed.

The inversion scheme conceived is shown in Fig. 1 and relies on a Least Square optimization procedure aiming to obtain:

\[ \hat{H}_i, x_p, \hat{\phi}_{i,p} = \argmin_{H_i, x_p, \phi_{i,p}} \left(\sum_i \sum_p \left| y_{i,p} - e^{j\phi'_{i,p}}H'_i x'_p \right|^2\right) \]

(10)

The processing chain includes a Generalized Likelihood Ratio Test (GLRT) based PPS detection which is performed in combination with the estimation steps in an iterative way. The aim is to jointly achieve a gradual refinement of the estimates \( \hat{H}_i, x_p, \hat{\phi}_{i,p} \) and the enlargement of the selected PPS subset. The reader is referred to [6] for further details. At the end of the estimation-detection iterations, the ambiguity \( K \) must be retrieved through external scene information.
III. RESOLVING THE AMBIGUITY BY DT ASSIMILATION

One or more calibrated images (thus known a-priori) would be a convenient and straightforward way to calibrate the other images of the stack. However, when no reference PDM is available the demanded information can be extracted from the scene itself. A convenient DT with azimuth symmetry must be delineated to this aim. The DT-based techniques extract the distortion parameters from second-order statistics (covariance matrix) of areas with expected orientation symmetry. The 4 × 4 model-based matrix, $\mathbf{H}^D = \mathbf{H}(\Theta_D)$, is returned, where $\Theta_D$ is a set of unambiguous distortion parameters. In case of no Faraday $\mathbf{H}$, is related to $\mathbf{H}^D$ through:

$$\mathbf{H} = \mathbf{A} \cdot \mathbf{H}^D \cdot \mathbf{P}_{4-3} \cdot \mathbf{F}$$  \hspace{1cm} (11)

$$\mathbf{H}^D = \begin{bmatrix} 1 & \alpha \delta' & \delta' & \alpha \delta'^2 \\ \delta_1 & \alpha & \delta_2 & \alpha \delta'_2 \\ \delta_3 & \alpha \delta_2 & \delta_1 & \alpha \delta'_3 \\ \delta_1 \delta_2 & \alpha \delta_3 & \delta_1 & \alpha \end{bmatrix}$$  \hspace{1cm} (12)

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_2^2 \end{bmatrix}$$  \hspace{1cm} (13)

with:

$$\alpha = \frac{f_1}{f_2}, \quad \delta'_2 = \frac{\delta_2}{f_1}, \quad \delta'_4 = \frac{\delta_4}{f_2}$$  \hspace{1cm} (14)

and $\mathbf{A}, \mathbf{F}$ represent undetermined components of the model. The parameters introduced in (12) and (14) form:

$$\Theta_D = \{\alpha, \delta_1, \delta'_2, \delta_3, \delta'_4\}$$  \hspace{1cm} (15)

i.e. the largest unambiguous set attainable through DT estimation. The calibration of the data by DT is possible up to an imbalance amplitude and phase factor changing from polarization to polarization and from time to time, if the distortion is not stable throughout the stack. The idea herehence promoted is that of exploiting $\mathbf{H}^D$ as external information to disambiguate the PolPSCal PDMs, $\mathbf{H}$. Either a single image or the full stack estimates can be used in the process. The case of the whole stack is here discussed. For a generic image $i$, the DT estimates $\hat{\mathbf{H}}^{D_T}$ and the ones from (10) are related through:

$$\mathbf{A}_i e^{j\psi_i} \cdot \hat{\mathbf{H}}^{D_T} \cdot \mathbf{P}_{4-3} \cdot \mathbf{F}_i = \hat{\mathbf{H}}_i \cdot \mathbf{K} + \varepsilon_i$$  \hspace{1cm} (16)

where $\varepsilon$ is the overall noise in the estimates and $\psi$ accounts for the phase ambiguity in (9). The unknowns of the non-linear system are $A(i), f_2(i), \psi(i)$ and $\mathbf{K}$, therefore $2 N_f$ real ($\mathbf{A}$ and $\psi$) and $N_f + 9$ complex coefficients ($f_2$ and the elements of $\mathbf{K}$). On the other hand, the number of real equations is $24 N_f$. Still, it is easy to notice that retrieving at the same time $\mathbf{A}, \psi, f_2$ and $\mathbf{K}$ is not possible. This is practically handled by constraining the parameters of a chosen image $i_0$ to the arbitrary values:

$$\hat{A}(i_0) = 1, \quad \hat{\psi}(i_0) = 0, \quad \hat{f}_2(i_0) = 1$$  \hspace{1cm} (17)

It is hence possible to retrieve $\mathbf{K}$ within the overall minimization process:

$$\hat{\mathbf{K}}, \hat{\mathbf{A}}(\neq i_0), \hat{\psi}(\neq i_0), \hat{f}_2(\neq i_0) = \arg \min_{\mathbf{K}, \mathbf{A}, \psi, f_2} \sum_{i=1}^{N_f} \|\varepsilon(n; \mathbf{K}, \mathbf{A}, \psi, f_2)\|_{\mathbf{F}}$$  \hspace{1cm} (18)

with $\|\cdot\|_F$ being the Frobenius norm. Under the assumption of small noise $\varepsilon$, it is easy to verify that (17) leads to the enlarged set:

$$\Theta_{PS+DT} = \left\{ A(i_0), f_2(i_0), f_2, \delta_1, \delta_2, \delta_3, \delta_4 \right\}$$  \hspace{1cm} (19)

which can be attained unambiguously from $\hat{\mathbf{H}}^{PS}$. In addition to $\Theta_D$ in (14), $\Theta_{PS+DT}$ accounts for the absolute gain and channel imbalances normalized to their values at time $i_0$. Therefore through PPS-DT assimilation the temporal evolution of every system internal parameter (gain, imbalances and cross-talks) is monitored during the whole mission life. The accuracy of such procedure obviously depends on the quality (number of looks and degree of symmetry) of the distributed target, on the PS polarimetric diversity (type and number $N_p$ of PSs), on the PS quality, and on the actual distortion parameters.

IV. RESULTS ON RS2 DATA

The data assimilation procedure has been validated on a stack of 26 Radarsat-2 Fine Quad-Pol acquisitions collected over Barcelona. More details are available in Table II. The performance has been tested with respect to several PS configurations, varying the number $N_p$ of selected PSs and their quality (through detection thresholding). Due to the absence of absolute reference data for accuracy validation, an analysis on the technique stability has been carried out instead. The stability for each PS configuration was evaluated by computing the (spatial) dispersion over the imagette estimates separately for each acquisition and then averaging this latter throughout the stack. The radiometric performance results in Fig. 2 indicate that 500 PSs of good quality are enough to provide image gain stability $< 0.5$ dB (3σ) and channel imbalance stability $< 0.25$ dB (3σ). The imbalance ratio is the most robust parameter with deviation $< 0.1$ dB (3σ). Two vegetated mountain areas in the background ($\sim 10^6$ looks) and a third smaller area with

Figure 1. Schematic representation of the iterative PS-based calibration solution, namely the PolPSCal technique.
Table II

<table>
<thead>
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<th>Site</th>
<th>Barcelona (ES)</th>
</tr>
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<tr>
<td>Acquisition mode</td>
<td>Radarsat-2 Fine Quad (FQ9)</td>
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<td>DC std.dev. Hz</td>
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</table>

cultivated fields (∼ 10^5 looks), shown in Fig. 3, provided then favorable locations for the DT-based Quegan [8] estimation. The corresponding time-series show very good consistency, with dispersion comparable if not better than the 5000 PS configuration. It is though easy to notice that the DT estimates are only available for the imbalance ratio, whereas temporal trends of the image gain and imbalances (in amplitude and phase) are a product of PPS-DT data assimilation.

V. CONCLUSIONS

In conclusion, the polarimetric PS technique provides an effective and cheap alternative to traditional ground-installation strategies for system monitoring and calibration. When one or more calibrated images are provided a full data calibration and absolute distortion estimation can be performed. Otherwise, the technique can be conveniently integrated with the DT-based partial estimates in order to unveil the whole temporal information, Θ_{PS+DT}, of the system distortion, while retaining comparable performance on the DT unambiguous set Θ_{DT}. Furthermore, though the C-Band is supposedly a favourable scenario thanks to the joint presence of polarimetric diversity and stability of the PPS and the availability of symmetric DT areas, it is important to remark that the technique general framework is deemed promising even for next generation lower frequency satellites. More validation on available stacks is therefore advisable for future research.

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