A model for the aqueous thermal boundary layer at an air-water interface

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Abstract—A fundamental understanding of the thermal characteristics of the air-water boundary is critical to applications such as remote sensing of the bulk sea temperature and modeling of the heat transfer through the interface. The objective of the current work is to develop a model of the thermal boundary layer on the aqueous side of an air-water interface. This model is based on the surface strain model Csanady[1] developed for gas transfer. The primary underlying assumption is that a quasi-steady straining exists at the air-water interface. This quasi-steady straining field maintains a steady state aqueous thermal boundary. With this approach the properties of the thermal boundary layer can then be related to a variety of hydrodynamic conditions. This approach is motivated by experimental evidence and validated with simulation results.

I. INTRODUCTION

The presence of a thin layer of relatively cool fluid on a water surface is well known. The characteristics of this ‘cool skin’ are of critical importance in a number of applications. For example, satellite sea surface temperature retrievals are typically calibrated with buoy bulk temperature measurements. The bulk measurements may have as much as a 1K bias, due to the presence of the cool skin.

The development of the first models for the thermal boundary layer at the air-water interface which were based on boundary layer theory are typically attributed to Saunders (1967) [2]. The temperature difference across the thermal boundary layer takes the form:

\[ \Delta T = \lambda \frac{Q}{u_\kappa k} \]  
(1)

where \( u_\kappa \) is the friction velocity, \( Q \) is the heat flux, \( \nu \) is the kinematic viscosity, \( k \) is the thermal conductivity of the water, and \( \lambda \) is to be a universal constant. Wick (1996) [3] provides a more complete review of the work following Saunders (1967). An attempt to extend the model to the low wind space was made by Fairall (1996) by offering a different expression for \( \lambda \) [4]. Fairall’s extension, which useful, still did not provide the framework to extend and accommodate other environmental conditions.

A model for the zero wind regime was offered by Katsaros, et al. (1977) and was of the form:

\[ \Delta T = \left( \frac{\alpha g}{\kappa \nu} \right)^{-1/4} \left( \frac{Q}{\rho c_p k} \right)^{3/4} \]  
(2)

where \( \alpha \) is the thermal expansion coefficient, \( \kappa \) is the coefficient of thermal diffusion, and \( \rho \) and \( c_p \) are the density and heat capacity of the water. However, this expression does not account for turbulent mixing due to wave breaking or wind stress, and also is not easily extensible to include these and other important phenomena.

A second class of thermal boundary layer models are based on surface renewal theory [5]. In this approach a separate surface renewal timescale is defined for each of the important hydrodynamic processes included in the model. This timescale characterizes the process by which eddies impact and disrupt the surface, and reside there, until displaced by the next event. The advantage to this approach is that as time scales are determined for hydrodynamic processes they can be included in the theory [6]. However, the definition of a renewal timescale is not straightforward for all processes, making a rigorous physics based model difficult to obtain.

The current work seeks to draw on ideas from both techniques to develop a more universal model of the thermal boundary layer at the air-water interface.

II. MOTIVATION

Figure 1 contains representative images extracted from three different sequences of infrared images taken at the Wind-Wave-Current facility at the Air-Sea Interactions laboratory at the University of Delaware, USA. In the results shown here there was no current in the test section aside from the wind induced flow. At the lowest wind speed the thermal structure present is characterized by broad regions of warm, ascending fluid separated by narrow bands of cool, descending fluid. Region A illustrates the more clearly defined leading edge of the upwelling and its longer, more diffuse down-wind boundary. A rising plume is also visible in the region marked B, but in this case it is being split by a descending sheet. Observation of the image sequences shows that these structures evolve slowly as they are convected downstream by the action of the wind.

On the down-wind side of the figure 1(b) the cellular structures have reduced in scale and been stretched along the axis of the wind. The lines B and C mark the locations of crests of wind generated waves. There is some evidence of small scale structures left in the wakes of the wave crests. In this case the thermal structure at the interface is in transition from an almost purely thermal convection driven state to a wave/wind stress driven state.

Figure 1 (c) shows much more small scale cellular structure in front of the stronger wind wave, indicating higher turbulence levels due to the wind stress, and very fine scales behind the wave, showing the even higher levels of turbulent mixing due to...
Fig. 1. IR image of water surface. The three wind speeds shown are, from top to bottom, 2 m s⁻¹, 4 m s⁻¹, 6 m s⁻¹. The wind is from right to left; the darker shades represent cooler surface temperatures.

III. Surface Strain Model

The surface strain model [7] is founded on the premise that the hydrodynamics near the surface act in a quasi-steady way to maintain a steady-state thermal boundary layer. In the case of a generic turbulent flow it is the fluctuating velocity fields which strain the thermal boundary layer, resulting in a steady-state condition for the thermal boundary layer.

Figure 2 shows a schematic of the thermal boundary layer. In this schematic a constant heat flux, \( Q \), through the interface is assumed. Molecular diffusion causes the thermal boundary layer to grow continuously in the negative \( z \) direction. This is balanced by the rate of strain, \( \sigma_{tt} \), at the surface. The strain compresses the thermal boundary layer in the \( z \) direction; when the rate of strain and thermal diffusion are balanced a steady state is reached.

The model itself is based on the convection-diffusion equation for the temperature field. First, this equation is assumed to be steady. Secondly, the straining field at the air-water interface is assumed to be locally steady, positive over a majority of the interface, and of the form:

\[
\mathbf{U} = (\sigma_{tt} x, -\sigma_{tt} z),
\]

where \( \sigma_{tt} \) is the local rate of strain. The governing equation becomes:

\[
-\sigma_{tt} z \frac{\partial \Theta}{\partial z} = \kappa \nabla^2 \Theta
\]

where \( \kappa \) is the coefficient of thermal diffusivity and \( \Theta = T(z) - T(z \to -\infty) \). The boundary condition for the temperature at the free-surface is a constant heat flux:

\[
\frac{\partial \Theta}{\partial z} = -\frac{Q}{\kappa}.
\]

The solution for 3 is:

\[
\Theta(z) = -\sqrt{\frac{\pi \kappa}{2\sigma_{tt}}} \frac{Q}{k} \text{erfc} \left( -z \sqrt{\frac{\sigma_{tt}}{2\kappa}} \right)
\]

From this the dimensional scales for the thermal boundary layer temperature difference, \( \Theta_{\text{rms}} \), and thickness, \( \Delta_{\text{rms}} \), given the heat flux and surface straining are:

\[
\Theta_{\text{rms}} = \sqrt{\frac{\pi \kappa}{2\sigma_{tt}}} \left( \frac{Q}{k} \right),
\]

and

\[
\Delta_{\text{rms}} = \sqrt{\frac{2\kappa}{\sigma_{tt}}},
\]

The critical aspect of this model is that the important thermal boundary layer scales are now based on the surface strain rate, \( \sigma_{tt} \), which must be modeled. The simplest model for \( \sigma_{tt} \) is to relate it to the turbulence dissipation at the interface. Since

\[
\sigma_{tt} \approx \frac{\partial u}{\partial x} \approx \left| \frac{\partial w}{\partial z} \right|,
\]

the following approximation can be made:

\[
\epsilon = \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \approx \nu \sigma_{tt}^2,
\]

where \( \epsilon \) is the isotropic dissipation function.

The rate of strain can then be approximated using the production of turbulent kinetic energy:

\[
\sigma_{tt} \sim \left( \frac{\epsilon}{\nu} \right)^{\frac{1}{2}} \sim \left( \frac{P}{\nu} \right)^{\frac{1}{2}}
\]
Models for the boundary layer thickness and temperature difference can be formulated in terms of the production of turbulent kinetic energy:

\[ \Theta_{str} \sim \sqrt{\frac{\pi}{2}} Pr^{\frac{1}{2}} (\nu \rho \bar{c}_p)^{-\frac{1}{4}} \left( \frac{Q}{\rho \bar{c}_p} \right), \]  

and

\[ \Delta_{str} \sim \sqrt{2} Pr^{\frac{1}{2}} (\nu \rho \bar{c}_p)^{-\frac{1}{4}}, \]  

The model provides the expected behavior: As the turbulence production and dissipation increase, the temperature and length scales, \( \Theta_{str} \) and \( \Delta_{str} \), decrease.

We have now obtained a model for the important thermal boundary layer parameters which is based on the production of turbulent energy. Models for the production of turbulent energy due to buoyancy, wind shear, and wave breaking can be developed and inserted to this model.

The total turbulent production is estimated as the linear combination of buoyant, shear stress, and wave breaking:

\[ P = C_b \rho g \frac{Q}{\rho \bar{c}_p} + C_s \frac{u_*^4}{\nu} + C_w u_* g, \]  

where \( C_b \), \( C_s \), and \( C_w \) are constants to be determined. The details of the development of each of the production models may be found in [7].

If the total production is normalized by shear production it can be rewritten as:

\[ P = \left( 1 - Ri + Ke^{-1} \right) \frac{u_*^4}{\nu} \]  

where \( Ri \) is the Richardson number: \( Ri = \frac{\rho g Q}{\rho \bar{c}_p u_*^3} \), and \( Ke \) is the Keulegan number: \( Ke = \frac{u_* L}{\nu} \).

If equation 12 is substituted into equations 9 and 10 the results are:

\[ \Theta_{str} = \sqrt{\frac{\pi}{2}} Pr^{\frac{1}{2}} (C_b - C_s Ri + C_w Ke^{-1}) \frac{1}{\nu \rho \bar{c}_p} \left( \frac{Q}{\rho \bar{c}_p} \right), \]  

and

\[ \Delta_{str} = \sqrt{2} Pr^{\frac{1}{2}} (C_b - C_s Ri + C_w Ke^{-1}) \frac{1}{u_*} \frac{1}{\nu}, \]  

which are expressions for the temperature difference across and thickness of the thermal boundary layer. Note that in the limit of zero waves or buoyancy these expressions revert to Saunders’ model. However, in this form the model can accommodate these effects, which Saunders’ model does not.

The advantage of the current approach over earlier work is the use of turbulent production as the scaling parameter. For many environmental configurations, models for the production currently exist. Additionally, it is straightforward to include improved or completely new models for different modes of production in this framework.

References


