Abstract—Fully polarimetric SAR data analysis has found wide application for terrain classification, land-use, soil moisture and ground cover classification. Critical to all analyses and applications is accurate calibration of the relative amplitudes of and phases between the various polarimetric channels. Here we develop a “minimalist” approach to polarimetric calibration, wherein only the weakest of constraints, reciprocity, is initially imposed upon the data. Additional parameters are self-consistently estimated from the constraints, reciprocity, is initially imposed upon the data. In this manner channel imbalances and cross-talk are estimated without unduly biasing the resulting polarimetric data.  In this manner channel imbalances and cross-talk are estimated without unduly biasing the resulting polarimetric data. Known ground targets or active radar calibrators may be incorporated, but they are not central to the method.

Planned deployments of new and recent upgrades to current fully polarimetric SAR systems have brought to the fore questions of polarimetric calibration. Present SAR systems achieve a high degree of separation between the polarization channels and often employ in-flight active calibration techniques to further enhance the accuracy of the polarimetric data. These improvements have, of course, been matched by advances in polarimetric decomposition techniques and new methods for extracting detailed geophysical information via polarimetric analysis. Hence the desire for SAR data that is well calibrated both in the relative amplitudes of and the relative phases between the polarimetric channels. Here we develop a new method to calibrate cross talk between channels and imbalances in the channel gains employing only the observed polarimetric SAR data. Known ground targets or active radar calibrators may be incorporated, but they are not central to the method.

There are several requirements for a posteriori calibration methods: First is a model that relates the observed polarimetric signals to the desired (true) calibrated signals. In principle, this model incorporates parameters for cross-talk, channel imbalance, system noise, systematic peculiarities, etc. In practice we employ a standard model that relates the transmitted horizontally (H) and vertically (V) polarized signals to the polarized received signals [1,2].

\[
\begin{bmatrix}
O_{11} & O_{12} \\
O_{21} & O_{22}
\end{bmatrix} = \begin{bmatrix}
r_{11} & r_{12} \\
r_{21} & r_{22}
\end{bmatrix} \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix}
\]

(1)

denotes how the actual scattering matrix, \([S]\), is transformed both by transmission and reception cross-talk and channel imbalances, \([t]\) and \([r]\), respectively. The subscripts 1 and 2 refer to two polarization states, typically H and V.

Converting the scattering matrix to a vector format and rewriting the previous model yields:

\[
\begin{bmatrix}
O_{HH} \\
O_{HV} \\
O_{VH} \\
O_{VV}
\end{bmatrix} = \begin{bmatrix}
k^2 \alpha & v k & w k \alpha & v w \\
zk^2 \alpha & k & w z k \alpha & w \\
uk^2 \alpha & u v k \alpha & k \alpha & v \\
uzk^2 \alpha & u k & z k \alpha & 1
\end{bmatrix} \begin{bmatrix}
S_{HH} \\
S_{HV} \\
S_{VH} \\
S_{VV}
\end{bmatrix}
\]

(2)

The values of \(k\) and \(\alpha\) relate to the channel imbalance and the values of \(u\), \(v\), \(w\) and \(z\) parameterize the channel cross-talk. The definitions are \(k = r_{11} / r_{22}, \alpha = r_{22} t_{11} / r_{11} t_{22}, u = r_{21} / r_{11}, v = t_{21} / t_{22}, w = r_{12} / r_{22}\) and \(z = t_{12} / t_{11}\). An overall gain factor, \(r_{22} t_{22}\), has been dropped, so absolute radiometric calibration is not considered. All of these parameters are complex.

Redundancy in the observed data is required so that information is available to estimate model parameters. The simplest assumption, reciprocity, sets the value of \(S_{HV}\) equal to the value of \(S_{VH}\). The differences between the observed HV and VH returns, \(O_{HV}\) and \(O_{VH}\), are then used to derive values for calibration parameters. Additional scattering symmetries can be imposed on the calibrated data. These may be determined by either theoretical or empirical considerations. However, if the assumed symmetry is not consistent with the data then the calibration process will try to enforce that symmetry, skewing the polarimetric information. Sufficient data redundancy is generated, assuming only reciprocity, by SAR systems that produce quad-polarization data, e.g. coherent HH, HV, VH and VV polarizations, with good channel separation. If redundant data is available then a posteriori calibration is possible.

Having outlined some requirements for an a posteriori calibration method, we note that there are two additional characteristics that are highly desirable: Repeated calibration of previously calibrated data should have no effect. And, the calibration method should not destroy or bias the polarimetric content of the data. While the first is easy to test and quantify, the second is more problematic. We will illustrate the second point employing simulated and anechoic chamber data where the “answer” is known. For actual polarimetric SAR imagery one only has plausibility arguments and empirical comparisons with known targets to support the contention that a calibration algorithm does not unreasonably degrade the polarimetric content of the SAR imagery. Several PI-SAR images contain corner reflectors for calibration testing.
CROSS-TALK CALIBRATION

The general method to determine cross-talk parameters involves averaging the polarimetric returns from many pixels. The averaged covariance matrix is employed to determine the cross-talk parameter values. Rewriting in terms of the covariance matrices the calibration (2) becomes

$$[C] = [M][\Sigma][M]^\dagger$$

(3)

where $[M]$ is the 4x4 calibration matrix of (2) and the observed covariance is given as $[C]$. The Hermitian conjugate is denoted by the $^\dagger$ symbol. The general form of the calibrated covariance matrix, in the linear basis, $[HH, HV, VH, VV]$, making the assumption that the scattering is reciprocal, is

$$\begin{bmatrix}
\sigma_{HHHH} & A^* & A^* & \sigma_{HHHV} \\
A & \beta & \beta' & B \\
A & \beta' & \beta & B \\
\sigma_{VVHV} & B^* & B^* & \sigma_{VVVV}
\end{bmatrix}$$

(4)

where $A^*$ is the complex conjugate of $A$, and similarly for $B^*$. The values of $\beta$ and $\beta'$ are real. Since reciprocity implies that the two cross-polarization channels, HV and VH, are identical, one requires for calibrated data that they, and all of their correlations, are in fact identical. This form of the covariance matrix is very general, permitting non-zero helicity, orientation angles, etc. Ideally, $\beta$ equals $\beta'$ but here we allow for the presence of system noise by letting $\beta \geq \beta'$. More stringent assumptions concerning scattering symmetries place additional requirements on the values of $A$ and $B$. Reflection symmetry implies that there is an orientation angle (a rotation about the radar line-of-sight) such that $A = B = 0$. However, explicitly setting $A = B = 0$ may be inappropriate since this forces reflection symmetry with fixed orientation angle equal to zero. Similarly, enforcing both reciprocity and zero helicity on the backscatter requires $\Im(A) = \Im(B)$. The question now becomes one of estimating the values of $A$ and $B$ in some reasonable and practical manner consistent with the assumptions of the scattering symmetry.

CHANNEL IMBALANCE

The values of $k$ and $\alpha$ determine the relative gains and phase delays between the polarimetric channels. $\alpha$ effects most directly the $\Sigma_{HHHV}$, $\Sigma_{VVHV}$, and $\Sigma_{HHVV}$ covariance elements. Assuming that the cross-talk is small, the value of $\alpha$ can be estimated by setting $\Sigma_{HHHV} = \Sigma_{VVHV} = \beta$ and $\arg(\Sigma_{HHHV}) = 0 = \arg(\Sigma_{VVHV})$. Now the average of $\Sigma_{HHHV}$ and $\Sigma_{HHVV}$ is an estimate for $A$, and similarly $\Sigma_{VVHV}$ and $\Sigma_{VVVV}$ estimates $B$.

The $k$ parameter is more troublesome. Merely counting the number of free parameters and the number of equations implied by (4) shows that the system of equations is under-determined. The direct effect of $k$ is on the co-polarized returns and correlations. Without additional information the value of $k$ is indeterminate. Traditionally, a trihedral corner reflector is used to fix both the phase and amplitude of $k$. Alternatively any region of smooth surface can provide a good estimate of the phase of $k$. For odd-bounce scattering the phase of the HH-VV correlation coefficient, $\rho_{HHVV}$, should be zero. The amplitude of $k$ is more problematic. Either a model of a presumed scattering mechanism can be employed to set the $|HH|/|VV|$ ratio, or an ad hoc assumption of the $|HH|/|VV|$ ratio can be employed. Other more or less ad hoc schemes to set $k$ can also be envisioned. In any case additional information, or an assumption, is required to fix $k$.

CALIBRATION METHOD

Typically, the required calibration is range dependent. The antenna gain patterns vary with depression angle. Also the relative displacement of the H and V antennas results in range-varying phases. These range dependencies suggest that each range line should be calibrated independently. An azimuth average of the covariance matrix is employed to determine the calibration parameters then these parameters are used to calibrate every pixel along the given range line.

In azimuth there may be slow phase drifts. The phase of $\rho_{HHVV}$ is one quantity that sometimes displays azimuth drift. These drifts can be identified and corrected by averaging the covariance along range lines and then applying the calibration to every pixel along that range line. Since this recalibration primarily involves $k$ compensating for azimuth drifts should have little effect on either $\alpha$ or the cross-talk parameters.

A comment should be made concerning the covariance matrix averaging: Approximately 7% of the brightest pixels are excluded from the averaging. The span (total power) is used to determine which pixels are dropped from the average. The point is that bright pixels tend to saturate the receivers and thus unrealistically skew the polarimetric averages. While the 7% solution may be somewhat arbitrary the results do not appear very sensitive to the cut-off value. Pixels returning very low total power may also have poor polarimetric information but they tend not to effect the average provided that some of the pixels in the average have significant power and well-defined polarimetric content.

NUMERICAL METHOD

The non-linear solution for the calibration parameters is solved by iteration. First (3) is linearized around the current values of $k$, $\alpha$, $u$, $v$, $w$ and $z$. The solution of this set of complex linear equations determines the updated values of $u$, $v$, $w$ and $z$. New estimates of $k$ and $\alpha$ are made and the
iteration repeats. For a given set of cross-talk parameter, \( \alpha \) is re-estimated, using the new \( [\Sigma] \) values, by setting \( \Sigma_{HHVV} = \Sigma_{VHVH} = B \) and \( arg(\Sigma_{HHVV}) = 0 = arg(B') \). Normally the estimates for \( A, B, B \) and \( B' \) are independent of \( k \). Only when the estimates of \( A \) and \( B \) are explicitly related, e.g. \( \Im(A) = \Im(B) \), does the value of \( k \) effect this part of the calibration.

This solution of (3) implies that the all calibration coefficients and the calibrated covariance matrix are determined self-consistently. Because of this self-consistency, re-calibrating already calibrated data produces no effect. For the re-calibration, all cross-talk parameters are zero and both \( k \) and \( \alpha \) are real and one.

The calibrated scattering matrix is immediately found from (2) and the calibration matrix, \([M]^{-1}\). The calibrated HV and VH returns are statistically equal. For any given pixel HV and VH returns may differ, however on average they will be equal provided the calibrated averaged covariance matrix is of the form in (4). The calibrated HV and VH returns should be straightforwardly averaged to produce the standard (calibrated) cross-polarized return.

**Simulated And Anechoic Chamber Results**

The calibration algorithm was tested on simulated data and on anechoic chamber data from EMSL. The simulated covariance matrices were constructed from simple scatterers (trihedrals, dihedrals, etc.). Then a variety of cross-talk, channel imbalances and orientation angle rotations were applied. (Since the value of \( k \) can not be determined from reciprocity constraints, in these test it was set to 1.) The resulting covariance matrices were calibrated to provide a first order test of the algorithm. Noise was not added to these simulated covariance matrices. Therefore, the calibration should compensate for the cross-talk and channel imbalance but leave the orientation angle alone. This was the case.

Covariance matrices were formed from the EMSL anechoic chamber data for “smooth”, “mixed” and “rough” surfaces at frequencies corresponding roughly to X-, C- and L-band. The orientation angles for these nine covariance matrices were calculated before and after our calibration. The differences were always less than \( \pm 2 \) degrees. The EMSL data corresponds to a more complicated and more realistic surface scattering than the simple simulations mentioned previously. Again the orientation angle test was performed. The EMSL covariance matrices were rotated by a known angle, calibrated and the orientation angle recalculated. The recalculated orientation angles matched the initial angles well within \( \pm 5 \) degrees across the full range of orientation angles (\( \pm 45 \) degrees) for all nine covariance matrices.

Repeat calibrations of several calibrated EMSL covariance matrices were performed with the predicted null result. These tests lend support to the general calibration procedure and show that at least for orientation angles the calibration procedure does not unduly bias the polarimetric information.

**PI-SAR and E-SAR Polarimetric Imagery**

We have applied the proposed calibration method to E-SAR and S-SAR polarimetric SAR imagery. Several specific methods for correcting range and azimuth dependencies are evaluated. The problems associated with determining \( k \) values directly from polarimetric imagery are assessed.

The E-SAR imagery was provided in both calibrated (at DLR) and uncalibrated format. This permits a direct comparison of the present calibration technique to the DLR calibration algorithm designed for E-SAR data. The E-SAR imagery covers a mountainous uniformly forested area.

The PI-SAR imagery is an uncalibrated image of a populated coastal area. A large area of fairly smooth water is in far range, while a mountainous region is in near range. An array of oriented dihedrals and trihedrals is located in the middle of the scene. These targets were not incorporated in our calibration. Instead we employ them to test our calibration procedures.

These two data sets from different SAR systems and imaging rather different scenes should provide reasonable benchmarks for any calibration procedure. Here we are emphasizing methods of calibrating polarimetric imagery employing only the image data and imposing only the weakest possible constraints on the scattering symmetry. We envision this calibration technique to be used to check (and if necessary correct) polarimetric calibration of SAR imagery.

By incorporating only very weak scattering symmetries and by virtue of the self-consistent determination of the calibration parameters, the \( a \) posteriori calibration method presented should not bias previously well-calibrated polarimetric imagery. Comparisons are made to assess the effects of calibration methods on standard polarimetric decomposition techniques.

**Acknowledgment**

We thank DLR and NASDA for providing us the imagery used in this work and EMSL for the anechoic chamber data.

**References**
