Abstract—Wireless sensor networks (WSN) that require bridging to other networks or extending sensor coverage must keep high link availability to reduce energy wasted on error control. This paper first details the relationship of individual link probability of availability to $E_b/N_0$ and to upper protocol layers. Then the efficient computation of the network probability of availability is shown, and applied to four extensible, planned formations of nodes. Finally, we demonstrate the use of these formations in examples of bridging and extensions of WSNs, and discuss how to deploy them.

Index Terms— network availability, network formations, probability of availability, wireless sensor networks

I. INTRODUCTION

A wireless sensor network (WSN) is composed of a large number of sensor nodes. Each of these nodes has hardware components for sensing, processing, memory, and communications. These nodes can network to achieving sensing of targets or phenomenon and transmit this information to a sink. But each node is limited in its amount of energy, so the efficiency of the communications network is very important to maintain the WSN throughout a planned lifetime. This efficiency is influenced by the upper layer protocols, wireless channel characteristics, and the physical distance between nodes.

A multi-hop WSN can have a random or planned topology. To expand the coverage of a WSN, a network can establish new nodes bridging to another WSN. These deployed nodes are at planned locations. These nodes are critically important in maintaining connectivity between an established WSN and another WSN (Fig. 1a). If this bridge has a “low” probability of availability, the combined network will have a decreased effectiveness. In this work, we establish the availability metric for a formation of nodes used to bridge one WSN to another WSN. It will be shown that particular formations of nodes best perform this purpose. While WSN A and/or B may be random in topology and arbitrarily complex, the formation used as a bridge is a planned, fixed topology. These formations could also be used (Fig. 1b) to simply extend an established sensor network in a chosen direction for additional coverage.

Considering related work, a majority of research papers in WSN availability address network reliability for random graphs of large numbers of nodes, and few address planned topologies.

A recent paper [1] analyzed planned mesh networks of fixed infrastructure, determining an availability metric change as redundant nodes were added to the initial topology. Others [2] derived analytical network availability expressions for a limited number of planned, bipartite formations, assuming each link had the same probability of availability. While [3] uses the factoring theorem, and vertex failures to generate network availability. Our research performs a similar analysis, also by complete state enumeration, but considers edge failures not vertex failures. Petingi [4] lately used a source-to-K-terminal reliability to optimize region reliability by using a 4-node random graph bridging two different geographical regions. Whereas the extension concept is applied to a random and linear WSN to develop minimum energy routing in [5].

The work reported in this paper is unique in computation and application. It presents an efficient computation algorithm of the exact network availability, which can use different values of probability of availability for each link. Using graph theory, we apply this network availability to a series of extensible formations of nodes. These different network formations are planned, increasingly connected, and yield...
different probability of availability for their purposes as bridges or extensions of WSNs.

This paper is organized as follows: Section 2 details the individual links between nodes, relating this to $E_b/N_o$ and upper layers of a protocol. Section 3 presents the network availability. With this computational tool, the network availability of four different formations are presented in Section 4. Finally, Section 5 discusses the deployment of these node formations and provides examples.

II. LINK PROBABILITY OF AVAILABILITY

Within a multi-hop WSN of $n$ nodes, the probability of link availability between two adjacent nodes is $p_a$. Graph theory models of networks are built upon this basic link probability of availability. For these models to be relevant to real-world deployment, the relationship between the link probability of availability, $p_a$ and a communication system parameter will be established.

A. Relationship to $E_b/N_o$

To briefly see how $p_a$ relates to $E_b/N_o$, consider a transmission between two nodes. The first node has a fixed location, the second node will be moving. The received power at each node decreases with increasing distance, $d$, between nodes, and as a function of the path loss exponent, $\alpha$. The path loss effects of this channel are modeled with $X_o$, a zero-mean Gaussian random variable with standard deviation $\sigma$, as [6]

$$P_r = P_t L_0 X_o \left( \frac{d_0}{d} \right)^\alpha$$  \hspace{1cm} (1)$$

where $P_r$ is the transmitted power, and $L_0$ is the path loss at a reference distance $d_0$. The received power, $P_r$, is used to compute the signal-to-noise ratio (SNR) by

$$SNR = \frac{P_r}{N}$$  \hspace{1cm} (2)$$

where $N$ is the noise power and is assumed constant.

For digital communications, the probability of bit error $P_e$ of a modulation scheme depends on the ratio of bit energy, $E_b$, to noise power spectral density, $N_o$, as

$$\frac{E_b}{N_o} = SNR \left( \frac{B}{R} \right)$$  \hspace{1cm} (3)$$

where $B$ is the channel bandwidth and $R$ the bit rate. Now demonstrating with a simple modulation scheme, note that the probability of bit error of the BPSK modulation is given by: [6]

$$P_e = Q \left( \sqrt{\frac{E_b}{N_o}} \right).$$  \hspace{1cm} (4)$$

Figure 2 shows this probability of bit error performance for the BPSK modulation scheme. This $P_e$ is the parameter indirectly related to the link probability of availability, $p_a$.

As an example, choosing a requirement $P_e$ e.g., $P_e < 10^{-4}$, then the minimum $E_b/N_o$, $E_b/N_o,_{min}$ is 8.4 dB using (4). The link is considered available when it meets or exceeds this requirement, which is $\frac{E_b}{N_o} > \frac{E_b}{N_o,_{min}}$. The $E_b/N_o$ received at a certain distance is modeled as a random variable, and is assumed to be Gaussian with non-zero mean, as in Figure 3. The standard deviation $\sigma$, represents the spread of the received $E_b/N_o$ around the mean, due to shadowing.

Figure 2. Probability of Bit Error for BPSK modulation as a function of $E_b/N_o$.

Since the link is available when it meets or exceeds this requirement, the link probability of availability is represented as:

$$p_a = \int_{E_b/N_o,_{min}}^{\infty} f_{\frac{E_b}{N_o}} \left( \frac{E_b}{N_o} \right) d \frac{E_b}{N_o}$$  \hspace{1cm} (5)$$

Figure 3. $E_b/N_o$ probability density function with threshold $E_b/N_o,_{min}$.

Now consider the second node moving away from the first node. As the distance between nodes increases, the signal strength decreases, so the mean $\frac{E_b}{N_o}$ decreases. As more of the Gaussian random variable is below the unchanging $E_b/N_o,_{min}$.
threshold, the \( p_a \) will decrease. Conversely, as the second node moves closer to the first node, the \( p_a \) will increase. The link probability of availability can be obtained in terms of a \( Q \)-function as:

\[
p_a = Q\left(\frac{E_p - E_n}{\sigma_E}\right).
\]

In summary, a communications system parameter can be related to the link probability of availability, \( p_a \), based on a requirement of probability of bit error for the link.

### B. Link Availability and Upper Layers

The reasoning so far has only considered some of the physical layer and channel characteristics. The physical layer includes frequency selection, modulation and detection. The channel characteristics include: attenuation with distance, multi-path effects, noise, interference and possible burst errors. It was shown that there will be some resultant bit errors (\( p_a < 1 \)). If a channel had perfect availability (\( p_a = 1 \)), there would be no need for error control at the data link layer! When these errors occur, the data link layer functions adequately handle the imperfect channel through error control mechanisms such as forward error correction (FEC), automatic repeat request (ARQ), and interleaving [7]. However, each of these represents additional energy used to remedy low link availability. For example, two nodes spaced very far apart, with a corresponding low \( p_a \), will have a higher need for retransmission of packets. Also, recall that each node has limited energy, and increased expenditure reduces node lifetime. Different protocols may handle these tasks differently, but in summary, keeping link availability high corresponds to less energy used for error control.

If the considerations for a single link are extended to the entire network, we would be considering the network availability. The network availability would similarly be improved above a computed value because of the functioning of upper layers. Changes to the network and transport layers of a protocol have demonstrated reliability improvements for WSNs. [8]

### III. Network Availability Computation

The network availability, \( P_A \), is defined as the probability that there exists a path between any two nodes in the network, whether direct or through a number of connected nodes. Relating this to graph theory, of the many reliability metrics for a graph \( G \), this has been referred to as the all-terminal reliability, \( R(G) \). [9] We concentrate on the all-terminal reliability because each node is an important communication and sensing node. This metric is used rather than the reliability between two nodes at the ends of formations, \( R(G) \), to relay multi-hop messages [9]. Each node in this paper’s development is both sensor and router.

In a wireless network, as compared to a wired network, the link failures are more common, due to the unreliability of the channel. Therefore, for this analysis, we assume the nodes are immune to failure, and concentrate on the link failures. Also, we assume new sensor nodes are placed at specific locations, and remain stationary. Link failures are also assumed to occur independently. Having explicitly stated our assumptions, we proceed with the network availability computation.

Representing each sensor node as a vertex, and the link between nodes as an edge, a mesh network of nodes is represented as a undirected graph, \( G = (V,E) \). Each of the \(|E|\) edges of \( G \) has a unique probability of availability, \( p_a \). An edge-cut in a graph \( G \) is a set of edges \( D \) such that \( G-D \) has more components than \( G \). The edge-connectivity of a connected graph \( G \), denoted \( \kappa(G) \) is the minimum number of edges whose removal can disconnect \( G \). Thus, if \( G \) is a connected graph, the edge connectivity, \( \kappa(G) \) is the size of a smallest edge-cut. [10]

Complete state enumeration requires generating all \( 2^{|V|} \) states of \( G \). For certain edges that are failed, a subgraph \( G' \) is formed. Each corresponding state has a probability of availability of

\[
P_{\text{state}} = \prod_{e \in G} p_{a,e} \prod_{e \in G}(1 - p_{a,e})
\]

The network probability of availability, \( P_A \), is the summation of all \( P_{\text{state}} \) that represent a connected subgraph \( G' \). For a number of edge failures, \( n \), this is

\[
P_A = \sum_{n=0}^{\infty} \binom{|E|}{n} C_{|n|,p} P_{\text{state}}
\]

where \( C_{|n|,p} = 1 \) or 0 determined by the state’s \( G' \) being connected or not connected, respectively.

Classically, \( p_a \) for all edges is assumed identical to \( p \) [9], not unique as above. In this case, if \( N_v \) is the number of connected subgraphs \( G' \), (7) and (8) can be simplified to:

\[
P_A = \sum_{i=0}^{N_v} N_v p^i (1 - p)^{|E|-i}
\]

To implement the computation of \( P_A \), a connected, undirected, planar graph \( G \) is fully described by an “edge list” of: from vertex \( v \), to vertex \( w \), and the \( p_a \) of the edge joining \( v \) and \( w \). Algorithm 1 can process the “edge list” in any order and only requires having \( 0 < p_a \leq 1 \); however, we have used \( p_a = 0.9 \) for ease of comparison throughout this paper.

The algorithm progresses through the number of edge failures from zero to \(|E|-(|V|-1)|\), which is the limit beyond which the graph is certainly disconnected. This is equivalent to constraining the upper limit of the summation in (2). For each state, a graph \( G' \) is formed consisting of the “edge list” minus the failed edges. The “edge list” is used to compile a new list of vertices connected to \( v \), namely \( H \). A modified depth first search (DFS) through \( G' \) occurs until the \( G' \) is
established as connected or not connected. If connected, the polynomial for that specific failure condition of edges is computed and added in to the case of (#fails) edges. The algorithm returns the network availability, \( P_A \), a sum of the (#fails) polynomials.

Algorithm 1. Availability

1: \( pa(0 \text{ fails}) \leftarrow \prod pa \)
2: for #fails 1:
3:   compute edge failure combinations
4:   for each edge failure combination do
5:     \( G' \leftarrow (\text{edge list} - \text{failed edges}) \)
6:     connected \leftarrow false
7:     changes \leftarrow true
8:     \( H \leftarrow \{v_1\} \)
9:     while (not connected) and (changes=true) do
10:       changes \leftarrow false
11:     for all v,w in G'
12:       if vi or wi in H then
13:         H \leftarrow append(vi,wi)
14:       end if
15:     end for
16:     if H contains all vi in G'
17:       connected \leftarrow true
18:     end if
19:   end for
20:   if H contains all vi in G'
21:     connected \leftarrow true
22:   end if
23:   \( pa(#\text{fails}) \leftarrow pa(#\text{fails}) + \text{state probability} \)
24: end if
25: end while
26: end for
27: end for
28: \( P_A \leftarrow \sum pa(#\text{fails}) \)

IV. NETWORK FORMATIONS

The availability algorithm is now applied to four different extensible formation models, which can be used as bridges or extensions as in Fig. 1. As shown in Fig. 4, they resemble a chain, ladder, truss, and full truss, which correspond to all “interior vertices” having a \( \kappa_e(G) = 2, 3, 4 \) or 5 respectively. These formations could be extended in a direction, and can be of variable size depending on the number of nodes used, as in Table 1. Because the formations’ purpose is to span some distance, a ‘minimum hop’ metric for each formation is computed as the minimum number of hops to reach the maximum distance across the formation. In graph theory, this corresponds to the maximum of the eccentricity of all vertices in \( G \). For Fig. 4a-c the minimum hop metric is 3, for Fig. 4d, the minimum hop metric is 2.

For each of the formation models, the number of nodes was changed, then the number of nodes, number of edges, minimum hop metric and resulting network availability, \( P_A \), were calculated, as shown in Table 1.

From the data of Table 1 plotted in Fig. 5, the network availabilities are highest where there is greatest path redundancy, that is, a higher \( \kappa_e \) generally yields a higher \( P_A \). The ladder and full truss formations have a linear decrease in \( P_A \) with an increase in the min hop metric. The chain formation \( P_A \) decreases exponentially with distance, a result of its \( \kappa_e = 2 \) vertices.
V. DEPLOYMENT CONSIDERATIONS OF THE FORMATIONS

Contrary to the prevailing thoughts on WSN deployment, this paper advocates the placement of additional nodes to create a planned formation. The $p_a$ of graph theory does correspond to a physical distance, and the distance is found by measurement of a communication system parameter. Section 2 showed the relationship between link $p_a$ and $E_b/N_0$. During deployment of the formation nodes, it may not be possible to measure $E_b/N_0$ directly. In that case, to determine a location for the formation node, the receiver’s power level can be used to test for a position that is correct for node deployment. He, for instance, analyzes existing WSN radios, graphing the relationship between BER and receiver input power level [11].

If a moving system were to deploy these formations defined by edge availability, $p_a$, using received power level, it would be found by periodic communication with already established nodes. The moving system must nearly simultaneously monitor two already established nodes for the $\kappa > 2$ formations (ladder, truss, full truss), only one for the $\kappa = 2$ (chain) formation.

Of course, one cannot expect the physical deployment of nodes to exactly replicate the drawings presented here. During deployment, if the noise power increases, for instance, this will shorten the physical link distances. In that case, the neatly drawn formations of Fig. 4 will be warped during deployment.

An example shows bridging two WSNs to sense the area between them and enable WSN to WSN communication. The two WSN are first separated, each WSN having its own $P_a$, as shown in Fig. 7a. A four node chain formation (two deployed nodes) between the WSNs is shown in Fig. 7b. Each WSN still has the same $P_a$ within its WSN, and the $P_a$ of the

![Figure 5](image1.png)

Figure 5. Formation probability of availability, $P_a$, as a function of minimum hop metric for four formation types

![Figure 6](image2.png)

Figure 6. Formation probability of availability $P_a$ as a function of number of nodes in four formations

Figure 6 shows the superiority of the full truss formation to the ladder formation, which is achieved by the same number of nodes, but with increased edge connectivity. Deployed nodes may change their degree of connectivity based on changing propagation conditions; in this case the full truss formation would represent an upper bound on $P_a$ and the chain formation would represent a lower bound on $P_a$. It should be remembered that although the chain formation $P_a$ decreases rapidly as the number of nodes increases, it may span the required distance in an effective manner as could be measured by another metric such as physical distance covered per nodes deployed.

![Figure 7](image3.png)

Figure 7. Bridging two Wireless Sensor Networks (WSN) with different node formations
supernetwork is 0.7091. Using a different bridge formation, a five node full truss formation (three deployed nodes), brings the $P_A$ of the supernetwork to 0.9497, as shown in Fig. 7c.

Another example shows the extension concept applied to a WSN. From the original WSN with $P_A = 0.9763$ in Fig. 8a, extensions are first added in the form of a 3 node chain (2 deployed nodes) and then a 6 node full truss (4 deployed nodes). The resulting extended WSN has a $P_A = 0.7918$. The change from Fig. 8b to 8c shows than an extension can increase the probability of availability of the WSN depending on how it modifies the topology.

![Figure 8. Extending a Wireless Sensor Network (WSN) with different node formations](image)

The deployment of the WSN cannot be made without regard to the sensor coverage. This paper made no assumption about the relationship between sensor range and communication range, nor the shape of the sensor coverage for a single node. Common models of the sensor coverage are disc, sector and polygon. The regular, planned formations just discussed are certainly synergistic with a polygon sensor model, and tiling-based deployment, as in [12].

VI. CONCLUSION

This paper first detailed the efficient computation of the exact network probability of availability. When this metric was applied to four different extensible formations, the relative merits of these different formations was shown. But the very basis of each formation is the communication link between two nodes. Having established the relationship between the link probability of availability, $p_{\alpha}$, and $E_b/N_0$ allows the possibility of reproducing the graph theory formations in physical formations. This research together with the many considerations of upper layer protocols will allow effective node formation deployment for bridging or extensions to WSNs.

REFERENCES