Repeating Contract Research on Nonperishable Product with Random Demand

Xiangming Li
College of Economics & Management
China Agricultural University
Beijing, China
lxmlyx@163.com

Yunxian Hou
College of Economics & Management
China Agricultural University
Beijing, China
houyunxian@163.com

Abstract—A two-stage supply chain consists of a manufacturer and a retailer. Manufacture acquire demand information through EDI. This make it possible for upstream enterprise to manage inventory and determine output. If supply chain member keep repeating trade relationship, the long-term supply chain performance can be improved through signing a wholesale price contract. Study contract coordination problem when supply chain sell a nonperishable product to end customers. The efficient or optimal output quantity can be achieved through introducing penalty contract.

Keywords: repeating contract; VMI; supply chain contract

I. INTRODUCTION

Supply chain contracts have been suggested as a coordination mechanism that provides incentives to all supply chain members so that the decentralized and uncoordinated supply chain behaves nearly or exactly the same as an integrated one (Wang, 2002; Cachon, 2002). Due to the increased competition between supply chains, being a way linking different enterprises together and effective measure reducing double-marginality, supply chain contracts have attracted much interest.

Many researches have applied game-theoretic analysis to prove that opportunistic behavior results in supply chain inefficiencies. A recent empirical study of the semiconductor equipment industry has confirmed these results by demonstrating that if forecast are incredible, they will be ignored and the supply chain performance suffers. Much powerful evidence suggests that information sharing among supply chain members by itself is not sufficient to achieve optimal supply chain performance.

Previous researches on our topic date back to the era of classical inventory based on newsvboy model. The relevant work pertaining to the topic can be divided into three categories. The first concerns the buyer-supplier coordination in a contract. Tsay Nahmias and Agrawal (1999) summarize model-based research on contract in a supply chain setting, they examine how the design of contracts affect the behavior and performance of the supply chain. Cachon and Lariviere study revenue sharing contract in a video rental industry, they compare revenue sharing contract with various types of other contracts. A comprehensive review on the contracts in a supply chain can be found in Cachon (2003).

The second category is the long-term contractual agreement between a buyer and a seller. Moinzadeh and Nahmias (2000) consider a long-term contractual agreement between a buyer and a seller, in which Q unit are delivered to the buyer at regular time intervals with an upward adjustment in delivery quantity. Xu (2005) consider a multi-period dynamic supplying contract problem, where a buyer orders a product from a supplier in each period. In this paper, the supplier allows the buyer to cancel a portion of an outstanding order with a penalty during a planning horizon.

The third category is about VMI mode. The so-called vendor managed inventory has become an important business pattern in supply chain. In a VMI program, the vendor is commissioned to manage the inventory held at the retailer site. The vendor is responsible for the replenishment of goods according to a specified inventory control policy. The literature on VMI-driven supply chain is fairly extensive. Yao et al, 2007 focus on measuring the benefits of adopting this inventory cooperative strategy. Lau et al (2007) consider a stochastic and asymmetric-information framework for a manufacturer as a leader in supply chain.

Wang (2009) investigate a traditional supply chain arrangement versus a VMI arrangement for a single manufacturer single distributor supply chain where the manufacturer produces a single short life cycle product with random yield and uncertain demand. The above shows that the VMI arrangement is preferred by the manufacturer when the wholesale price is relatively high, only if the manufacturer is a price taker, while if the manufacturer is a price setter, the VMI arrangement is preferred by both the manufacturer and supply chain. Other literature related this paper is about profit and risk. Many scholars find a well-designed contract can make supply chain coordinating with sharing profit, risk and losses in proportion to respective contribution.

This article discusses a two-stage supply chain model. What the supply chain produce and sell is a seasonal products, the special time of year is season of sales. Once the season of sales past, the partner would wait for it to come next year, such as fireworks, air conditioning, and agriculture harvesting, planting equipment. These products are functional products, so this year's stock can continue to be used next year. With the expansion of retailer's power and development of information technology, VMI mode becomes increasingly popular.

Considering supply chain collaboration is a long-term
relationship, this paper discuss the repeating contract in supply chain under VMI environment.

II. BASIC ASSUMPTION

A two-stage supply chain consists of a manufacturer and a retailer. Manufacture knows the demand distribution of final product through information sharing system. Manufacture produce goods and manage retailer's inventory according to demand forecast. Each period the risk-neutral retailer sells its output at a predetermined price \( P \). Stochastic Demand is \( D \) on the interval \([a, b]\), where \( 0 \leq a < b \leq \infty \). The density function for is denoted \( f(y) \), where \( f(y) > 0 \) for \( y > 0 \).

The cumulative density function is denoted \( F(y) \), where \( F(a) = 0 \) and \( F(b) = 1 \). The Manufacture must choose an output before the start of a selling season that has stochastic demand. The manufacture's production cost per unit is \( c \) and selling cost per unit is \( c_s \) (such as packaging cost and transporting cost). The transfer price is \( w \) within supply chain.

The manufacture's decision variable is \( q \), and retailer's decision variable is \( w \). The stock unsold in this period can be sold in the future period. The manufacture is risk-neutral also.

Assume goodwill cost and salvage value are zero.

III. ONE PERIOD PROBLEM

According to the basic assumption above, the retailer's expected sales in a period then equal

\[
S(q) = \int_{0}^{a} yf(y)dy + y(1-F(y)) - q - \int_{0}^{q} F(y)dy
\]

A. The Optimal Solution to Integrated Supply Chain

An efficient supply chain is one that chooses \( q \) to maximize total profit, which equal the combined profits of the manufacture and the retailer. The profit of integrated supply chain is

\[
\pi = (p-c)q - \int_{0}^{q} F(y)dy + c_q q - c_q q
\]

(2)

According to The first order condition of (2), we can have

\[
F(q^*) = 1 - c / (p - c_m)
\]

Because \( F(y) \) is strictly increasing there is a unique solution to this optimal output problem. The optimal output for the supply chain, \( q^* \), is termed efficient output.

B. The Solution to Decentralized Supply Chain

The efficient output would be chosen if the manufacturer and retailer act as a single firm, without concern for the division of profits between them. However, the manufacturer and retailer act to maximize their individual profits, then the pursuit of their self-interests can cause a deviation from the interest of supply chain as a whole. Because the information is perfect, so we can use Stackelberg game to solve in which the retailer's decision variable is wholesales \( w \) and manufacturer's is output quantity \( q \).

The final product price \( P \) is exogenous, so the retailer offers a contract to the manufacture, specifying wholesale price \( w \). The manufacture accept it or reject it. If accept it, the manufacture uses \( w \) to determine its output quantity.

The problem is solved by first determining the manufacture's optimal output quantity in term of \( w \). The manufacture's profit function is

\[
\pi_m = (w-c_q)q - \int_{0}^{q} F(y)dy - c_q q
\]

According to (4), the first order condition for the manufacture's optimal output quantity is

\[
(1 - F(q_m))(w-c_q) = c
\]

(5)

The right-hand of (5) is the marginal cost to the manufacture of increasing output quantity, and the left-hand side is the marginal expected revenue from increasing output quantity. For analytical convenience condition (5) is rewrite as follows

\[
F(q_m) = 1 - c / (w-c_q)
\]

(6)

Compare (5) with (3), we will find: if \( w = P \), then

\[
q_m = q^*
\]

Decentralized supply achieve coordination. However, if \( w = P \), then the retailer's profit is zero, so any rational retailer can't accept it anyway. Furthermore, if retailer raises the wholesale price \( w \), then the manufacture would increase output quantity. In addition, from (6), we can find when \( w \) increase, manufacture's optimal output quantity increase also.

Given the manufacture's optimal response to \( w \), the retailer's profit function becomes

\[
\pi_r = (q_m - \int_{0}^{q_m} F(y)dy)(P - w)
\]

(7)

From first order condition, we can obtain

\[
\frac{\partial q_m}{\partial w}(P - w)(1 - F(q_m)) = q_m - \int_{0}^{q_m} F(y)dy
\]

(8)

The right-hand side of (8) represents the marginal loss to the retailer's profits resulting directly from an increase in wholesale price \( w \). The left-hand side represents the marginal gain in profits resulting from the positive effect that an increase in \( w \) has on output quantity. Stackelberg Equilibrium solution pair simultaneously satisfies condition (6) and (8).

This section mainly illustrate the manufacture's output quantity in decentralize supply chain is less than in integrated
supply chain. If the manufacturers improve supply quantity, supply chain profit has enhancing potential.

IV. LONG-TERM CONTRACT WITH REPEATED GAME

There are two way that the retailer can induce manufacture to increase output quantity. The one is to increase the manufacture’s revenue if manufacture produces more. The other is to increase the manufacture’s cost if stockout occurs. The simplest method is to increase wholesale price, but will be at a cost to retailer and cause overproducing.

The first contract below is designed according to the nonperishable product features, which is only sold in sales reason in a year, and leftover can be sold next sales reason.

The second contract is set if supply quantity is less than market demand, then the manufacture will be penalized.

A. Long-term Contract

Suppose the retailer offer a contract to manufacture that specifies a wholesale price \( w \) as well as a number of periods \( n \) in which the transaction is conducted. Demand is \( D \) each period, so the expected value of demand never changes. This period inventory can be resold in next period, so by the end of \( n \) periods, the total units of output quantity the manufacture has produced equals

\[
q_n = (n-1) \int_0^q F(y)dy + \int_0^q F(y)dy \tag{9}
\]

The expected amount of the total sales over \( n \) periods is

\[
S_T = n(q - \int_0^q F(y)dy) \tag{10}
\]

a) Integrated supply chain

The total supply chain profit of \( n \) periods is

\[
\Pi_s = n(q - \int_0^q F(y)dy)(p - c_s) - c(nq - (n-1)\int_0^q F(y)dy) \tag{11}
\]

The first order condition for the efficient output simplifies to

\[
F(q^*) = \frac{p - c_s - c}{p - c_s - \frac{c}{n}} \tag{12}
\]

Theorem 1. As \( n \) increases, the efficient output increases, thereby increasing the profit potential of the supply chain.

Proof: If \( n \) increase, the right-hand side of (12) increase. Since \( F(y) \) is strictly increasing in \( y \), the value \( q^* \) must increase to restored equality.

The total profits per period equal \( \Pi/n \)

This is positive, meaning that a higher \( n \) increases total profit period if \( q^* \) remains constant. But from (12) we can see as \( n \) increase, \( q^* \) increases also. The profit potential per period thus unambiguously increases as \( n \) increase.

b) Decentralized supply chain

If the manufacturer and the retailer act to maximize their own profit, the manufacture’s profit function is

\[
\Pi_n = n(q - \int_0^q F(y)dy)(w - c_s) - c(nq - (n-1)\int_0^q F(y)dy) \tag{13}
\]

Its first order condition simplifies to

\[
F(q_n) = \frac{w - c_s - c}{w - c_s - \frac{c}{n}} \tag{14}
\]

The retailer’s profit function is

\[
\Pi_r = n(q - \int_0^q F(y)dy)(p - w) \tag{15}
\]

Its first order condition for the optimal wholesale price equals

\[
\frac{\partial q_n}{\partial w} = \frac{c + c_s}{n(w - \frac{c_s}{n} - 1)(c + c_s)^2 f(q_n)} \tag{16}
\]

Theorem 2. When \( n \) increases, the manufacturer’s equilibrium output increases, but remain less than the efficient level for any value of \( n \).

Proof: Denote \( q_1 \) and \( w_1 \) as the equilibrium output and wholesale price when the number of period is \( n \). This implies that the first order condition for the manufacturer and retailer, (14) and (16) simultaneously hold.

By differentiating both sides of (14) with respect to \( w \) we can find that

\[
\frac{\partial q_1}{\partial w} = \frac{c + c_s}{n_1(w - n_1 - 1)(c + c_s)^2 f(q_1)} \tag{17}
\]

This value is positive, meaning that the equilibrium output increases as \( w \) increases.

Suppose that \( n \) from \( n_1 \) to \( n_2 \) and \( w \) remains unchanged for the moment. The right-hand side of (14) increases, and so the manufacturer increase output to restore equality. Suppose now that the manufacturer lowers \( w \) to the value, denoted \( w_0 \), at which the manufacturer reduces output to \( q_1 \), from (14) it can be found that

\[
w_0 = \frac{n_1 c + n_1 (w_1 - c_s) - n_2 c}{n_1 + n_1 n_2 - n_1^2 c < w_1} \tag{18}
\]

From (18) and (19) it can be seen that

\[
\frac{\partial q_1}{\partial w} \bigg|_{w=w_0} > \frac{\partial q_1}{\partial w} \bigg|_{w=w_0} \tag{19}
\]

Because of (19) and the fact \( w_0 < w_1 \), it is evident from (16) that at output \( q_1 \) and wholesale \( w_0 \),

\[
\frac{\partial q_1}{\partial w} \bigg|_{w=w_0} = \frac{c + c_s}{n_1(w - n_1 - 1)(c + c_s)^2 f(q_1)} \tag{20}
\]
The inequality in (20) implies that the marginal benefit from increasing \( w \) exceeds the marginal loss, and so the retailer responds by increasing output to a value higher than \( q_1 \), through raising \( w \). A higher \( n \) therefore results in a higher equilibrium output quantity.

The condition (12) and (14) indicate the efficient output \( q^* \) is not achievable in decentralized supply chain.

**B. The Penalty Contract**

In a penalty contract the retailer specifies the wholesale \( w \) and the penalty \( s \), which the manufacturer pays to the retailer per unit that demand exceeds output. The expected value of the total penalty paid per period equals

\[
s\left(\int_y f(y)(y-q)dy\right) = s(b - q - \int_y F(y)dy)
\]

The manufacturer's total profit over \( n \) periods is

\[
\Pi_m = n(q - \int_0^q F(y)dy)(w - c_s) - c(nq - (n-1)\int_0^q F(y)dy) - ns(b - q - \int_0^b F(y)dy)
\]

From the first order condition we can have

\[
F(q) = \frac{w - c_s + s - c}{w - c_s + s - \frac{n-1}{n}c}
\]  

(22)

Note from (12) and (22) that the manufacturer chooses efficient output if \( p = w + s \).

**V. CONCLUSIONS**

A two-stage supply chain consists of a manufacturer and a retailer. Manufacturer acquire demand information through EDI. This makes it possible for upstream enterprises to manage inventory and determine output. In the era of "Controlling the terminal is the key action", retailer has more control power, and so it is a usual mode that manufacturers bear the risk of inventory. However, retailers will face more stockout losses when under VMI. If supply chain member keep long and stable relationship, the performance of supply chain that sell a nonperishable product to end customers can be improved. The efficient or optimal output quantity can be achieved through introducing penalty contract. The cost structure of manufacturer has no influence on coordination result.

**REFERENCES**


