COMPRESSIVE DEMOSAICING FOR PERIODIC COLOR FILTER ARRAYS

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ABSTRACT

The utility of Compressed Sensing (CS) for demosaicing of images captured using random panchromatic color filter arrays (CFA) has been investigated in [1]. Meanwhile, most camera manufacturers employ periodic CFAs such as the popular Bayer CFA. In this paper, we derive a CS-based solution to demosaicing images captured using the general class of periodic CFAs. It is well known that periodic CFAs can be designed to effectively separate luminance and chrominance frequency bands [2, 3]. We employ this ability to reduce artifacts associated with luminance-chrominance overlap at the solver side. We show that the modified compressive demosaicing method coupled with the additional constraint that chrominance channels have smooth surfaces achieves further improved results for most periodic CFAs.

Index Terms—color filter array, compressed sensing

1. INTRODUCTION

Most low-cost digital cameras are designed to capture only one of the red (R), green (G) or blue (B) color components at each pixel. This is done by passing the input light through a color filter array (CFA) before the analog to digital conversion. Alternatively, a linear combination of RGB intensities may be captured at each pixel, using a panchromatic CFA [2]. Fig. 1 shows a pure color (Fig. 1(a)) and two panchromatic CFA patterns (Fig. 1(b) and (c)).

(a) Bayer (b) Spatio-Spectral [2] (c) random [1]

Fig. 1: A few CFA patterns.

The process of estimating missing RGB components is widely known as demosaicing. Most demosaicing algorithms exploit the correlations that exist among RGB planes (inter-channel correlations) and the correlations that exist among pixels of each RGB plane (inter-pixel correlations) to interpolate CFA images [4]. Yet some demosaicing approaches estimate the discrete Fourier transform coefficients of different color planes by applying filters that are specifically tailored for luminance and chrominance frequency bands [2, 3]. Meanwhile, the theory of Compressed Sensing (CS) [5] has been increasingly used in many imaging applications, and more recently Compressive Demosaicing (CD) [1] was introduced to exploit CS tools and algorithms for image demosaicing. Also authors in [6] tackle demosaicing as a non-uniform denoising problem, where the mosaic effect of the Bayer CFA is reduced by finding a feasible sparse color image using a trained – hence CFA dependent – dictionary.

Unlike [1], where only random panchromatic CFAs such as the one shown in 1(c) were considered, in this paper we develop a unified CS framework for demosaicing images captured through the general class of periodic CFAs. We believe that in addition of being dominant in their employment by virtually all consumer cameras, periodic CFAs could have a salient edge over random ones. As opposed to random CFAs, the periodic CFAs impose certain structures on the output image’s discrete Fourier spectrum which will be explained in detail in Section 2. Moreover, this work introduces other new contributions relative to our previous framework in [1] including the following two. First, in [1], we assumed that the three-dimensional color of a pixel can be expressed sparsely according to an over-complete frame (e.g. a redundant equiangular tight frame (ETF) in $\mathbb{R}^3$). Such setting suffers from a high level of overlap between luminance and chrominance atoms and leads to color washing. Contrarily, in here, we assume the chrominance channel is sparse in an over-complete color frame while the luminance channel is treated separately. Second, in this work, the chroma surfaces are modeled as locally smooth random processes. This paper is organized as follows: In Section 2 we analyze periodic CFAs in a systematic way using the notion of modulation. In Section 3 we briefly review the CD framework. In Section 4 we describe our approach for producing incoherent projection matrices for periodic CFAs. In Section 5 we show our demosaicing results and compare them with other leading demosaicing methods and conclude in Section 6.
2. PERIODIC COLOR FILTER ARRAYS

In general, the captured CFA intensity at each pixel is a linear combination of that pixel’s red (R), green (G) and blue (B) components: \( y_i = \alpha_i R_i + \beta_i G_i + \gamma_i B_i \) for the \( i \)th pixel. If \( R, G \) and \( B \) represent vectorized forms of the corresponding color planes (by stacking columns of each plane), the collection of \( \alpha_i \)'s, \( \beta_i \)'s and \( \gamma_i \)'s form the \( \alpha, \beta \) and \( \gamma \) vectors, respectively. Therefore, the captured image vector can be expressed as:

\[
y = \alpha \circ R + \beta \circ G + \gamma \circ B
\]  
(1)

where \( A \circ B \) denotes the element-wise multiplication of two vectors \( A \) and \( B \). From this point, a CFA resembles three modulators followed by an adder. For periodic CFAs, \( \alpha, \beta \) and \( \gamma \) can be expressed as linear combinations of (usually) three carriers: \( m_1, m_2 \) and \( m_3 \). Therefore, for the \( i \)th pixel:

\[
y_i = [\alpha_i \beta_i \gamma_i] \begin{bmatrix} R_i \\ G_i \\ B_i \end{bmatrix} = [m_1 \ m_2 \ m_3] K \begin{bmatrix} R_i \\ G_i \\ B_i \end{bmatrix}
\]  
(2)

where \( K \in \mathbb{R}^{3 \times 3} \). Alternatively, \( K \) can be regarded as a transformation matrix for the RGB vectors prior to modulation by the \( m_i \) carriers. For instance, for the Bayer CFA [3]:

\[
K_{Bayer} = \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/8 & 0 & -1/8 \\ 1/16 & -1/8 & 1/16 \end{bmatrix}
\]  
(3)

The first row constitutes the average \( (R + 2G + B)/4 \), while the second and third rows make the difference channels \( (R - B)/8 \) and \( (R - 2G + B)/16 \). It is known that since RGB planes share similar detail information [7], their differences (chrominance planes) are constant/smooth over small regions. In the frequency domain, this translates into narrow chrominance bands. The architecture of (2) can be designed to separate the chrominance bands from the luminance band by selecting \( m_2 \) and \( m_3 \) carriers to have sufficient distance from the luminance band, which is centered around the \( m_1 \) carrier. However, this separation is not complete and usually demosaicing color artifacts happen due to overlapping of luminance and chrominance bands.

The authors in [2] computed a number of optimal carrier sets based on lattice theory. The demosaicing approach adopted in there is basic filtering/demodulation. Although the complexity of linear filtering is low, it does not provide the best results. This motivates our employment of a CS-based general non-linear method for such CFAs. In the following section, we briefly describe the aforementioned CD framework before proceeding.

3. COMPRESSIVE DEMOSAICING REVIEW

The point-wise multiplication of two column vectors \( u \) and \( v \) can be expressed using linear algebra by \( u \circ v = \pi uv \) where, for a vector \( u, \pi \) is defined as

\[
\pi = \begin{bmatrix} u_1 & 0 & \ldots & 0 \\ 0 & u_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & u_n \end{bmatrix}
\]

Using the above notation, (1) can be written as:

\[
y = \pi R + \bar{\beta} G + \bar{\gamma} B = [\pi \ \bar{\beta} \ \bar{\gamma}][R^T \ G^T \ B^T]^T
\]  
(4)

In a CS context, (4) is usually expressed as \( y = \Phi x \), where \( \Phi = [\pi \ \bar{\beta} \ \bar{\gamma}] \) and \( x = [R^T \ G^T \ B^T]^T \). The theory of CS originates from the fact that \( x \) can be compressed using proper linear transforms. Every linear transform has an analysis and synthesis pair of matrices. Suppose \( \Psi \) is the synthesis matrix (also called the dictionary) that contains the synthesis functions (atoms) in its columns. Hence: \( x = \Psi \zeta \) where \( \zeta \) is the coefficient vector. The simplest structure for \( \Psi \) of RGB images is:

\[
x = \begin{bmatrix} \Psi_R & 0 & 0 \\ 0 & \Psi_G & 0 \\ 0 & 0 & \Psi_B \end{bmatrix} \begin{bmatrix} \zeta_R \\ \zeta_G \\ \zeta_B \end{bmatrix}
\]  
(5)

where \( \Psi_R, \Psi_G \) and \( \Psi_B \) can be any two-dimensional (planar) dictionaries such as DCT’s orthogonal synthesis matrix. In [1] a general formulation was proposed for \( \Psi \):

\[
x = \Theta \begin{bmatrix} \Psi_1 & 0 & \ldots & 0 \\ 0 & \Psi_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \Psi_q \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_q \end{bmatrix}
\]  
(6)

where \( \Theta = \theta \otimes I_n \), and

\[
\theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \ldots & \theta_{1q} \\ \theta_{21} & \theta_{22} & \ldots & \theta_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{q1} & \theta_{q2} & \ldots & \theta_{qq} \end{bmatrix}
\]  
(7)

defines the color dictionary. \( \otimes \) denotes the Kronecker product operator and \( I_n \) is the \( n \times n \) identity matrix. \( \theta \) is called the color frame and contains \( q \geq 3 \) RGB vectors in its columns. Apparently, (5) is a special case of (6) when \( \theta = I_3 \). Redundant color frames, i.e. ones with \( q > 3 \), are crucial for sparse color representation. These \( q \) vectors are as incoherent as possible, preferably in the positive octant of the RGB color space. The derivation algorithm of incoherent redundant color frames is discussed in [8].

By combining (4) and (6) we get:

\[
y = \Phi \Theta \Psi \zeta = P \zeta
\]  
(8)

The rectangular \( n \times qn \) matrix \( P = \Phi \Theta \Psi \) is called the projection matrix. Now, the objective of CD is to find the sparsest \( \zeta \)
that satisfies (8). It is shown [5] that if $P$ satisfies certain incoherence constraints, then convex optimization solvers such as Basis Pursuit (BP) can be employed to find the desired $\zeta$. In the next section, we discuss the required adjustments for CD when a periodic CFA is employed at the camera. The challenge is to maximize the incoherence of the projection matrix in spite of the coherent periodic sampling.

4. CD FOR PERIODIC CFA’S

We define $\theta' = K \theta$. We can now combine the previous expressions from (2) and (6) to obtain:

$$y = [\pi \beta \gamma](\theta \otimes I_n) \left[ \begin{array}{c} \Psi_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{array} \right] [\zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_q]$$

Using the notation in (8), the final projection matrix $P$ is:

$$P = \left[ \begin{array}{c} \left[ m_1 \theta_1 + m_2 \theta_2 + m_3 \theta_3 \right] \\ \left[ m_1 \theta_1' + m_2 \theta_2' + m_3 \theta_3' \right] \end{array} \right]$$

As can be seen above, $\Psi_1$ appears solely at the first macro-column of $P$ and is associated with $\theta_1, \theta_2$ and $\theta_3$ which are linear functions of $\theta_1, \theta_2$ and $\theta_3$. Also $\Psi_2, \ldots, \Psi_q$ are associated with the rest of $\theta'$'s columns that represent the chrominance components. As mentioned before, chrominance channels are constant/smooth over a small region. Therefore, over blocks of $8 \times 8$ or $16 \times 16$ pixels, it is reasonable to employ constant or very smooth functions to approximate the chrominance surface. For instance, $\Psi_2, \ldots, \Psi_q$ can comprise low frequency two-dimensional discrete cosine functions. $\Psi_1$ can be chosen differently from the chrominance bases to promote incoherence. For instance, most wavelet transforms and trained dictionaries [9] are incoherent with cosine functions. In Section 5, we will show some of the results when such dictionaries are utilized for lumiance in the decoder.

In the study of periodic CFAs, additional constraints are to be satisfied by the color frame. Suppose as in the Bayer pattern, $m_1$ corresponds to (0, 0) carrier in a 2D frequency grid, $m_2$ combines two $(\bar{0}, \pi)$ and $(\pi, 0)$ carriers and $m_3$ corresponds to $(\pi, \pi)$ carrier. From (9) it follows that each dictionary $\Psi_i$ is multiplied by a linear combination of $m_1, m_2$ and $m_3$: $(m_1 \theta_1 + m_2 \theta_2 + m_3 \theta_3)$. In the following, we will show that $\theta'$ should have the below structure for an incoherent

$$\theta'^* = \begin{bmatrix} \theta'_{11} & \theta'_{12} & \ldots & \theta'_{1q} \\ 0 & \theta'_{22} & \ldots & \theta'_{2q} \\ 0 & \theta'_{32} & \ldots & \theta'_{3q} \end{bmatrix}$$

If $\theta' = \theta'^*$, then $\Psi_1$ (the luminance basis) will be associated only with the $m_1$ carrier. Otherwise, if $\Psi_1$ is associated with all $m_1, m_2$ and $m_3$ carriers, then three (modulated) lumiance spectra would overlap due to the common aliasing effect and hence decrease incoherence. However, since $\Psi_2, \ldots, \Psi_q$ are narrow-band, there will be no chrominance aliasing.

5. SIMULATION RESULTS

The objectives of this section are: i) designing a proper color frame that satisfies (10), and ii) designing luminance and chrominance spatial dictionaries that conform to our color image model. Our results will be compared with other state-of-the-art denoising algorithms at the end of this section.

5.1. Designing the color frame

Consider the Bayer CFA for now. The constraint in (10) implies that $\theta$’s first column, i.e. $[\theta_{11} \theta_{21} \theta_{31}]^T$ should be orthogonal to the second and third rows of $K$. From (3) it follows that $K_{Bayer, \theta} = \theta'^*$ for any $\theta$ that has $[1 1 1]^T/\sqrt{3}$ (lumiance) at its first column. As another example, consider the Spatio-Spectral CFA [2] (Fig. 1(b)):

$$K_{SGS} = \begin{bmatrix} \mu_\alpha/\gamma & 1 - \mu_\alpha/\gamma - \mu_\beta/\gamma & \mu_\alpha/\gamma \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

where $\mu_\alpha$, $\mu_\beta$ and $\gamma$ are parameters of the CFA. Again $K_{SGS}$ has the structure denoted in (10) for any $\theta$ that has $[1 1 1]^T/\sqrt{3}$ as its first column. Therefore, a relevant color frame for such CFAs would a) contain the lumiance atom and b) be incoherent, i.e. columns of the color frame should have minimal inner products. Note that, the vectors in the color frame do not have to lie in the positive octant of $R^3$ (true colors). However, our experiments show a true color frame performs better than an artificial frame (that has negative RGB values) when BP solver [10] is utilized at the decoder. We use Incoherent Color Frames (ICF) [8] for $\theta$, which satisfy the above constraints. It should be noted that, the authors in [11], who propose to employ a single-pixel CS camera for acquiring Bayer samples, use a similar structure for spectrally decorrelating RGB quarter-planes.

5.2. Designing the spatial dictionary

The presented algorithm is run over $8 \times 8$ patches. Our experiments show that there are only a few possibilities for a
chrominance channel over such a small region; it can be constant, or have a vertical, horizontal or a diagonal step. For our simulations, we have used the 6 first DCT atoms (in zigzag order) for an $8 \times 8$ patch, as shown in Fig. 2.

![Fig. 2: The chrominance spatial atoms for an $8 \times 8$ patch.](image)

Fig. 3 shows our demosaicing results when applied on the fence region of the ‘lighthouse’ image, among other leading demosaicing results. Note that aliasing artifacts are mainly suppressed in the proposed scheme (Fig. 3(f)).

![Fig. 3: A cropped patch from different demosaicing methods using the Bayer CFA.](image)

We also applied our proposed framework to images shown in Fig. 4, when the Spatio-Spectral CFA no.1 of [2], Fig. 1(b), is employed. The results are summarized in Table 1, along with the results of [2].

![Fig. 4: Sample images (numbered from left to right, and top to bottom).](image)

### 6. CONCLUSION

In this paper, we have shown that the luminance and chrominance information need different configurations both in color and spatial domains. Periodic CFAs, specifically the ones that maximally distinguish luminance and chrominance bands, allow separate handling of the two components. The results would be enhanced by training spatio-spectral dictionaries similar to [6], within the proposed framework, which is a future work.

### 7. REFERENCES


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# Table 1: Comparison of PSNR results (dB) between the proposed method and the method of [2] when the Spatio-Spectral CFA no.1 of [2] is employed.