Tests of Sommerfeld Ground Wave Theory using Ground-Penetrating Radar Pulses

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Abstract —We have used ground-penetrating radar pulse waveforms within moveout profiles that we recorded over dielectric ground, to test the geometric amplitude attenuation rates of interfacial surface air waves predicted by classic Sommerfeld theory. The pulses allow separation of interfacial wave modes. We used horizontally polarized pulses centered from 37–390 MHz, frozen ground, and an intermediate range of tens of wavelengths over which radial polarization is predicted to exhibit anomalously low rates. As predicted, azimuthal broadside polarization at 37–45 MHz attenuated in proportion to the square of range, but the radial endfire rate was closer to the 1.6 power of range. At 360–390 MHz, both rates exhibited predicted range-squared amplitude attenuation. Not readily extractable from the theory are the rates for subsurface direct waves, for which we found, from numerical experiments, that both polarizations attenuate in proportion to the square of range. This suggests that geometric rates for all types of subsurface interfacial waves attenuate in proportion to range squared and could be subtracted from actual rates to determine material loss rates.

Keywords—ground waves; surface waves; Sommerfeld; dielectric ground; attenuation rates; GPR

I. INTRODUCTION

Antennas situated on the ground surface communicate primarily via surface waves (also known as ground waves). Above about 10 MHz, their attenuation rates asymptotically approach range-squared (1/R^2; where α = 2) dependency regardless of polarization, where range for these waves is known as the “far field.” After only one or two free space wavelengths (λ₀; λ is in situ wavelength), which is known as the “near field”, azimuthal (broadside) horizontal polarization (E₀ electric field strength) follows α = 2 dependency. However, within an intermediate range (IR) of several 10s λ₀, radially (endfire), horizontally polarized (E₀), surface air waves exhibit a lesser α, possibly near unity. Essentially, it appears as if the ground absorbs energy to cause eventual, α = 2 dependency.

The classic Sommerfeld theory was originally developed for the conductive earth case [1–3]. It was later also applied to dielectric earth [4, 5]. Integral solutions apply to ground of complex permittivity, ε*, and for antennas at any height above or below ground [3]. King et al. [6] numerically investigated the radial dielectric case and found such lesser rates existed for the IR, but gave no detail regarding rate of decay. To date, no field observations have been made to test these theories for horizontal polarization above 10 MHz, probably because the theory was developed for fixed frequencies and thus, the many complex modes that are excited along a layered earth cannot be separated.

Our objective was to measure these rates over dielectric ground in the IR. We used ground-penetrating radar (GPR) pulses, recorded in moveout profiles, to separate the air wave from subsurface modes that had returned to the surface. We compared the rates found against numerical integrations of the Sommerfeld theory for both radial (endfire) and azimuthal (broadside) horizontal polarization at 37–390 MHz. We used our calculations of α to describe the rates, although the actual dependency is likely to be slightly more complicated. We chose ground of medium dielectric constant ε = 6.8 because at such values there is a pronounced effect upon E₀ below about 100 MHz. As ε increases the attenuation rate becomes even closer to α = 1 at 37 MHz, and at less than about 20 λ₀, ε as it would for increasing conductivity [6]. Our pulse dominant frequencies were fixed by the available antennas.

Surface waves may be direct or indirect. Direct waves are received in a direct line from a transmitter antenna and are accompanied by direct subsurface waves (Fig. 1a). Both waves are matched on their respective opposite sides of the surface interface by head and evanescent waves, respectively. Indirect, also known as “lateral” or “refracted” surface waves, are generated by a subsurface antenna or by reflections that refract along the surface (Fig. 1b). After reaching the surface, indirect refracted waves are predicted to behave the same as direct waves [3, 6, 7]. It is the direct subsurface waves or modes within a near-surface layer that are generally exploited to derive the real part of ground dielectric permittivity, ε', from moveout profiles [8–10]. These subsurface waves can be trapped or guided [8, 9]. Such guided waves do not affect the direct wave but generate multiple indirect lateral air waves [8, 9], as we encountered. In our case below, we analyzed the leading cycles of these waves.

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subsequently refracted along the surface. We did this because we argue that our 37 MHz, radially polarized case was a lateral wave. We performed the calculation by placing the antenna at its mirror image below the subsurface interface, to account for the two-way propagation down to the interface and back (Fig. 1b), and then added finite ground conductivity to eliminate any further propagation of the subsurface wave beyond the point of refraction.

We performed modeling with a pseudospectral, finite-difference time domain (FDTD) 3-D model [13], using a point dipole source (as well as points of computation) placed at one-half grid element (0.25–0.40 m) above the interface. The point approximation is reasonable because real antennas are shorter than a half wavelength, and the current distribution is approximately a tapered cosine [14]. Our source was a Ricker wavelet, with spectrum centered at our pulse center frequency (about 38% lower than the apparent center frequency).

III. RESULTS

Our site was situated on the Donnelly glacial moraine, within Fort Greely, and near Delta Junction in the interior of Alaska (Fig. 2 inset). The transect was on a flat road, comprised of ice-bonded sand and gravel, with clast sizes less than a few centimeters. We estimated the rms surface height to have been < 10 cm, and some cobbles may have approached 7–8 cm maximum dimension. A 314-MHz reflection profile (Fig. 2) showed the interface between the layered road construction and the underlying unstratified moraine at 1–1.5 m depth, based on the \(\varepsilon' = 6.8\) derived from the 360–390 MHz ground wave slopes discussed below. The polarity of the layer reflections indicated a higher \(\varepsilon'\) beneath the road. The ground conductivity was likely < 0.0005 S m\(^{-1}\).

An \(\varepsilon' = 6.8\) gives \(\theta_{ca} = 22.5^\circ\). An average layer depth of 1.3 m then provides 2.8 m of round trip, subsurface propagation for an indirect wave before it would begin surface refraction 1.1 m from the transmitter and then, follow the direct air wave by 5.8 ns.

In Fig. 3a, the \(E_\rho\) profile of the air wave is direct because its 46 MHz, dominant frequency of the leading 1½ cycles is close to the antenna design frequency of 50 MHz. The wavelet is well isolated (Fig. 3c). The \(E_\rho\) profile (Fig. 3b) shows an unusual 71 MHz precursor followed by a stronger, 2½-cycle, 37 MHz air wave. Consequently, we interpret these two \(E_\rho\)

\[\text{Figure 1. Diagram a: Representation of waves launched by a surface antenna lying on homogeneous ground [5] described by a cylindrical coordinate system. A and B are spherical waves, and C and D are matching evanescent and head waves, respectively. The phase front of D propagates at the critical angle, } \theta_c = \sin^{-1}(1/c), \text{ with respect to vertical.} \]

\[\text{Diagram b: Representation of direct air and subsurface waves (dashed lines), and air and ground refractions (solid lines) propagating from a transmitter (Tx), to a receiver antenna (Rx). The air and ground refractions are launched at their critical angles, } \theta_{ca} \text{ and } \theta_{cg}, \text{ respectively. We used a mirror image source for Tx to simulate the down-up-over mode with numerical integration of the Sommerfeld solutions.} \]

II. METHODS

We used the Geophysical Survey Systems, Inc. (GSSI) System 10B, Model 5103 “400 MHz” antenna unit and 50 MHz (nominal design frequencies) antennas that we constructed ourselves for our moveout profiles. The actual center frequencies were either close for air waves (e.g., 46 MHz), or significantly less for waves launched into the subsurface (e.g., 37 MHz) because of ground impedance loading [11]. We separated the antennas by 0.2 m (400 MHz) and 1 m (50 MHz) increments and recorded with a 128-fold stack. Our maximum ranges (20 m at 400 MHz; 120 m at 50 MHz) encompassed the intermediate ranges. For post-processing, we removed the recorded range gain, then band-pass and used a background removal filter to reduce noise and clutter, respectively. We then Hilbert transformed (in amplitude) the traces, leaving only amplitude envelopes to alleviate local perturbations in waveform. We checked this procedure against using peak amplitudes and found no difference in attenuation rates.

We calculated Sommerfeld attenuation rates (\(\alpha_0\)) by using the standard National Electric Code (NEC)-IV code [12] at frequencies that corresponded with our pulse center frequencies. This theory applies to single frequencies and does not account for direct subsurface waves. It does account for antenna depth, and so we also calculated the attenuation rate for a lateral wave launched by a subsurface reflection that

\[\text{Figure 2. A 314 MHz reflection profile of our transect with depth scale calibrated using } \varepsilon' = 6.8. \text{ The reflection from the bottom of this frozen active layer at about 1.0–1.5 m depth is probably generated by a higher value of permittivity below the interface. The inset locates our area in interior Alaska.} \]
events to be direct and indirect, respectively; this interpretation suggests that the frequency of direct endfire radiation can differ from that of the broadside. The weaker, direct air wave precursor at 71 MHz is resolved in the Hilbert transformation (Fig. 3d, inset). Both moveout profiles in Fig. 3 contain 37 MHz ground reflection events (labeled with “2”) which do not last long enough to calculate their slopes because they lose energy through the subsurface interface.

The $E_\phi$ attenuation rate exponent (Fig. 3e) shows $\alpha = 1.93$ from $1.5–18\lambda_o$ (10–119 m). The Sommerfeld calculation gives $\alpha_S = 2.07$; close to the theoretical $\alpha = 2.00$. In the 37 MHz indirect $E_\rho$ case $\alpha = 1.64$ from $2.0 \left(0.9\lambda + 1.1\lambda_o\right)$ to 15 wavelengths, while $\alpha_S = 1.54$. For the direct $E_\rho$ case, the 71 MHz precursor gives $\alpha = 1.95$ from $1.7–29.0\lambda_o$ of propagation, while $\alpha_S = 1.76$. All fits have an $r^2$ value close to 1.00. This range of $29.0\lambda_o$ may be considerably farther than...
the IR. However, decreasing the 71 MHz range to 19\(\lambda_o\) actually increased \(\alpha\) to 2.01; decreasing to 10\(\lambda_o\) put \(\alpha\) at 2.00.

Figs. 4a–4b show 360 MHz \(E_\phi\) and 390 MHz \(E_\rho\) moveout profiles. The leading air waves are direct transmissions because their leading edges project to intersect those of the ground events at 0 m (beyond the vertical axes) and because their dominant frequencies are close to the antenna design frequency of 400 MHz. As in the previous example, the frequency for \(E_\rho\) is higher. The ground waves have similar dominant frequencies, and slopes that give \(\epsilon' = 6.8\). Fig. 4c shows good isolation of the direct wave for the \(E_\phi\) profile; the isolation for the \(E_\rho\) event is best seen in its Hilbert transform (Fig. 4d inset). For \(E_\phi\), \(\alpha = 2.09\) and \(\alpha_S = 2.06\) beyond 1.9\(\lambda_o\) (Fig. 4e); equation (5) gives \(\alpha = 2.00\). For \(E_\rho\), \(\alpha = 2.14\) and \(\alpha_S = 1.78\) from 1.9–26.0\(\lambda_o\) (Fig. 4f). Again, the \(r^2\) values are close to 1.000. This range of 26.0 \(\lambda_o\) may be considerably farther than

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**Figure 4.** \(E_\phi\) 360 MHz (a) and \(E_\rho\) 390 MHz (b) polarization moveout profiles; sample traces showing air wavelets (c, d) with labeled distances at which they were recorded (inset is the Hilbert transform); and respective plots of air wave field strength vs. range (e, f). Events 1 and 2 (in a, b) are the direct air and subsurface waves, respectively. The arrows (in d) indicate the event of interest. In (e, f), the black triangles are field data and the open boxes are Sommerfeld numerical data, the latter of which are offset to allow comparison with the field values. The \(\alpha\) (field) and \(\alpha_S\) (Sommerfeld) values apply to the best linear fits (dashed lines) over the indicated range of \(R\).
the IR. However, \( \alpha \) decreases only to 2.00 after decreasing the data range to 10.0\( \lambda_o \).

IV. MODELS

In Fig. 5 we modeled 37 MHz field strength for \( E_\phi \) and \( E_\rho \), and for both air and subsurface direct waves on homogeneous dielectric ground with \( \varepsilon' = 6.8 \). The legends give the ranges over which we calculated \( \alpha \). For \( E_\rho \) the \( \alpha \) values are slightly greater than predicted by Sommerfeld theory or than we measured. We tried several ranges and still found \( \alpha \) approached 2.00 more rapidly than expected. The results for \( E_\phi \) deviate slightly from perfect \( \alpha = 2.00 \) dependency, which suggests there may be an error. In both cases, the subsurface \( \alpha \) is very close to 2.00 with steady dependency. Consequently, use of subsurface waves to calculate ground attenuation rates should be able to assume \( \alpha = 2.00 \) geometric rates; at higher frequencies this rate should remain.

V. CONCLUSIONS

Our field observations corroborate the Sommerfeld predictions. Our model results also show a significant difference in behavior between the two polarizations, but the failure to reach an exact \( \alpha = 2 \) dependency for \( E_\phi \) at all ranges needs further investigation. They do, however, strongly suggest that \( \alpha = 2 \) behavior exists for all ranges and polarizations for any subsurface interfacial wave.

Our lower frequency observations were limited by cable length to 120m range. High power antennas are now available with fiber optic connections that can possibly be increased to several hundred meters. This would allow more accurate and complete observation of transitions to the eventual, “far field” \( \alpha = 2 \) dependency, and eliminate any possible modes propagating along the cable (such as the events in Fig. 3 indicated by large arrows).

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REFERENCES


