Adaptive Immersion and Invariance Control for a Class of Electromechanical Systems

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Abstract—This paper proposes a control approach for a class of electromechanical systems based on immersion and invariance method. Two cases are considered: full information system and unknown parameters existence. Thereafter, two controllers are, respectively, proposed: stabilization control and adaptive control. The effectiveness of the obtained control schemes is illustrated via simulation of an example.

Keywords—Adaptive control, Electromechanical system, Immersion and invariance, Nonlinear system.

I. INTRODUCTION

The control of the nonlinear systems represents an unceasing defy for automatic control community because of complexity related to the mathematical models as well as the absence of the unified approaches [1], [2],[3]. Indeed, the nature of nonlinearities and the diversity of the classes of systems increase extra constraints on the applicability and the effectiveness of the control approaches [4], [5], [6]. Consequently, research in the field of the nonlinear systems does not cease developing in order to promote new more convenient and more reliable control. One of the recent and promising approaches is that of Immersion and Invariance (I&I) [7]. This control method is based on two fundamental concepts: invariance and the immersion. The first concept, based on the theory of differential geometry, is exploited in the stability analysis of the nonlinear systems by the research of the invariant manifolds [1]. The second concept allows the immersion of the dynamic behavior of a reduced order system towards high order system without loss of the properties of the first one.

Recently, in [7], these two concepts have been combined to create the approach I&I which allowed the synthesis of a nonlinear control law guaranteeing the stabilization of the system on the basis of a subsystem of a reduced nature and whose dynamic characteristics are preset. Another advantage of this approach is the not-need for Lyapunov candidate function in the phase of the controller design [8], [9].

This control approach has been, also, exploited within the adaptive control framework of nonlinear systems with unknown parameters. In this field, the I&I approach is fortified with a principal advantage which is the perfect decoupling between closed loop system dynamics and those of the adaptation law. So that, more freedom in the choice of the adaptation law is allowed without generating undesirable mutual influences.

In this context, the present work is devoted to the proposition of a control approach based on immersion and invariance for a class of electromechanical systems often encountered in practice. The system with full information knowledge will be, firstly, considered to carry out an I&I stabilization controller. Secondly, the case of unknown parameters existence in the dynamical model will be relaxed by the design of an adaptive I&I controller. The efficacy of suggested controllers is evaluated through simulation of an example.

II. SYSTEM DESCRIPTION AND PRELIMINARY RESULTS

A. System Description

In this paper a particular class of third order electromechanical systems will be considered. This class is given by the following state equation:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + f_1(x_1, x_2) \\
\dot{x}_3 &= u + f_2(x_1, x_2, x_3)
\end{align*}
\] (1)

The main objective, in the present work, is the I&I approach application to the control of the above system class. The first task is the system stabilization with the assumption that all parameters are known. The second is the consideration of unknown system parameters existence. Indeed, the adaptive I&I control will be investigated to attend the synthesis goal.

B. Preliminary results

In this section the fundamental theorem, serving as principal tool of the Immersion and Invariance approach, is recalled.

Theorem 1 [7]. Consider the system

\[
\dot{x} = f(x) + g(x) u
\] (2)

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \) and an equilibrium point \( x^* \in \mathbb{R}^n \) to be stabilized. Let \( p < n \), and assume that we can find mapping

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\[
\alpha(\cdot): \mathbb{R}^p \to \mathbb{R}^p , \quad \pi(\cdot): \mathbb{R}^p \to \mathbb{R}^n , \quad \phi(\cdot): \mathbb{R}^p \to \mathbb{R}^m , \quad \psi(\cdot): \mathbb{R}^{n \times (n-p)} \to \mathbb{R}^m
\]

such that the following hold

**H1** (Target system). The system

\[
\dot{\xi} = \alpha(\xi)
\]

with state \( \xi \in \mathbb{R}^p \), has a globally asymptotically stable equilibrium at \( \xi_\ast \in \mathbb{R}^p \) and \( x_\ast = \pi(\xi_\ast) \).

**H2** (Immersion condition). For all \( \xi \in \mathbb{R}^p \)

\[
f(\xi) + g(\xi)\psi(x,z) = \frac{\partial \pi}{\partial \xi} \alpha(\xi)
\]

**H3** (Implicit manifold). The following set identity holds:

\[
\{ x \in \mathbb{R}^n / x = \pi(\xi) \} = \{ x \in \mathbb{R}^n / x = \pi(\xi), \text{pour} \xi \in \mathbb{R}^p \}
\]

**H4** (Manifold attractivity and trajectory boundedness). All trajectories of the system

\[
\dot{z} = \partial_{\xi}[f(x) + g(x)\psi(x,z)]
\]

are bounded and satisfy:

\[
\lim_{t \to \infty} z(t) = 0
\]

Then \( x_\ast \) is a globally asymptotically stable equilibrium of the closed-loop system

\[
\dot{x} = f(x) + g(x)\psi(x,\phi(x))
\]

This control approach has been used for adaptive control of nonlinear systems. The adaptive immersion and invariance framework has been developed, firstly, in [4]. Indeed, consider a nonlinear system with unknown parameter vector \( \theta \):

\[
\dot{x} = f(x) + g(x)u, \quad f(x) = f_0(x) + \varphi^T(x)\theta,
\]

If there exists \( u = \psi(x,\theta_\ast) \) ensuring a globally asymptotically stable equilibrium at \( x = x_\ast \), then the I&I adaptive control problem is formulated as follows:

**Definition** [7]. The system (2) is said to be adaptively I&I stabilizable if the system

\[
\begin{cases}
\dot{x} = f(x) + g(x)\psi(x,\bar{\theta} + \beta_1(x)) \\
\dot{\bar{\theta}} = \beta_2(x,\bar{\theta})
\end{cases}
\]

with extend state \( x,\bar{\theta} \) and controls \( \beta_1 \) and \( \beta_2 \) is I&I stabilizable with target dynamics \( \dot{\xi} = f_\xi(x) \).

This method has been, later, considered for nonlinear systems in feedback form [10], [11], [12]. Specific adaptive I&I control laws have been, consequently, developed. These controllers are, also, based on the systematic procedure of backstepping control naturally suitable for this class of systems. One of the essential control structures is given in [11]. This adaptive control scheme maintains the same four conditions, needed for the application of theorem 1, with some particularities due to unknown parameter vector. The major difference is the statement of the fourth hypothesis given as follows:

**H'4** All trajectories of the system

\[
\dot{z} = \omega(x,\xi,\bar{\theta}) + \frac{\partial \beta}{\partial x} f(x,u(x,\xi,\bar{\theta}),\theta) \]

are bounded and satisfy:

\[
\lim_{t \to \infty} \xi(t) = 0
\]

III. I&I STABILIZATION OF THE FULL-INFORMATION SYSTEM

In this section we consider the stabilization control via I&I approach of the electromechanical system class given by (1). All system parameters are supposed known. The proposed control law is formulated in the following theorem.

**Theorem 2.** Consider system (1), the following control law

\[
u = -\gamma(x_3 + k_1x_1 + k_2x_2 + f_1(x_1, x_2)) - k_4x_2
\]

\[
- f_2(x_1, x_2, x_3) - k_3x_3 - k_2f_1(x_1, x_2) - \frac{\partial f_1}{\partial x_1}x_1
\]

where: \( \gamma, k_1 \) and \( k_2 \) are positive design parameters, ensures that the origin is an asymptotically globally stable equilibrium of the closed loop system.

**Proof.** The demonstration is done by checking the four assumptions imposed in Theorem 1:

- Verification of (H1): let us consider the following target system:

\[
\begin{align}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= -k_1\xi_1 - k_2\xi_2
\end{align}
\]
where \((k_1, k_2)\) are positive parameters. It is obvious that the considered system has \(\xi_* = (0, 0)\) as the only globally asymptotically stable equilibrium.

- Verification of (H2): by application of (4) we obtain the mapping \(\pi(\xi)\) as follows:

\[
\begin{align*}
\pi_1 &= \xi_1 \\
\pi_2 &= \xi_2 \\
\pi_3 &= -k_1\xi_1 - k_2\xi_2 - f_1(\xi_1, \xi_2)
\end{align*}
\]

(19)

Then, the immersion condition is checked.

- Verification of (H3): the application of the set identity (5) permits the derivation of the following implicit manifold:

\[
M = \{ x_3 + k_1x_1 + k_2x_2 + f_1(x_1, x_2) = 0 \} \quad (20)
\]

- Verification of (H4): applying (4) we get:

\[
\begin{align*}
\dot{z} &= u + f_2(x_1, x_2, x_3) + k_1x_2 + k_2x_3 + k_2f_1(x_1, x_2) \\
&\quad + \left( \frac{\partial f_1}{\partial x_1} \right) \dot{x}_1 + \left( \frac{\partial f_1}{\partial x_2} \right) \dot{x}_2
\end{align*}
\]

(21)

in order to satisfy that the perturbation variable \(z\) converges to zero it is sufficient to impose \(\dot{z} = -\gamma z\), with \(\gamma > 0\). So that, we choose:

\[
\begin{align*}
u &= \psi(x, z) = -\gamma z - f_2(x_1, x_2, x_3) - k_1x_2 - k_2x_3 \\
&\quad - k_2f_1(x_1, x_2) - \left( \frac{\partial f_1}{\partial x_1} \right) \dot{x}_1 - \left( \frac{\partial f_1}{\partial x_2} \right) \dot{x}_2
\end{align*}
\]

(22)

Substituting (22) into (1), we obtain the following state equation:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + f_1(x_1, x_2) \\
\dot{x}_3 &= -\gamma z - k_1x_2 - k_2x_3 - k_2f_1(x_1, x_2) \\
&\quad - \left( \frac{\partial f_1}{\partial x_1} \right) \dot{x}_1 - \left( \frac{\partial f_1}{\partial x_2} \right) \dot{x}_2
\end{align*}
\]

(23)

To complete the test of the hypothesis H4, we set the following change of variable:

\[
\eta = x_3 + f_1(x_1, x_2)
\]

(24)

Thus, in the new base \((x_1, x_2, \eta)\) we obtain:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \eta \\
\dot{\eta} &= -\gamma z - k_1x_2 - k_2\eta
\end{align*}
\]

(25)

By considering the autonomous sub-system \((x_2, \eta)\) and choosing the Lyapunov candidate function \(V(x_2, \eta) = \frac{1}{2}k_1x_2^2 + \frac{1}{2}\eta^2\), holds that \(\dot{V}(x_2, \eta) = -k_2\eta^2 < 0\). So that, \((x_2, \eta)\) converge to zero since \(z\) converges to zero. In addition, \(x_1 = x_2\), then \(x_1\) is bounded.

Consequently, the four hypothesis of theorem 1 are valid. Therefore, the control law given by (17) is obtained from (22) by replacing \(z\) by \(\phi(x)\), which achieves the proof.

IV. ADAPTIVE I&I CONTROL OF THE SYSTEM WITH UNKNOWN PARAMETERS

At this phase, we will consider the existence of unknown parameters in the system model. The adaptive I&I control scheme will be applied.

Since considering the presence of unknown parameters, the system state equation can be rewritten as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + \varphi_1(x_1, x_2)^T \theta \\
\dot{x}_3 &= u + \varphi_2(x_1, x_2, x_3)^T \theta
\end{align*}
\]

(26)

The proposed adaptive I&I control scheme is expressed in the following theorem.

**Theorem 3.** Consider system (26), the following adaptive I&I control law:

\[
\begin{align*}
u &= -\gamma(x_3 + k_1x_1 + k_2x_2 + \varphi_1^T \hat{\theta}) - k_1x_2 - k_2x_3 \\
&\quad - \frac{\partial \varphi_1^T}{\partial x_1} x_2 - \frac{\partial \varphi_1^T}{\partial x_2} x_3 \hat{\theta} - \varphi_1^T \omega(x, \hat{\theta}) \\
&\quad - \varphi_2^T (\hat{\theta} + \beta) - \left( k_2 + \frac{\partial \varphi_1^T}{\partial x_2} \theta \right) \varphi_1^T (\hat{\theta} + \beta)
\end{align*}
\]

(27)

with the adaptive law given by:

\[
\dot{\hat{\theta}} = \omega(x, \hat{\theta}) = -\frac{\partial \beta}{\partial x_2} x_2 - \frac{\partial \beta}{\partial x_2} (x_3 + \varphi_1^T (\hat{\theta} + \beta))
\]

(28)

and:

\[
\beta(x_1, x_2) = \lambda \int_0^{x_2} \varphi_1(x_1, \lambda) d\lambda
\]

(29)

where: \((\gamma, k_1, k_2, \lambda)\) are positive design parameters,
ensures that the origin is an asymptotically globally stable equilibrium of the closed loop system.

**Proof.** Similarly to the proof of theorem 2, we proceed by checking the four conditions allowing the design through I&I approach of adaptive control as depicted in the second part of the preliminary results.

- Verification of (H1): for this purpose we take the same target system given by (18).
- Verification of (H2): by application of (4) we obtain the mapping \( \pi_{1} \), having the same form as in (19):
  \[
  \begin{align*}
  \pi_1 &= \xi_1 \\
  \pi_2 &= \xi_2 \\
  \pi_3 &= -k_2 \xi_1 - k_2 \xi_2 - \varphi_1 (x_1, x_2)^T \theta 
  \end{align*}
  \]
  Then, the immersion condition is checked.
- Verification of (H3): the application of the set identity (5) permits the derivation of the following implicit manifold:
  \[
  \phi(x, \theta) = x_3 + k_1 x_1 + k_2 x_2 + \varphi_1 (x_1, x_2)^T \theta = 0
  \]
- Verification of (H’4): applying (12) we get:
  \[
  \begin{align*}
  \dot{\zeta} &= \dot{\phi}(x, \theta) \\
  &= u + \varphi_2^T \theta + k_1 x_2 + k_2 (x_3 + \varphi_1^T \theta) + \frac{\partial \varphi_1^T}{\partial x_1} \dot{x}_1 \\
  &= u + \varphi_2^T \theta + k_1 x_2 + k_2 (x_3 + \varphi_1^T \theta) + \varphi_1^T \omega(x, \zeta, \theta)
  \end{align*}
  \]

Define: \( z = \dot{\theta} - \theta + \beta(x_1, x_2) \), so we can reformulate (32) as:

\[
\dot{\zeta} = u + k_1 x_2 + k_2 x_3 + \varphi_1^T \omega(x, \zeta) + \left( \frac{\partial \varphi_1^T}{\partial x_1} - x_2 + \frac{\partial \varphi_1^T}{\partial x_2} x_3 \right) \dot{\theta} \\
+ \left( \varphi_2^T + \left( k_2 + \frac{\partial \varphi_1^T}{\partial x_2} \dot{\theta} \right) \varphi_1^T \right) (\theta + \beta) \\
- \left( \varphi_2^T + \left( k_2 + \frac{\partial \varphi_1^T}{\partial x_2} \dot{\theta} \right) \varphi_1^T \right) z 
\]

The selection of the controller expression as follows:

\[
\begin{align*}
  u &= -\gamma \zeta - k_1 x_2 - k_2 x_3 - \varphi_1^T \omega(x, \zeta) \\
  &= -\left( \frac{\partial \varphi_1^T}{\partial x_1} x_2 + \frac{\partial \varphi_1^T}{\partial x_2} x_3 \right) \dot{\theta} \\
  &= -\left( \varphi_2^T + \left( k_2 + \frac{\partial \varphi_1^T}{\partial x_2} \dot{\theta} \right) \varphi_1^T \right) (\theta + \beta)
  \end{align*}
\]

transforms (33) as:

\[
\dot{\zeta} = -\gamma \zeta - \left( \varphi_2^T + \left( k_2 + \frac{\partial \varphi_1^T}{\partial x_2} \dot{\theta} \right) \varphi_1^T \right) z
\]

In addition, the time derivative of \( z \) is given by:

\[
\dot{z} = \omega(x, \theta) + \frac{\partial \beta}{\partial x_1} x_2 + \frac{\partial \beta}{\partial x_2} (x_3 + \varphi_1^T (\dot{\theta} + \beta)) - \frac{\partial \beta}{\partial x_2} \varphi_1^T z
\]

So, if we select \( \omega(x, \theta) \) and \( \beta(x_1, x_2) \) as given respectively by (28) and (29), then (36) becomes in the novel form:

\[
\dot{z} = -\lambda \left( \varphi_1^T \right) z
\]

This expression proves that \( z \) converges to zero. Furthermore, \( \zeta \), also, converges to zero.

Consequently, the system (23) can be expressed in the \( \zeta \), \( z \), \( x_1 \), \( x_2 \) coordinates as:

\[
\begin{align*}
  \dot{\zeta} &= -\gamma \zeta - \left( \varphi_2^T + \left( k_2 + \frac{\partial \varphi_1^T}{\partial x_2} \dot{\theta} \right) \varphi_1^T \right) z \\
  \dot{z} &= -\lambda \left( \varphi_1^T \right) z \\
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= -k_1 x_1 - k_2 x_2 - \varphi_1 (x_1, x_2)^T z + \zeta
  \end{align*}
\]

So, since \( \zeta \) and \( z \) converge to zero, \( (x_1, x_2) \) converge to zero.

Finally, we can deduce that the system (38) has a globally asymptotically stable equilibrium at the origin. Then, the proof is achieved.

**V. EXAMPLE**

The obtained results are applied, in this section, to ensure the control of an electromechanical system composed by a DC motor which drives a manipulator arm with a load [13].
The dynamic model of the considered system is given by the following:

$$\begin{align*}
M\ddot{q} + B\dot{q} + N\sin(q) &= I \\
L\dot{I} &= V_0 - RI - KB\dot{q}
\end{align*}$$

(39)

where:

$$M = \frac{J}{K_T} + \frac{mL_0^2}{3K_T} + \frac{M_0L_0^2}{K_T} + \frac{2M_0R_0^2}{5K_T},$$

$$N = \frac{mL_0G}{2K_T} + \frac{M_0L_0G}{K_T} \quad \text{et} \quad B = \frac{B_0}{K_T}.$$  

**TABLE I. SYSTEM PARAMETERS AND THEIR VALUES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>Rotor inertia</td>
<td>$1.625 \times 10^{-3}\text{Kg.m}^2$</td>
</tr>
<tr>
<td>$m$</td>
<td>Link mass</td>
<td>$0.506\text{ Kg}$</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Load mass</td>
<td>$0.434\text{ Kg}$</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Link length</td>
<td>$0.305\text{ m}$</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Radius of the load</td>
<td>$0.023\text{ m}$</td>
</tr>
<tr>
<td>$G$</td>
<td>Gravity coefficient</td>
<td>$9.8 \text{ /ms}$</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Coefficient of viscous friction</td>
<td>$16,25.10^{-3}\text{Nms/ rad}$</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Motor torque constant</td>
<td>$0.90\text{N.m / A}$</td>
</tr>
<tr>
<td>$R$</td>
<td>Armature resistance</td>
<td>$5.0\Omega$</td>
</tr>
<tr>
<td>$L$</td>
<td>Armature inductance</td>
<td>$25.10^{-3}\text{H}$</td>
</tr>
<tr>
<td>$K_B$</td>
<td>Back-emf constant</td>
<td>$0.90\text{N.m / A}$</td>
</tr>
</tbody>
</table>

Choosing: $x_1 = q$ , $x_2 = \dot{q}$ , $x_3 = \frac{I}{M}$ and $u = (\frac{1}{LM})V_0$ , 

the state space representation is derived from (39) by the following expression:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 - a_1\sin(x_1) - a_2x_2 \\
\dot{x}_3 &= u - b_1x_2 - b_2x_3
\end{align*}$$

(40)

where: $a_1 = \frac{N}{M}$ , $a_2 = \frac{B}{M}$ , $b_1 = \frac{K_B}{LM}$ and $b_2 = \frac{R}{LM}$.

1) **I&I stabilization:**

We consider the full information system. The system state equation has the same form introduced in (1) with:

$$f_1(x_1,x_2) = -a_1\sin(x_1) - a_2x_2$$

$$f_2(x_1,x_2,x_3) = -b_1x_2 + b_2x_3$$

Then, according to Theorem 1 the control law is set as

$$u = -\gamma(x_3 + k_1x_1 + k_2x_2 - a_1\sin(x_1) - a_2x_2) + [b_1 - k_1 + a_1\cos(x_1) - a_2(-k_2 + a_2)]x_2$$

$$+ a_1(-k_2 + a_2)\sin(x_1) + (b_2 - k_2 + a_2)x_3$$

with $\gamma$ , $k_1$ and $k_2$ are positive design parameters.

2) **Adaptive I&I control:**

For adaptive I&I control, we consider the system where parameters $a_1$ , $a_2$ , $b_1$ and $b_2$ are supposed unknown.

Consequently, the system state equation can be expressed as in (26) where:

$$\varphi_1^T = [-\sin(x_1) -x_2 0 0], \quad \varphi_2^T = [0 0 -x_2 -x_3]$$

and $\theta^T = [a_1  a_2  b_1  b_2]$

The application of the theorem 3 permits the derivation of the following control law:

$$u = -\gamma(x_3 + k_1x_1 + k_2x_2 + \varphi_1^T\dot{\theta}) - k_2x_2 - k_2x_3$$

$$-\varphi_2^T\omega(x,\dot{\theta}) - \left[\frac{\partial\varphi_1^T}{\partial x_1}x_2 + \frac{\partial\varphi_1^T}{\partial x_2}x_3\right]\dot{\theta}$$

$$- \left[\varphi_2^T + \left[k_2 + \frac{\partial\varphi_1^T}{\partial x_2}\dot{\theta}\right]\varphi_1^T\right](\dot{\theta} + \beta)$$

where, $\dot{\theta}$ is the estimated parameter vector accordingly to (28) and $\beta$ is obtained from (29) as follows:

$$\beta^T = \lambda\left[-x_2\sin(x_1) - \frac{1}{2}x_2^2 0 0\right], \quad \lambda > 0$$

The controlled system has been simulated according to the adaptive backstepping available in [4] and [6], the design parameters are chosen to make the comparison of the system behavior possible, $k_1 = 10$ , $k_2 = 10$ , $\gamma = 5$ and $\lambda = 1$.

Figure1 shows the state variable responses using the I&I stabilization control (when all parameters are supposed known), the I&I adaptive control and the adaptive
backstepping. Figure 2 illustrates the corresponding armature voltage (control input) evolution. It is obvious that the proposed controllers (I&I stabilization and adaptive I&I) outcome on a more good transient behavior compared to the adaptive backstepping approach. Moreover, the time response of the system can be decreased simply by tuning the design parameters $\gamma$ and $\lambda$. The first parameter acts on the speed of the immersion surface reachability, while the second controls the adaption law speed.

VI. CONCLUSION

In this paper, an immersion and invariance based control approach of a class of electromechanical system has been proposed. Both full information and unknown parameters cases have been considered; indeed I&I stabilization and I&I adaptive controllers has been carried out, respectively. The proved theoretical results have been also illustrated through simulation of an example of electromechanical system.

REFERENCES