Estimation of the parameters for a complex repairable system with preventive and corrective maintenance

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Abstract—Estimation of reliability and maintainability parameters is essential to modeling repairable systems and determining maintenance policies. However, this estimation becomes more difficult when system failure times are neither identically nor independently distributed. This is due to the aging of repairable systems under imperfect maintenance. In this paper, reliability and maintainability RAM parameters are estimated in the maximum likelihood sense based on historical RAM data and using the virtual age model of Kijima Type I and Type II. A Weibull distribution for the first system failure is assumed. Kijima Type I and II imperfect corrective and preventive maintenance are also considered. Using the maximum-likelihood approach, four parameters of this repairable system are estimated. The proposed method is illustrated with simulated data.

Index Terms—Maximum-likelihood estimation, Imperfect maintenance, Preventive maintenance, Corrective maintenance, Parameter estimation, Virtual age, Kijima type I and II, Reliability and Maintainability.

I. INTRODUCTION

In the field of reliability, maintenance activities were initially few or not formalized. In addition, they consisted in repairing equipment once it was faulty, but few incorporated the concept of preventive maintenance PM that is to say interventions to prevent failure. Formalized notion of maintenance is relatively recent. It appeared with the automation of production, the growing economic and industrial issues and the stringent regulations for the protection of the individual and the environment. In this paper we will focus on maintenance strategies which aim to prevent, avoid or repair diverse and often complex system malfunctions (electronic, network,...), this includes the timing and nature of interventions (perfect repair, minimal repair, imperfect repair,...). To address this problem, it is often necessary to rely on mathematical modeling of reality, capable of capturing the complexity resulting system subject to the maintenance strategy. We introduce two types of maintenance, corrective imperfect maintenance CM and preventive imperfect maintenance PM in repairable system RS. The moment of failure for a repairable system depends on both the distribution of life and the effect of maintenance actions performed on the corresponding system. Whereas there are some physical models to describe the moment of failure for some RS, statistical models are also useful for modeling and evaluating the performance of repairable systems based on historical reliability and maintainability data without knowing the failure mechanism, these models have been discussed in [1], [2] and [3]. Maintenance can be classified into several categories such as perfect, minimal and imperfect maintenance. In this paper we will be interested in the imperfect maintenance often encountered in industry, where maintenance actions are not completely efficient. We required a mathematical model to describe the impact of imperfect maintenance in order to analyze the attitude of RS under imperfect maintenance. There is a large variety of imperfect maintenance models. We will mention the most known in the literature: The model of Brown and Proschan [4], the model of Chan and Shaw [5], the model of Doyen [6], the quasi-renewal [7] and models of virtual age Kijima [7],[8]. In this paper, we will discuss the Kijima type I and type II models. Kijima Type I model: The effect of the i-th maintenance is to reduce the virtual age just before the moment of failure, by an amount proportional to the time elapsed since the previous maintenance.

\[ V_i = V_{i-1} + a X_i \]  \hspace{1cm} (1)

When \( a = 0 \) this means that we are in the case of perfect maintenance and when \( a = 1 \) we are in the case of minimal maintenance.

Kijima type II model: The effect of the i-th maintenance is to reduce the system’s global virtual age of a quantity which is proportional.

\[ V_i = a \left( V_{i-1} + X_i \right) \]  \hspace{1cm} (2)

\( X_i \): designates the length of the i-th period of RS function, \( (i = 1, 2, 3, \ldots). \) \( V_i \): designates the virtual age of RS after the i-th maintenance action, \( (V_0 = 0) , (i = 1, 2, 3, \ldots). \) \( a \): represents the efficiency factor of imperfect maintenance \( (0 \leq a \leq 1) \). Maintenance actions performed on the RS can be classified into two groups: corrective maintenance actions and preventive maintenance actions. Corrective actions will be triggered after the system failure and may correspond to activities such as repairs or replacements. Preventive actions are imperfect maintenance actions intended to delay or prevent system failures, but they are not performed. We apply the maximum likelihood method to estimate reliability parameters \( (\beta, \eta) \) and maintainability parameters \( (a_r, a_p) \) using new forms.
of virtual age model: Kijima type I and type II. Some recent studies have been made to estimate the parameters of reliability and maintainability. In [7] the authors estimated the parameters of reliability and maintainability under imperfect CM and PM using the virtual age Kijima Type I model. In 2000 Chen et al. [9] discussed the estimation of reliability parameters using minimal corrective maintenance and perfect preventive maintenance of a Bayesian point of view. In [10] Dayanik, Savas and Gurler estimated reliability parameters using minimal corrective maintenance and perfect preventive maintenance. Seo et al. [11] treated the parameter estimation of reliability under minimal corrective maintenance and perfect preventive maintenance. In 2003 Gasmi et al. [12] considered reliability and maintainability parameters estimation as two types of corrective maintenance one is minimal CM and the other is imperfect CM with the virtual age model of Kijima Type I. Mattes and Zhao [13] in 2005 have estimated the parameters of reliability and maintainability under imperfect maintenance with Kijima Type II virtual age model. In [14] Pingjian et al. estimated the parameters of reliability and maintainability under imperfect CM and PM with Kijima type I model. In this paper, we consider a repairable system that undergoes corrective and preventive imperfect maintenance by using the new forms of virtual age models for Kijima Type I and type II. Our aim is to estimate the parameters of reliability and maintainability simultaneously, we remark that we simulate 100 reliability and maintainability data from a repairable system with parameters $\beta = 2.2$, $\eta = 1$, $a_r = 0.3$, $a_p = 0.8$ [14].

II. M A XIMUM LIKELIHOOD ESTIMATION

The weibull intensity is an extremely important intensity to characterize the probabilistic behavior of a large number of real phenomena. This intensity is especially used as a failure model in analyzing the reliability and maintainability of different types of repairable systems. We assume that the first failure of the system follows a Weibull distribution. The probability density function PDF of RS is given by:

$$f(t, \beta, \eta, V) = \frac{\beta}{\eta} \left(\frac{V + t}{\eta}\right)^{\beta-1} \exp \left[\left(\frac{V}{\eta}\right)^{\beta} - \left(\frac{V + t}{\eta}\right)^{\beta}\right]$$

The reliability of RS at time $t$ with the initial age of the system is given by:

$$R(t, \beta, \eta, V) = \exp \left[\left(\frac{V}{\eta}\right)^{\beta} - \left(\frac{V + t}{\eta}\right)^{\beta}\right]$$

Where $\eta$ is the scale parameter of the Weibull distribution and $\beta$ is the shape parameter of the Weibull distribution. We assume that $C$ denotes CM action, and $P$ denotes PM action. When the $i$-th action of maintenance is CM we have $i \in P$ and when the $i$-th action of maintenance is CM we have $i \in C$. Then we can write the joint PDF of $(X_1, X_2, \cdots, X_{i-1})$:

$$f(x_1, x_2, x_3, \ldots, x_i | \beta, \eta, a_r, a_p) = f(x_1; \beta, \eta, V_{i-1}) f(x_2, x_2, \ldots, x_{i-1} | \beta, \eta, a_r, a_p)$$

$$= \prod_{i=1}^{n} f(x_i; \beta, \eta, V_i)$$

When we have preventive maintenance actions, the data is right censored and we have to use reliability function in place of PDF. The likelihood function becomes:

$$f(x_1, x_2, \ldots, x_i | \beta, \eta, a_r, a_p) = L(x_1, x_2, \ldots, x_i | \beta, \eta, a_r, a_p)$$

$$= \prod_{i \in C} f(x_i; \beta, \eta, V_{i-1}) \prod_{i \in P} R(x_i; \beta, \eta, V_{i-1})$$

Using (3) and (4) we obtain the following likelihood-function:

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**TABLE I: SIMULATED DATA OF RELIABILITY AND MAINTENABILITY.**
\[ L(x_1, x_2, \ldots, x_i \mid \beta, \eta, a_r, a_p) = \prod_{i=1}^{n} \frac{\beta [(V_i - 1 + X_i)/\eta]^{\beta - 1}}\eta \prod_{i=1}^{n} \exp \left( \frac{(V_i - 1)\beta - (V_i - 1 + X_i)/\eta}{\eta} \right) \]  

(7)

We cannot estimate the parameters of reliability and maintainability because of the complexity of the likelihood function (7) in the presence of index C that represents corrective maintenance actions. To avoid this problem, we propose a new expression of the likelihood function by introducing the indicator function \( \delta_i \).

\[ \delta_i = \begin{cases} 1 & \text{If } CM \\
0 & \text{If } PM \end{cases} \]

(8)

We obtain:

\[ L(x_1, x_2, \ldots, x_i \mid \beta, \eta, a_r, a_p) = \prod_{i=1}^{n} \left[ \frac{\beta [(V_i - 1 + X_i)/\eta]^{\beta - 1}}\eta \right]^{\delta_i} \prod_{i=1}^{n} \exp \left( \frac{(V_i - 1)\beta - (V_i - 1 + X_i)/\eta}{\eta} \right) \]

Now, we proceed to the second step and apply the log function to the expression of the likelihood function:

\[ \log(L) = \log(\beta) \sum_{i=1}^{n} \delta_i + (\beta - 1) \sum_{i=1}^{n} \delta_i \log(V_i - 1 + X_i) \]

\[ - \beta \log(\eta) \sum_{i=1}^{n} \delta_i + \sum_{i=1}^{n} \left( \frac{(V_i - 1)/\eta}{\eta} \right)^\beta \]

(10)

A. Parameter estimation using Kijima type I model:

Let \( V_i \) represent Kijima Type I model

\[ V_i = V_{i-1} + a X_i \]

(11)

With \( a \) denotes the efficiency factor of imperfect maintenance. When we have two types of maintenance, corrective and preventive, the expression of the virtual age of Kijima type I model becomes:

\[ V_i = \begin{cases} V_{i-1} + a_r X_i & \text{If } CM \\
V_{i-1} + a_p X_i & \text{If } PM \end{cases} \]

(12)

\( a_r \): efficiency factor of corrective maintenance. \( a_p \): efficiency factor of preventive maintenance.

In presence of two types of maintenance we can not use the classical expression of the virtual age model of Kijima type I, so we propose a new form to facilitate the estimation.

\[ V_i = \sum_{i=1}^{n} a_r^{\delta_i} a_p^{1-\delta_i} X_i \]

(13)

Now, we continue the steps of the estimation using maximum likelihood method. First we must calculate the derivatives of the log function with respect to the parameters of reliability and maintainability. We obtain a system of nonlinear equations consisting of four equations in four parameters. The resolution of this system is not explicit it requires the use of numerical methods. To maximize the likelihood function we solve the system of the score functions: The derivative of the Log(L) with respect to \( \beta \):

\[ \sum_{i=1}^{n} \left( (V_i - 1)/\eta \right)^\beta \log(V_i - 1)/\eta \sum_{i=1}^{n} \delta_i \log(V_i - 1 + X_i) - \frac{n}{\eta} \sum_{i=1}^{n} \delta_i = 0. \]

The derivative of the Log(L) with respect to \( \eta \):

\[ -\beta \eta^{-\beta-1} \sum_{i=1}^{n} [(V_i - 1)^\beta - (V_i - 1 + X_i)^\beta] - \frac{2}{\eta} \sum_{i=1}^{n} \delta_i = 0. \]

The derivative of the Log(L) with respect to \( a_r \):

\[ \left[ \beta \sum_{j=1}^{n} \delta_j a_r^{\delta_j} a_p^{1-\delta_j} X_j \right] \left( \sum_{j=1}^{n} a_r^{\delta_j} X_j + X_i \right)^{\beta-1} - \beta \sum_{j=1}^{n} \delta_j a_r^{\delta_j} a_p^{1-\delta_j} X_j \left( \sum_{j=1}^{n} a_r^{\delta_j} X_j + X_i \right)^{\beta-1} \]

\[ (\beta - 1) \sum_{j=1}^{n} \delta_j \sum_{j=1}^{n} a_r^{\delta_j} a_p^{1-\delta_j} X_j + X_i + \frac{n}{\eta^{\beta-1}} \sum_{i=1}^{n} \delta_i = 0. \]

The derivative of the Log(L) with respect to \( a_p \):

\[ \left[ \beta \sum_{j=1}^{n} (1 - \delta_j) a_r^{\delta_j} a_p^{1-\delta_j} X_j \right] \left( \sum_{j=1}^{n} a_r^{\delta_j} a_p^{1-\delta_j} X_j + X_i \right)^{\beta-1} - \beta \sum_{j=1}^{n} (1 - \delta_j) a_r^{\delta_j} a_p^{1-\delta_j} X_j \left( \sum_{j=1}^{n} a_r^{\delta_j} X_j + X_i \right)^{\beta-1} \]

\[ (\beta - 1) \sum_{j=1}^{n} (1 - \delta_j) a_r^{\delta_j} a_p^{1-\delta_j} X_j + X_i + \frac{n}{\eta^{\beta-1}} \sum_{i=1}^{n} \delta_i = 0. \]

Now, we will try to solve this system of equations to find the optimal parameters that maximizes the likelihood function. Based on the moments of failure and maintenance types of Table I, we proceed to estimate the parameters of the likelihood function. To find the optimum maintainability and reliability parameters values that maximize the likelihood function we need to introduce the initial values of \( (\beta, \eta) \) and \( (a_r, a_p) \) at the beginning of the program. We just need to take initial values that belong to the domain of convergence, \( \eta > 0, \beta > 0, 0 \leq a_r \leq 1 \) and \( 0 \leq a_p \leq 1 \). The system converges to the global maximum and gives as a result: \( a_r = 0.449, a_p = 0.732, \beta = 2.259 \) and \( \eta = 0.997 \). Note that the authors of [14] estimated these parameters using a Bayesian perspective and they had as a result \( a_r = 0.468 \) and \( a_p = 0.745, \beta = 2.339 \) and \( \eta = 1.031 \).

To verify the results numerically, we will try to represent the likelihood function according to the parameters of maintainability and reliability.
The figure 1 illustrates the log-likelihood function in 3D, it varies depending on the maintainability parameters $a_r$ and $a_p$. The optimum values of $a_r$ and $a_p$ that maximizes the log likelihood are checked by curves in figure 2 and 3, we have here the log-likelihood function is maximum for $\hat{a}_r = 0.44$ and $\hat{a}_p = 0.73$. We notice that these values verifies the results found previously.

The figure 4 represents the log-likelihood function in 3D, it varies depending on the reliability parameters $\eta$ and $\beta$. The curves in figure 5 and 6 are maximum respectively for $\hat{\eta} = 1$ and $\hat{\beta} = 2.21$, these results represent the optimum values of $\eta$ and $\beta$ that maximizes the log likelihood and also confirms the previous results found numerically.

### B. Parameter estimation using Kijima type II model

The second model proposed by [8] based on the same idea as the virtual age model Kijima type I. However the age reduction is not proportional to the time elapsed since the last maintenance, but the virtual age itself. The general expression of virtual age model in presence of corrective and preventive maintenance is given by:

$$V_i = \begin{cases} 
  a_r (V_{i-1} + X_i) & \text{If CM} \\
  a_r (V_{i-1} + X_i) & \text{If PM}
\end{cases} \quad (14)$$

The proposed new virtual age model Kijima type II is:

$$V_i = \sum_{j=1}^{i} a_r^{(i-j+1)\delta_i} a_p^{(i-j+1)(1-\delta_i)} X_j \quad (15)$$
In this section, we will proceed in the same way to estimate the parameters of reliability and maintainability by using the virtual age model Kijima type II. Then we keep the same log-likelihood function previously found and we update the expression of the virtual age \( V_t \) given by (15). As before we had a system of nonlinear equations with four parameters, but this time with the virtual age model Kijima type II. We apply to this system the same steps to achieve optimum values that maximize the log likelihood function, and we choose the initial values so that \( \eta > 0, \beta > 0, 0 \leq \alpha_r \leq 1 \) and \( 0 \leq \alpha_p \leq 1 \). So we got as a result: \( \hat{\alpha}_r = 0.86, \hat{\alpha}_p = 0.933, \hat{\beta} = 1 \) and \( \hat{\eta} = 3.55 \).

We will show below the transition system consisting of four equations to a system consisting of three equations with three unknowns \( \beta, \alpha_r \) and \( \alpha_p \). From the derived log function relative to \( \eta \) we extract the expression of the scale parameter \( \eta \) and we replace it in the function (10) as follows:

\[
\frac{d \log (L)}{d \eta} = 0.
\]

\[
-\frac{\beta}{\eta} \sum_{i=1}^{n} \delta_i - \beta \eta^{-\beta-1} \sum_{i=1}^{n} [(V_{i-1})^\beta - (V_{i-1} + X_i)^\beta] = 0.
\]

\[
\eta = \left( \frac{\sum_{i=1}^{n} [(V_{i-1})^\beta - (V_{i-1} + X_i)^\beta]}{\sum_{i=1}^{n} \delta_i} \right)^{\frac{1}{\beta}}. \tag{16}
\]

We obtain the following form of \( \log (L) \):

\[
\log (L) = \log(\beta) \sum_{i=1}^{n} \delta_i + (\beta - 1) \sum_{i=1}^{n} \delta_i \log(V_{i-1} + X_i)
\]

\[
- \log\left( \sum_{i=1}^{n} [(V_{i-1})^\beta - (V_{i-1} + X_i)^\beta] \right) \sum_{f=1}^{n} \delta_f
\]

\[
+ \log(\sum_{i=1}^{n} \delta_i) \sum_{f=1}^{n} \delta_f - \sum_{i=1}^{n} \delta_i. \tag{17}
\]

We apply to this system the same steps to achieve optimal values that maximize the log likelihood function. The system converges to the following values: \( \hat{\alpha}_r = 0.808 \) and \( \hat{\alpha}_p = 0.817 \). The proper functioning of the equipment is characterized from during their life cycle. The second way is to estimate the parameters in the case of bivariate distributions, which means that the proper functioning of the equipment is characterized from specific data using the method of maximum likelihood. The specific problems that we have treated, are largely inspired from situations encountered in industry which is required to maintain production tools in working condition by controlling time, cost and effectiveness of maintenance. We placed this work in the practical context in which it often happens that maintenance operations are carried out in an imperfect way. The aim of the work presented in this paper is to optimizing maintenance strategies, to predict the nature and timing of interventions and to keep the system in good condition. To solve this problem we relied on new form of the virtual age models of Kijima Type I and II to estimate the parameters of reliability and maintainability of the system simultaneously from specific data using the method of maximum likelihood. It would be very interesting to use these models in the development and the study of new policies of optimum product warranty taking into account the imperfect repair they suffer from during their life cycle. The second way is to estimate the parameters in the case of bivariate distributions, which means that the proper functioning of the equipment is characterized by two random variables which can be for example time and usage.

### IV. Conclusions

This paper is part of the overall problem concerning the maintenance of repairable systems subject to random failures.

### III. Results

The following table shows the results of parameters estimation using maximum likelihood estimation compared with results obtained by using the Bayesian method. We note from the results of the estimations using the virtual age of Kijima type I model, that the degree of repair of corrective maintenance is lower than that of preventive maintenance. Also we can conclude that the values obtained by our estimator are closer to the real values than those given in [14], note that the authors of [14] used a Bayesian perspective to estimate reliability and maintainability parameters and they had almost similar results. Concerning the results of the estimation using the virtual age of Kijima type II model, the values of \( \alpha_r \) and \( \alpha_p \) are higher than the values found with virtual age model Kijima Type I and they become closer to 1 means that we need a minimal degree of repair for the two types of maintenance.

When the system has three parameters, the value of \( \beta \) is equal to 1, i.e. a constant failure rate, so we are in the case of an exponential distribution like in [12] and the maintenance factors \( \alpha_r \) and \( \alpha_p \) are approaching to the minimal state like for the system depends on 4 parameters.

### IV. Conclusions

This paper is part of the overall problem concerning the maintenance of repairable systems subject to random failures. The specific problems that we have treated, are largely inspired from situations encountered in industry which is required to maintain production tools in working condition by controlling time, cost and effectiveness of maintenance. We placed this work in the practical context in which it often happens that maintenance operations are carried out in an imperfect way. The aim of the work presented in this paper is to optimizing maintenance strategies, to predict the nature and timing of interventions and to keep the system in good condition. To solve this problem we relied on new form of the virtual age models of Kijima Type I and II to estimate the parameters of reliability and maintainability of the system simultaneously from specific data using the method of maximum likelihood. It would be very interesting to use these models in the development and the study of new policies of optimum product warranty taking into account the imperfect repair they suffer from during their life cycle. The second way is to estimate the parameters in the case of bivariate distributions, which means that the proper functioning of the equipment is characterized by two random variables which can be for example time and usage.

### References


