Spatial shape variability analysis of T-waves in the ECG signal

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Abstract—This work aims to study the shape variability of the T-waves using multi channel records. The classification between a group of subjects with Myocardial Infarction (MI) and a group of healthy subjects is obtained using a synthetic signal called Integral Shape Averaging (ISA). Our methodology is based on calculation of projection coefficients corresponding to each electrode for both groups of subjects. These coefficients represent a distance between each subject and one of the groups. This latter allows us to compare each subject to each population. These coefficients are obtained by a surface difference between the signal of each electrode in each column and the associated ISA signal. In addition, we seek for the electrode that represents the best separation between the two populations. In this study we improve previous studies and make a comparison with another approach which is a combination between the ISA technique and the calculation of shape difference.

Keywords—Shape; T-wave; ISA; classification; the best electrode of separation

I. INTRODUCTION

In our modern world, the majority of causes of mortality come from heart disease. For this reason, doctors need powerful tools to establish their diagnosis. Among the possible cardiological examinations, we found the ElectroCardioGram (ECG) which is a recording of the electrical activity of the heart. The ECG contains several repetitive waves: The P-wave reflects the atrial depolarization, the QRS complex corresponds to the ventricular depolarization and the T-wave represents the ventricular repolarization.

In this work, we are interested only on the T-wave. In fact, the pathology studied, Myocardial Infarction (MI) affects only the morphology of this wave. The T-waves were recorded by a team in Warsaw with a system of 64 electrodes (Fig.1). The electrodes are disposed all around the thorax as a succession of columns. These data comes from 15 healthy subjects and 12 patients. In previous works [1,2] the two groups were separated using the shape variability of the T-wave in each column of electrodes, using a shape difference approach. The aim of this paper is to present the same work using a different approach and to compare it to the previous one. First, we study the difference between T-wave surfaces to classify normal subject and MI population. The T-wave surface is given by an approximation of area rectangle. Secondly to select in each column the electrode that represents the best separation between them.

Fig. 1. The system of 64 electrodes

II. METHODS

We propose to use a technique called Integral Shape Averaging (ISA) which is an averaging technique invariant by translation and scaling. This method was presented in [3] and [4]. Its main idea is as follows:

Assuming \( x_i(t) \) (i = 1 to N) are strictly positive signals, their normalized integral functions \( X_i(t) \) are strictly increasing from zero to one.

For each \( y, 0 < y < 1 \), there exists one and only one value such that:

\[ X_i(t_y) \]  \hspace{1cm} (1)

The main idea of ISA is to associate the ordinate \( y \) to the average \( \bar{T} \):

\[ \bar{T} = \sum_{i=1}^{N} t_i \]  \hspace{1cm} (2)

This average value is a function of \( y \); reciprocally, plotting \( y \) in function of \( \bar{T} \) leads to the mean normalized integral \( \bar{X} \).
The ISA signal is then given by:

$$\tilde{X}(t) = y$$  \hspace{1cm} (3)

The ISA signal is then given by:

$$\tilde{x}(t) = \frac{d\tilde{X}(t)}{dt}$$  \hspace{1cm} (4)

The shape of this signal can be considered as the average of the shapes of $x_i(t)$.

When all the $x_i(t)$ are the same shape, each one can be viewed as derived from the same signal $s(t)$

$$x_i(t) = k_i s(\alpha_i t - d_i)$$  \hspace{1cm} (5)

Here $k_i$, $\alpha_i$, and $d_i$ are respectively the magnitude coefficient, the scale coefficient and the delay. In this case, we will see that this shape is also the same shape of $s(t)$.

The normalized integral functions $X_i(t)$ and $S(t)$ of $x_i(t)$ and, respectively, are related by the following formula:

$$X_i = S \circ \varphi_i$$  \hspace{1cm} (6)

$$S = X_i \circ \psi_i$$  \hspace{1cm} (7)

Where:

$$\psi_i = \varphi_i^{-1}$$  \hspace{1cm} (8)

For each $y$, $0 < y < 1$, we can write:

$$y = S(t) = X_i(t) \text{ with } t_j = \psi_i(t)$$  \hspace{1cm} (9)

Then, to find the ISA for $i$ varying from 1 to N, we have to average the $\psi_i(t)$, giving:

$$\overline{\psi}(t) = \frac{1}{N} \sum_{i=1}^{N} \psi_i(t)$$  \hspace{1cm} (10)

Using the form (5), we have:

$$\psi_i(t) = \frac{t + d_i}{\alpha_i}$$  \hspace{1cm} (11)

Also, using the from (10) and (11), the average time $\overline{T}$ has the form

$$\overline{T} = \gamma t + \beta$$  \hspace{1cm} (12)

Where:

$$\gamma = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\alpha_i} \quad , \quad \beta = \frac{1}{N} \sum_{i=1}^{N} d_i$$  \hspace{1cm} (13)

Replacing $t$ in (12) by $S^{-1}(y)$, $\overline{T}$ becomes:

$$\overline{T} = \gamma S^{-1}(y) + \beta$$  \hspace{1cm} (14)

This gives:

$$y = S(\frac{\overline{T} - \beta}{\gamma}) = S(\theta \overline{T} - \tau) = S(\overline{\varphi}) = \tilde{X}(\overline{T})$$  \hspace{1cm} (15)

Where:

$$\tau = \frac{\beta}{\gamma} = \frac{1}{N} \sum_{i=1}^{N} d_i \quad , \quad \theta = \frac{1}{\gamma} \approx \frac{1}{N} \sum_{i=1}^{N} \alpha_i$$  \hspace{1cm} (16)

This shows that $\tilde{x}(t)$ derives from $s(t)$ by an affine function, i.e., shape is preserved by ISA.

To illustrate this technique, we present five signals that have the same shape and the signal averaging ISA respectively in figures (Fig.2) and (Fig.4) where it is noted that the ISA signal retains the shape of the five signals which is not the case with the classical signal averaging (Fig.3).
III. CLASSIFICATION APPROACH

To make the classification between the two populations, we propose to use the ISA technique combined with a calculation of surface difference. The method of this classification is as follows:

For each healthy subject and for each sick subject, we characterize each column with a signal averaging obtained by the technique ISA presented in section 2.

For each column and for each healthy subject, we define the following parameter:

\[ d_1(p,e) : \text{The difference between the surface of the electrode signal } e \text{ of a healthy subject } p \text{ and the surface of the ISA signal of the same column.} \]

Similarly, for each column and for each sick subject, we define the following parameter:

\[ d_2(p,e) : \text{The difference between the surface of the electrode signal } e \text{ of a sick subject } p \text{ and the surface of the ISA signal of the same column.} \]

From \( d_1(p,e) \) and \( d_2(p,e) \), we define two averages relative to the 15 healthy subjects and the 12 sick subjects:

\[ D_1(e) = \frac{1}{15} \sum_{p=1}^{15} d_1(p,e) \quad (17) \]
\[ D_2(e) = \frac{1}{12} \sum_{p=1}^{12} d_2(p,e) \quad (18) \]

For each healthy subject \( p \) and for each electrode \( e \), we define:

\[ \text{coefs}_1(p,e) = |d_1(p,e) - D_1(e)| \quad (19) \]
\[ \text{coefs}_2(p,e) = |d_2(p,e) - D_2(e)| \quad (20) \]

Where:

\( \text{coefs}_1 \), is given as the projection coefficient of a healthy subject on the data base of healthy subjects.

\( \text{coefs}_2 \), is given as the projection of a healthy subject coefficient on the data base of sick subjects.

Similarly, for each sick subject \( p \) and for each electrode \( e \), we define:

\[ \text{coefm}_1(p,e) = |d_2(p,e) - D_2(e)| \quad (21) \]
\[ \text{coefm}_2(p,e) = |d_1(p,e) - D_1(e)| \quad (22) \]

Where:

\( \text{coefm}_1 \), is the projection coefficient of a sick subject on the data base of sick subjects.

\( \text{coefm}_2 \), is the projection coefficient of a sick subject on the data base of healthy subjects.

From \( \text{coefs}_1(p,e) \) and \( \text{coefs}_2(p,e) \), we define two averages relative to 15 healthy subjects:

\[ \alpha s_1(e) = \frac{1}{15} \sum_{p=1}^{15} \text{coefs}_1(p,e) \quad (23) \]
\[ \alpha s_2(e) = \frac{1}{15} \sum_{p=1}^{15} \text{coefs}_2(p,e) \quad (24) \]

Also, from \( \text{coefm}_1(p,e) \) and \( \text{coefm}_2(p,e) \), we define two averages relative to 12 sick subjects:

\[ \alpha m_1(e) = \frac{1}{12} \sum_{p=1}^{12} \text{coefm}_1(p,e) \quad (25) \]
\[ \alpha m_2(e) = \frac{1}{12} \sum_{p=1}^{12} \text{coefm}_2(p,e) \quad (26) \]

For each column \( c \), we seek for the best electrode which classifies the two groups of subjects. Using the previous coefficients \( \text{coefs}_1(p,e) \), \( \text{coefs}_2(p,e) \) we calculate:

\[ r(p,e) = |\text{coefs}_1 - \text{coefs}_2| \quad (27) \]

And

\[ R(e) = \frac{1}{15} \sum_{p=1}^{15} r(p,e) \quad (28) \]

The selected electrode is given by:

\[ E_{\text{ref}} = \text{Max}(R(e)), \quad e \in \text{the same column} \quad (29) \]

To compare these results with another method, we propose to use the ISA method combined with a shape difference method. This method is obtained by the Distribution Function Method (MFR) [6].

IV. RESULTS

In this work, we aim to find for each column, \( \alpha s_1 \) bigger than \( \alpha s_2 \) and \( \alpha m_1 \) bigger than \( \alpha m_2 \). Globally this aim is achieved and good separation of the two populations is well observed on certain columns more than others and on certain
electrodes more than others. In figure (Fig.4), we represent the projection coefficients $\alpha s_1$, $\alpha s_2$, $\alpha m_1$ and $\alpha m_2$ of the column 5 using the surface approach. In figure (Fig.5), we represent the same coefficients using the second approach and the shape difference. We notice that using the difference shape approach, better classification is obtained. In fact, the surface method depends on the shape but it doesn’t exploit the whole information about shape as the difference in shape [2].

Using the parameters given in (27) and (28), we represent in figure (Fig.7), the curves of the selected electrodes in each column for the healthy subjects obtained on one hand by ISA technique and the surface difference, on the other hand by ISA technique and the shape difference.

According to the results shown in figure (Fig. 7), we observe that the electrodes found using the two methods are close to each other. We notice that these selected electrodes coincide with the standards electrodes. These electrodes are generally well chosen to make the diagnosis of the ECG.

I. CONCLUSION

In this work, we addressed two problems: The first is the classification between a group of subjects with Myocardial Infarction and a group of healthy subjects and the second is the choice of the electrode that represents the best separation between the two populations.

To make a comparison, this problem of classification is done on one hand by the application of the ISA technique and the calculation of surface difference, on the other hand by the application of the ISA technique and the calculation of the shape difference. For both methods separation between the two populations is achieved. We found that the best electrodes of separation between the two populations coincide with standard electrodes which are generally well chosen to make the diagnosis of ECG.

REFERENCES


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