Fuzzy c-regression models based on the BELS method for nonlinear system identification

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Abstract—A fuzzy c-regression model clustering algorithm based on Bias-Eliminated Least Squares method (BELS) is presented. This method is designed to develop an identification procedure for noisy nonlinear systems. The BELS method is used to identify consequent parameters and eliminate the bias. The proposed approach has been applied to benchmark modeling problem which proved a good performance.

Keywords—Takagi-Sugeno fuzzy model; fuzzy c-regression models; Bias-Eliminated Least-Squares.

I. INTRODUCTION

Fuzzy modeling algorithms are studied in many research areas. It is known as one as better describing nonlinear systems. Takagi and Sugeno (T-S) system is a most fuzzy model studied in literature [19]. In [6], the authors present a modified version of Fuzzy C-Means (FCM) clustering algorithm called fuzzy C-Regression Models (FCRM). The FCRM algorithm develops hyper-plane-shaped clusters, while FCM algorithm develops hyper-spherical-shaped clusters. However, in [11], the authors prove that the FCRM using the Weighted Regressive Least Square (WLRS) algorithm made it sensitive to initialization. Hence, different initializations may lead, easily, to different results. To overcome this problem, the gradient descent algorithm was used to adjust the parameters of fuzzy models [9]. Recently, the orthogonal least squares (OLS) is used to identify the consequent parameters [11- 12].

The affine T-S fuzzy model structure architecture is frequently applied to analysis and synthesis of nonlinear systems ([18], [8]). The (T-S) model has been widely studied without constant parameter ([20], [22]). In [25], the authors shown that the model is assumed as a quasi-linear model and the stability analysis ensured, if the scalar term is eliminated.

In this paper, a nonlinear system modeling with T-S fuzzy models without taking into account the constant term in consequent parameter is considered. The BELS method is used to identify the consequent parameters of local linear model. The BELS method has a robust capability against noise and is quite successful in improving the robustness of a variety of modeling problem. In other word, the BELS method is applicable to arbitrary colored disturbances acting on the plant ([14-15], [26]).

This paper is organized as follows. In Section 2, fuzzy c-regression models clustering algorithm is briefly introduced. In Section 3, the proposed method is given. Some example is given to illustrate the proposed approach. Finally, the conclusion is presented in Section 5.

II. FUZZY C-REGRESSION MODELS CLUSTERING ALGORITHM

The affine T-S fuzzy model based on FCRM belongs to the range of clustering algorithms with linear prototype. Let \( S^c = \{(x_1, y_1), \ldots, (x_N, y_N)\} \) be a set of input-output sample data pairs. Assume that the data pairs in \( S \) are drawn from \( c \) different fuzzy regression models, the hyper-plane-shaped cluster representative will be adopted has the following structure:

\[
y^c_k = f^c_i(x_k, \theta^c_i) + E^c_{ik}(\theta^c_i)
\]

where

\[
E^c_{ik}(\theta^c_i) = a_{i1}x_{k1} + a_{i2}x_{k2} + \ldots + a_{iM}x_{km} + b_{i0} + E^c_{ik}(\theta^c_i)
\]

(1)

The distances \( E^c_{ik}(\theta^c_i) \) are weighted with the membership values \( \mu_{ik} \) in the objective function that is minimized by the clustering algorithm and formulated as:

\[
J(U, \theta) = \sum_{k=1}^{N} \sum_{i=1}^{c} (\mu_{ik})^m (E^c_{ik}(\theta^c_i))^2
\]

(2)

with

\[
E^c_{ik}(\theta^c_i) = \|y^c_k - x^c_k \cdot \theta^c_i^T\|
\]

(3)

where \( m \) is the weighting exponent and \( \mu_{ik} \) is the membership degree of \( x_k \) to \( i^{th} \) cluster. The membership values \( \mu_{ik} \) have to satisfy the following conditions:

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\[ \mu_{ik} \in [0, 1]; \quad i = 1, 2, \ldots, c; \quad k = 1, 2, \ldots, N \]
\[ 0 < \sum_{k=1}^{N} \mu_{ik} < N; \quad i = 1, 2, \ldots, c \]
\[ \sum_{i=1}^{c} \mu_{ik} = 1; \quad k = 1, 2, \ldots, N \]

The identification procedure of the FCRM algorithm is summarized as follows [6].

Given data \( S \), set \( m > 1 \) and specify regression models Equation (1), choose a measure of error Equation (3). Pick a termination threshold \( \varepsilon \) and initialize \( U^{(0)} \) (e.g. random).

**Repeat** for \( l = 1, 2, \ldots \)

**Step 1.** Calculate values for \( c \) model parameters \( \theta^{(l)} \) in Equation (1) that globally minimize the restricted function Equation (2).

**Step 2.** Update \( U^{(l)} \) with \( E_{ik}(\theta^{(l)}) \), to satisfy:

\[ U_{ik}^{(l)} = \begin{cases} 
\frac{1}{\sum_{j=1}^{c} \left( \frac{E_{ik}}{E_{jk}} \right)^{m-1}} & \text{if } E_{ik} > 0 \text{ for } 1 \leq i \leq c \\
0 & \text{otherwise}
\end{cases} \]

Until \( \left\| U^{(l)} - U^{(l-1)} \right\| \leq \varepsilon \) then stop; otherwise set \( l = l+1 \) and return to step 1.

**III. THE NEW FCRM CLUSTERING ALGORITHM**

Many authors have shown that the clustering performance severely distorted when the noisy data is presented ([24], [13], [3], [10]). To overcome this problem, noise clustering (NC) algorithm is widely used ([11], [12]). In this approach, noise is considered as a separate class. It is represented by a fictitious prototype that has a constant distance \( \delta \) from all the data points. The membership \( \mu_{ik} \) of point \( x_k \) in the noise cluster is given by:

\[ \mu_k = 1 - \sum_{i=1}^{c} \mu_{ik} \]

Thus, the membership constraint for the good clusters is effectively relaxed to:

\[ \sum_{i=1}^{c} \mu_{ik} < 1 \]

Dave’s objective function is given by:

\[ J_{NC}(U, V) = \sum_{i=1}^{c} \sum_{k=1}^{N} \left( \mu_{ik} \right)^{m} \left( D_{ik} \right)^{2} + \sum_{k=1}^{N} \delta^{2} \left( \mu_{ik} \right)^{m} \]

for any input \( x_k \) in subspace \( i \) denoted by center \( v_i \), \( D_{ik} = \| x_k - v_i \| \).

The combination of noise clustering algorithm with FCRM algorithm can lead to New FCRM (NFCRM) objective function as follows [17]:

\[ J_{new}(S; U, \theta) = \sum_{i=1}^{N} \sum_{k=1}^{c} \left( \mu_{ik} \right)^{m} \left( E_{ik}(\theta) \right)^{2} \]

+ \sum_{k=1}^{N} \delta^{2} \left( \mu_{ik} \right)^{m} \]

Where \( \delta \) is a scale parameter and may be used on the idea presented in [4] as:

\[ \delta^{2} = \gamma \frac{1}{c \cdot N} \sum_{k=1}^{N} \sum_{i=1}^{c} \left( E_{ik}(\theta) \right)^{2} \]

where \( \gamma \) is a user-defined parameter depending on the example’s type.

To solve constrained problem \( J_{new} \) with respect to \( \mu_{ik} \), we introduce \( N \) Lagrange multipliers \( \lambda_{ik}, k = 1, \ldots, N \). The minimization of \( J_{new} \) is as follows:

\[ F(\mu_{ik}, \lambda_{ik}) = J_{new} - \sum_{i=1}^{N} \lambda_{ik} \left( \sum_{j=1}^{c} \mu_{jk} + \mu_{ik} - 1 \right) \]

By differentiating the Lagrangian with respect to the \( \mu_{ik} \), \( \mu_{ik}^{*} \) and \( \lambda_{ik} \) and setting the derivatives to zero, we obtain:

\[ \frac{\partial F}{\partial \mu_{ik}} = m \left( \mu_{ik}^{*} \right)^{m-1} - \lambda_{ik} = 0 \]

\[ \frac{\partial F}{\partial \mu_{ik}} = m \delta^{2} \left( \mu_{ik}^{*} \right)^{m-1} - \lambda_{ik} = 0 \]

\[ \frac{\partial F}{\partial \lambda_{ik}} = \sum_{j=1}^{c} \mu_{jk} + \mu_{ik} - 1 = 0 \]

From Equations (11-12), we get:

\[ \mu_{ik} = \left[ \frac{\lambda_{ik}}{m} \right]^{1/(m-1)} \left[ \frac{1}{E_{ik}^{2}} \right]^{1/(m-1)} \]

and

\[ \mu_{ik} = \left[ \frac{\lambda_{ik}}{m} \right]^{1/(m-1)} \left[ \frac{1}{\delta^{2}} \right]^{1/(m-1)} \]

Using Equations (13-15), we get:

\[ \left[ \frac{\lambda_{ik}}{m} \right]^{1/(m-1)} \left[ \frac{1}{\delta^{2}} \right]^{1/(m-1)} = \sum_{j=1}^{c} \left( \frac{1}{E_{jk}} \right)^{2/(m-1)} + \left( \frac{1}{\delta^{2}} \right)^{2/(m-1)} \]

And then substituting it into Equation (16), the following equation can be obtained:

\[ \mu_{ik} = \frac{1}{\sum_{j=1}^{c} \left( \frac{E_{ij}}{E_{jk}} \right)^{2/(m-1)} + \left( \frac{E_{ij}}{\delta^{2}} \right)^{2/(m-1)}} \]

From Equations (1-8), the objective function of NFCRM is defined as:

\[ J_{new} = \sum_{k=1}^{N} \sum_{i=1}^{c} \left( \mu_{ik}^{*} \right)^{m} \left( y_{k} - \sum_{j=1}^{M} \theta_{j}^{T} \hat{x}_{ij} \right)^{2} \]

+ \sum_{k=1}^{N} \delta^{2} \left( 1 - \sum_{i=1}^{c} \mu_{ik} \right)^{m} \]
where \( \hat{x}_k = [x_k, 1]. \)

The partial derivation of the object Equation (18) is:

\[
\frac{\partial J_{\text{new}}}{\partial \theta_j} = -2 \sum_{k=1}^{N} (\mu_{ik})^m \left( y_k - \sum_{i=1}^{M} \theta_{ij} \hat{x}_{ki} \right) \hat{x}_{kj} = 0
\]

and then,

\[
\theta_{ij} = \frac{\sum_{k=1}^{N} (\mu_{ik})^m (y_k - \sum_{i=1}^{M} \theta_{ij} \hat{x}_{ki}) \hat{x}_{kj}}{\sum_{k=1}^{N} (\mu_{ik})^m \hat{x}_{kj}^2} \quad (20)
\]

\( i = 1, 2, ..., c; \ j = 1, 2, ..., M + 1 \)

As mentioned in [23], the non-Euclidean distance is more robust than the Euclidean distance. By transforming Equation (3), the measure of error is defined as:

\[
E_{ik}(\theta) = 1 - \exp(-\rho |y_k - [x_k, 1] \cdot \theta^T|)
\]

where \( \rho \) is a positive constant. Then the NFCRM objective function Equation (8) is rewritten as:

\[
J_{\text{new}}(S; U, \theta) = \sum_{k=1}^{N} \left( \sum_{i=1}^{c} (\mu_{ik})^m (E_{ik}(\theta)) \right)^2 + \sum_{k=1}^{N} \delta^2 (1 - \sum_{i=1}^{c} (\mu_{ik})^m)
\]

Using Equations (9) and (17) can be respectively rewritten as:

\[
\mu_{ik} = \frac{c}{\sum_{j=1}^{c} \frac{E_{ik}^2}{E_{jk}} + \left( \frac{E_{ik}}{\delta} \right)^m - 1}
\]

and \( \delta^2 = \gamma \frac{1}{c \cdot N} \sum_{k=1}^{N} \sum_{i=1}^{c} (E_{ik}(\theta))^2 \)  

Using Equation (20) can be respectively rewritten as:

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\mu_{ik} = \frac{c}{\sum_{j=1}^{c} \frac{E_{ik}^2}{E_{jk}} + \left( \frac{E_{ik}}{\delta} \right)^m - 1}
\]

A. Estimation of consequent parameters by the BELS method

The novel fuzzy c-regression models for decomposition of the input-output space into multiple linear structures is used. Gaussian membership functions are usually chosen to represent the fuzzy sets in the premise part of each fuzzy rule. The premise parameters can be easily obtained using \( \mu_{ik}. \)

The partial derivation of the object Equation (18) is:

\[
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\]

and then,

\[
\theta_{ij} = \frac{\sum_{k=1}^{N} (\mu_{ik})^m (y_k - \sum_{i=1}^{M} \theta_{ij} \hat{x}_{ki}) \hat{x}_{kj}}{\sum_{k=1}^{N} (\mu_{ik})^m \hat{x}_{kj}^2} \quad (20)
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J_{\text{new}}(S; U, \theta) = \sum_{k=1}^{N} \left( \sum_{i=1}^{c} (\mu_{ik})^m (E_{ik}(\theta)) \right)^2 + \sum_{k=1}^{N} \delta^2 (1 - \sum_{i=1}^{c} (\mu_{ik})^m)
\]

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\[
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\]

and \( \delta^2 = \gamma \frac{1}{c \cdot N} \sum_{k=1}^{N} \sum_{i=1}^{c} (E_{ik}(\theta))^2 \)  

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\]

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The novel fuzzy c-regression models for decomposition of the input-output space into multiple linear structures is used. Gaussian membership functions are usually chosen to represent the fuzzy sets in the premise part of each fuzzy rule. The premise parameters can be easily obtained using \( \mu_{ik}. \)
By using the BELS method, the solution of Equation (27) is given by the following expression:

\[ \theta_{BELS}(N) = \theta_{LS}(N) - R_{opp}^{-1}DR_{yen}(N) \]  

(35)

with \( R_{yen} = E[y(k)a_0(k)] \): noise covariance vector.

Under condition which the noise estimates covariance vector \( R_{yen} \) is attainable in some manner.

The BELS algorithm is described as follows:

**step 1.** Initialize the parameters vector 

\[ \theta_{BELS} = 0 \]

**step 2.** From the sampled input-output data \( \{u(k), y(k), N \geq 1\} \) calculate the estimates of the covariance matrices and vectors:

\[ \hat{R}_{opp}(N) = \frac{1}{N} \sum_{k=1}^{N} \varphi(k)\varphi^T(k) \]

\[ \hat{R}_{y}(N) = \frac{1}{N} \sum_{k=1}^{N} \varphi(k)\varphi^T(k) \]

(36)

\[ \hat{R}_{rr}(N) = \frac{1}{N} \sum_{k=1}^{N} \varphi(k)y(k) \]

\[ \hat{R}_{e}(N) = \frac{1}{N} \sum_{k=1}^{N} \varphi(k)y(k) \]

**step 3.** Estimate the parameter vector \( \theta \) by LS method using Equation (30)

**step 4.** Compute the estimate of the noise covariance \( R_{yen} \)

\[ \hat{R}_{yen}(N) = \left( \hat{R}_{opp}(N)\hat{R}_{opp}^{-1}(N)D \right)^{-1} \]

\[ \left\{ \hat{R}_{opp}(N)\theta_{LS}(N) - \hat{R}_{r}(N) \right\} \]

(37)

**step 5.** Calculate the BELS estimate of the system parameter vector \( \theta \)

\[ \theta_{BELS}(N) = \theta_{LS}(N) - R_{opp}^{-1}DR_{yen}(N) \]  

(38)

**IV. EXPERIMENTAL RESULTS**

In this section, the performance algorithm is tested. Mean square error MSE was used as a performances index, which is defined as:

\[ MSE = \frac{1}{N} \sum_{k=1}^{N} (y_k - \hat{y}_k)^2 \]  

(39)

**A. Benchmark problem**

Considering a nonlinear system given by equation Equation (40) [1]

\[ y_{k+1} = \frac{y_k (y_{k-1} + 2) (y_k + 2.5)}{8.5 + y_k^2 + y_{k-1}^2} + u_k + v_k \]  

(40)

where \( y_k \) is system output, \( u_k \) is input which is uniformly distributed in the interval [-1 1] and \( v_k \) is white noise with zero mean and variance \( \sigma^2 \), which is added to the output system at different Signal-to-Noise Ratio (SNR) levels.

Choosing SNR as follow: SNR=1 dB, 5 dB, 10 dB, 15 dB and 20 dB.

The simulations results are given for two experimental cases. The training data set contains 500 samples for input-output pairs while for the testing 1000 data pairs are generated by the following input signal:

\[ u_k = \begin{cases} \sin \left( \frac{2k\pi}{250} \right) & \text{if } k \leq 500, \\ 0.3 \sin \left( \frac{2\pi k}{25} \right) + 0.1 \sin \left( \frac{2\pi k}{25} \right) & \text{otherwise}. \end{cases} \]  

(41)

Tables I–VI present results obtained with different algorithms such as: Gustafson-Kessel (GK) [5], NFCRM algorithm (NFCRMA) [11], FCM [7], fuzzy model identification (FMI) clustering algorithm [2] and NFCRM1 [16], et NFCRM2 [17].

Choosing \( \{y(k-1), y(k-2), u(k), u(k-1)\} \) as output-input variables, and the number of fuzzy rules is four. The parameter settings are: \( \gamma=0.1 \) and \( \rho=0.1 \) for NFCRM_BELS. MSE\(_{Tr}\) and MSE\(_{Ts}\) are the MSE for training and testing data, respectively.

In Case 1, Comparing results with those cited above with regard to the noisy data. Table 1 shows that the best MSE is obtained by our method during training and test phases. In addition, the consequent parameters are obtained without constant terms unlike to other approaches mentioned in these tables. This shows that our model is better than others techniques reported in literature taking into account the few number parameters for each rule.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>MSE(_{Tr})</th>
<th>MSE(_{Ts})</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCM</td>
<td>0.0090</td>
<td>0.2220</td>
</tr>
<tr>
<td>GK</td>
<td>0.0046</td>
<td>0.1347</td>
</tr>
<tr>
<td>FMI</td>
<td>0.0013</td>
<td>0.0181</td>
</tr>
<tr>
<td>NFCRMA</td>
<td>5.20 e-4</td>
<td>0.0096</td>
</tr>
<tr>
<td>NFCRM1</td>
<td>4.76 e-4</td>
<td>0.0052</td>
</tr>
<tr>
<td>NFCRM2</td>
<td>3.94e-4</td>
<td>0.0045</td>
</tr>
<tr>
<td>NFCRM_BELS</td>
<td>2.65e-4</td>
<td>0.0028</td>
</tr>
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</table>

In Case 2, the noise influence is analyzed with different SNR levels (SNR=1, 5, 10, 15 and 20dB). The parameter settings are: \( \gamma=0.1 \) and \( \rho=1 \) for the NFCRM_BELS algorithm. As shown in Tables II–VI, we can note that, whatever the noise level is, only the proposed algorithm retained good performance. Consequently, our algorithm is more resistant to noise compared with other existing algorithms in the literature.
### TABLE II.
COMPARISON RESULTS WITH SNR=20DB

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>$\text{MSE}_{tr}$</th>
<th>$\text{MSE}_{ts}$</th>
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</thead>
<tbody>
<tr>
<td>FCM</td>
<td>0.0285</td>
<td>0.2417</td>
</tr>
<tr>
<td>GK</td>
<td>0.0258</td>
<td>0.1867</td>
</tr>
<tr>
<td>FMI</td>
<td>0.0134</td>
<td>0.0802</td>
</tr>
<tr>
<td>NFCRMA</td>
<td>0.0133</td>
<td>0.0621</td>
</tr>
<tr>
<td>NFCRM1</td>
<td>0.0126</td>
<td>0.0473</td>
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<tr>
<td>NFCRM2</td>
<td>0.0116</td>
<td>0.0442</td>
</tr>
<tr>
<td>NFCRM_BELS</td>
<td>0.0093</td>
<td>0.0276</td>
</tr>
</tbody>
</table>

### TABLE III.
COMPARISON RESULTS WITH SNR=15DB

<table>
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<th>Algorithms</th>
<th>$\text{MSE}_{tr}$</th>
<th>$\text{MSE}_{ts}$</th>
</tr>
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<tbody>
<tr>
<td>FCM</td>
<td>0.0533</td>
<td>0.3164</td>
</tr>
<tr>
<td>GK</td>
<td>0.0471</td>
<td>0.2212</td>
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<tr>
<td>FMI</td>
<td>0.0373</td>
<td>0.1110</td>
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<tr>
<td>NFCRMA</td>
<td>0.0363</td>
<td>0.0806</td>
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<tr>
<td>NFCRM1</td>
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<td>NFCRM2</td>
<td>0.0342</td>
<td>0.0770</td>
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<tr>
<td>NFCRM_BELS</td>
<td>0.0256</td>
<td>0.0574</td>
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### TABLE IV.
COMPARISON RESULTS WITH SNR=10DB

<table>
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<th>$\text{MSE}_{ts}$</th>
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<tbody>
<tr>
<td>FCM</td>
<td>0.1155</td>
<td>0.5505</td>
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<td>GK</td>
<td>0.1100</td>
<td>0.3042</td>
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<tr>
<td>FMI</td>
<td>0.1068</td>
<td>0.2136</td>
</tr>
<tr>
<td>NFCRMA</td>
<td>0.1039</td>
<td>0.1926</td>
</tr>
<tr>
<td>NFCRM1</td>
<td>0.0963</td>
<td>0.1836</td>
</tr>
<tr>
<td>NFCRM2</td>
<td>0.0947</td>
<td>0.1762</td>
</tr>
<tr>
<td>NFCRM_BELS</td>
<td>0.0688</td>
<td>0.1130</td>
</tr>
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### TABLE V.
COMPARISON RESULTS WITH SNR=5DB

<table>
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<th>$\text{MSE}_{tr}$</th>
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<td>GK</td>
<td>0.3364</td>
<td>0.8094</td>
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<tr>
<td>FMI</td>
<td>0.3356</td>
<td>0.4859</td>
</tr>
<tr>
<td>NFCRMA</td>
<td>0.3309</td>
<td>0.4465</td>
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<tr>
<td>NFCRM1</td>
<td>0.3158</td>
<td>0.4276</td>
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<tr>
<td>NFCRM2</td>
<td>0.3094</td>
<td>0.4261</td>
</tr>
<tr>
<td>NFCRM_BELS</td>
<td>0.2605</td>
<td>0.3273</td>
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### TABLE VI.
COMPARISON RESULTS WITH SNR=1DB

<table>
<thead>
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<th>Algorithms</th>
<th>$\text{MSE}_{tr}$</th>
<th>$\text{MSE}_{ts}$</th>
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<tbody>
<tr>
<td>FCM</td>
<td>2.1292</td>
<td>2.1640</td>
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<td>GK</td>
<td>1.0079</td>
<td>1.3765</td>
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<td>FMI</td>
<td>0.9171</td>
<td>1.1649</td>
</tr>
<tr>
<td>NFCRMA</td>
<td>0.8904</td>
<td>1.1395</td>
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<tr>
<td>NFCRM1</td>
<td>0.8505</td>
<td>0.9491</td>
</tr>
<tr>
<td>NFCRM2</td>
<td>0.8092</td>
<td>0.9194</td>
</tr>
<tr>
<td>NFCRM_BELS</td>
<td>0.8041</td>
<td>0.8934</td>
</tr>
</tbody>
</table>

### V. CONCLUSION
In this paper, a new fuzzy c-regression model clustering algorithm based on the BELS (NFCRM_BELS) method has been presented. The application of BELS method improves the robustness of the FCRM method in noisy data. The proposed fuzzy modeling approach is applied to benchmark modeling problem. The experimental results show that the NFCRM-BELS algorithm has better performance than other methods in terms of the accuracy and the compactness.

### REFERENCES


