New Allied Fuzzy C-Means algorithm for Takagi-Sugeno Fuzzy model Identification

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Abstract— Takagi-Sugeno (TS) fuzzy model have received particular attention in the area of nonlinear identification due to their potentialities to approximate any nonlinear behavior [1]. In literature, several fuzzy clustering algorithms have been proposed to identify the parameters involved in the Takagi-Sugeno fuzzy model, as the Fuzzy C-Means algorithm (FCM) and the Allied Fuzzy C-Means algorithm (AFCM). This paper presents the New Allied Fuzzy C-Means algorithm (NAFCM) extension of the AFCM algorithm. Then an optimization method using the Particle Swarm Optimization method (PSO) combined with the NAFCM algorithm is presented in this paper (NAFCM-PSO algorithm). The simulation’s results on a nonlinear system shows that the New Allied Fuzzy C-Means algorithm combined with the PSO algorithm gives results more effective and robust than the Allied Fuzzy C-Means algorithm.

Keywords— nonlinear system, TS fuzzy model, Fuzzy identification, fuzzy clustering, non-Euclidean distance, Particle Swarm Optimization

I. INTRODUCTION

Fuzzy model identification is an effective tool for the approximation of uncertain nonlinear systems on the basis of measured data [2]. Among the different fuzzy modeling techniques, Takagi-Sugeno (TS) fuzzy model drawn the attention of several research, this is to their effectiveness in the nonlinear system modeling [1]. In this context, the fuzzy clustering technique [2] [3] constitute one of the best approaches used for the representation of such process. Indeed, this technique is to approximate the nonlinear system overall by Takagi-Sugeno local linear models, in this case, each model represents by a fuzzy rule [4]. The number of rules (clusters) is fixed by an expert according to the type of application considered and the performances required by this last. Several clustering algorithms exist in literature allowing the identification of the parameters intervening in the TS fuzzy model. We can quote as an example the Fuzzy C-Mean algorithm (FCM) [4]. However, FCM is sensitive to noises. To resist the noises some fuzzy clustering algorithms have been proposed. A novel fuzzy clustering model, called allied fuzzy c-means (AFCM) clustering [5], has been proposed by Wu and Zhou to deal with noisy data. AFCM can produce memberships and possibilities simultaneously and it overcomes the noise sensitivity shortcoming of FCM. However, both FCM and AFCM are all based on Euclidean distance in their objective functions. In real world, the Euclidean distance is not complex enough to deal with more sophisticated problems. Wu and Yang have proposed a non-Euclidean distance to replace the Euclidean distance [6] in FCM. Inspired by Wu and Yang’s algorithm, we introduce the new distance into AFCM to replace the Euclidean distance in it and propose a new fuzzy clustering algorithm called New Allied Fuzzy C-Means algorithm (NAFCM) but the application is limited because of their convergence to local optima and their sensitivity to initialization (random choice of number of clusters). To remedy this problem, a combination of the New Allied Fuzzy C-Means algorithm (NAFCM) and the PSO algorithm, is used. The effectiveness of this algorithm (NAFCM-PSO) compared to the AFCM algorithm is tested on a noisy nonlinear system.

This paper is organized as follows:

The second part of this work is devoted to formulating a Takagi-Sugeno fuzzy model and identifying the premise parameters of this model using the new Allied Fuzzy C-Means algorithm (NAFCM), and identifying the consequent parameters by RWLS. The third part is dedicated to presenting the particle swarm optimization and the NAFCM-PSO algorithm. The forth part is dedicated to presenting the results of simulation and model validity of AFCM, NAFCM, NAFCM-PSO algorithms. Finally, we conclude this paper with a conclusion.

II. TAKAGI-SUGENO FUZZY MODEL

Takagi-Sugeno fuzzy model (TS) is one of the best approaches for modeling and identifying a nonlinear system, defined by the recurrent equation \( y(k) = g_{nl}(x_k) \).

TS model is constructed by a rule-based type If ... Then in which the consequent uses numeric variables rather than linguistic variables (case of Mamdani).

In general, a Takagi-Sugeno fuzzy model is based on rules of the form:
The "if" rule function defines the premise part, while the "then" rule function constitutes the consequent part of the TS fuzzy model. Where $i \in [1, \ldots, C]$,

$$R_i : if \ x_1, \ldots, x_n \ is A_{i_1}, \ldots, A_{i_n} \ then \ y_i = a_i^T x_i + b_i,$$  \hspace{1cm} (1)

The estimated output of the nonlinear model is given by the following equation:

$$\hat{y} = \sum_{i=1}^{C} \beta_i(k) y_i = \sum_{i=1}^{C} \beta_i(k) \left[a_i^T x_i + b_i \right]$$  \hspace{1cm} (2)

As

$$\beta_i(k) = \frac{\prod_{j=1}^{n} \mu_j(x_j)}{\sum_{i=1}^{C} \prod_{j=1}^{n} \mu_j(x_j)}$$  \hspace{1cm} (3)

$\mu_i$ : Membership functions.

The estimated output of the Takagi-Sugeno fuzzy model can be expressed by:

$$\hat{y} = \sum_{i=1}^{C} \beta_i[k] \left[a_i^T x_i + b_i \right]$$  \hspace{1cm} (4)

### III. IDENTIFICATION ALGORITHM FOR PREMISE PARAMETERS

To identify the premise parameters of a Takagi-Sugeno fuzzy model described by equation (1), the Allied fuzzy C-Means algorithm (AFCM) is used.

#### A. Allied Fuzzy C-Means algorithm (AFCM)

The Allied Fuzzy C-Means algorithm (AFCM), which uses Euclidean distance, finds the partition of the collection $X = \{x_1, \ldots, x_N\} \subseteq \mathbb{R}^P$ of $N$ measures, specified by $k$-dimensional vectors $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T$, into $C$ fuzzy subsets by minimizing the following objective function [5]:

$$J_{AFCM}(U,t,V) = \sum_{i=1}^{N} \sum_{k=1}^{C} \left(a_k \mu_i^m + b_k \right) D_{ik}^2 + \sum_{i=1}^{N} \sum_{k=1}^{C} \eta_i \log(t_k) - t_k$$  \hspace{1cm} (5)

Where

$1 \leq C \leq N$ : The number of clusters,

$\mu_{ik}$ : The membership of $x_k$ in cluster $i$ satisfying

$$0 < \mu_{ik} < 1 \quad 1 \leq i \leq C, \quad 1 \leq k \leq N$$  \hspace{1cm} (6)

$$\sum_{i=1}^{C} \mu_{ik} = 1 \quad 1 \leq k \leq N$$  \hspace{1cm} (7)

$$0 < \sum_{i=1}^{N} \mu_{ik} < N \quad 1 \leq i \leq C$$  \hspace{1cm} (8)

$t_{ik}$ : The typicality of $x_k$ in class $i$.

$$D_{ik}^2 = \|x_k - v_i\|^2$$  \hspace{1cm} (9)

$V$: the set of cluster centers $(v_i \subseteq \mathbb{R}^P)$.

$\eta_i$ : The suitable positive numbers choosing by Krishnapuram and Keller such as:

$$\eta_i = K \frac{\sum_{k=1}^{N} \mu_{ik}^m D_{ik}^2}{\sum_{k=1}^{N} (\mu_{ik}^m)^m}, \ K > 0$$  \hspace{1cm} (10)

Typically, $K$ is chosen to be 1.

$\mu_{a,rcm}$ is the terminal membership values of FCM.

$m$ represent the degree of weighting, this parameter directly influences the form clusters in data space.

To minimize equation (5), we take its partial derivative of variables, $\mu_{ik}, t_{ik}$ and $v_i$, equal to zero and obtain the following equations:

$$\mu_{ik} = \left[\sum_{i=1}^{C} \left(\frac{D_{ik}^2}{D_{ij}^2}\right)^{\frac{2}{m-1}}\right]^{-1}$$  \hspace{1cm} (11)

$$t_{ik} = \exp \left(-\frac{b D_{ik}^2}{\eta_i}\right), \forall i, k$$  \hspace{1cm} (12)

$$v_i = \frac{\sum_{k=1}^{N} (a_k \mu_{ik}^m + b_k) x_k}{\sum_{k=1}^{N} (a_k \mu_{ik}^m + b_k)}, \forall i$$  \hspace{1cm} (13)

#### AFCM algorithm

Given a set of observations $X = \{x_1, \ldots, x_N\}$, the AFCM algorithm is described by the following steps:

**Initialize** $l = 0$

Set the number of clusters $C, 1 \leq C \leq N$

Set the level of weighting $m : 2 < m \leq 4$

Set the stopping criterion $\epsilon, \ \epsilon > 0$

Run the FCM algorithm and use equation (10) to get $\eta_i$

**Repeat** $l = l + 1$

**Step 1**

Update the cluster centers by equation (13).
Step 2
Update the membership matrix \( U = [\mu_{ik}] \) by equation (11)

Step 3
Update the typicality matrix \( T = [t_{ik}] \) by equation (12)

If \( \|U - U^{t-1}\| < \epsilon \), return to step 1, if not stop.

B. New Allied Fuzzy C-Means algorithm (NAFCM)

The allied c-means algorithm uses the Euclidean distance to calculate the fuzzy membership \( \mu_{ik} \) by equation (11) and the typicality \( t_{ik} \) by equation (12). However, in real world, the Euclidean distance is not complex enough to deal with more sophisticated problem. In this paper, we use a non-Euclidean distance to replace the Euclidean distance used in AFCM. The new distance is more robust than the Euclidean distance used in AFCM. The new distance is defined as [6]:

\[
D_{A,ik} = \sqrt{1 - \exp(-\rho \|x_i - v_j\|^2)}
\]

(14)

Here \( \rho \) is a positive constant.

Considering equation (14), the objective function (5) is transformed into as follows

\[
J_{NAFCM}(U,T,V) = \sum_{i=1}^{C} \sum_{k=1}^{N} \left( a\mu_{ik}^m + bt_{ik} \right) \left( 1 - \exp(-\rho \|x_i - v_j\|^2) \right)
\]

\[+\sum_{i=1}^{C} \sum_{k=1}^{N} \eta \sum_{k=1}^{N} (t_{ik} \log(t_{ik}) - t_{ik}) \]

(15)

The minimization of criterion (15) using the centers of the classes is obtained directly by canceling the gradient \( J_{NAFCM} \) with respect to different centers:

\[
\frac{\partial J_{NAFCM}(U,T,V)}{\partial v_j} = \sum_{i=1}^{C} \sum_{k=1}^{N} [2\rho(a\mu_{ik}^m + bt_{ik}) \exp(-\rho \|x_i - v_j\|^2)](x_i - v_j) = 0
\]

(16)

\[
\sum_{i=1}^{C} \sum_{k=1}^{N} [2\rho(a\mu_{ik}^m + bt_{ik}) \exp(-\rho \|x_i - v_j\|^2)] x_i
\]

\[= \sum_{i=1}^{C} \sum_{k=1}^{N} [2\rho(a\mu_{ik}^m + bt_{ik}) \exp(-\rho \|x_i - v_j\|^2)] v_j
\]

(17)

Yields:

\[
v_j = \frac{\sum_{i=1}^{C} (a\mu_{ik}^m + bt_{ik}) \exp(-\rho \|x_i - v_j\|^2) x_i}{\sum_{i=1}^{C} (a\mu_{ik}^m + bt_{ik}) \exp(-\rho \|x_i - v_j\|^2)}
\]

(18)

With \( 1 \leq i \leq C, 1 \leq k \leq N \)

Using the Lagrange multiplier, the relationship of updating fuzzy coefficients \( \mu_{ik} \) is obtained by minimizing the following criterion:

\[
J_{NAFCM}(U,T,V) = \sum_{i=1}^{C} \sum_{k=1}^{N} (a\mu_{ik}^m + bt_{ik}) D_{A,ik}^2 + \sum_{i=1}^{C} \sum_{k=1}^{N} \eta \sum_{k=1}^{N} (t_{ik} \log(t_{ik}) - t_{ik}) - \lambda \]

\[= -\sum_{k \neq i} \left( \lambda \sum_{j=1}^{C} \left( \mu_{jk} - 1 \right) \right) \]

(19)

As the columns of the partition matrix \( U \) are independent, we can reduce the problem of minimization at each observation (column).

\[
\frac{\partial J_{NAFCM}(U,T,V)}{\partial \lambda} = -\left( \sum_{i=1}^{C} \mu_{ik} - 1 \right) = 0
\]

(20)

\[
\frac{\partial J_{NAFCM}(U,T,V)}{\partial \mu_{ik}} = a m(\mu_{ik})^{n-1} D_{A,ik}^2 - \lambda = 0
\]

(21)

From the expression (21), we can write \( \mu_{ik} \) this form:

\[
\mu_{ik} = \left( \frac{\lambda}{a m D_{A,ik}^2} \right) \frac{1}{m-1}
\]

(22)

Substituting expression (22) in expression (20):

\[
\frac{\partial J_{NAFCM}(U,T,V)}{\partial \mu_{ik}} = \sum_{i=1}^{C} \mu_{ik} = \sum_{i=1}^{C} \left( \frac{\lambda}{a m D_{A,ik}^2} \right) \frac{1}{m-1} = 1
\]

(23)

The two expressions (22) and (23) give the following expression:

\[
\mu_{ik} = \left( \frac{1}{D_{A,ik}^2} \right) \frac{1}{m-1}
\]

(24)

Hence the relationship of updating the partition matrix:

\[
\mu_{ik} = \left( \sum_{i=1}^{C} \left( \frac{1}{D_{A,ik}^2} \right) \frac{1}{m-1} \right)
\]

(25)

\[
\mu_{ik} = \left[ \sum_{i=1}^{C} \left( \frac{1}{D_{A,ik}^2} \right) \frac{1}{m-1} \right]^{-1}
\]

(26)
With \( D_{\Delta,ik} = \sqrt{1 - \exp(-\rho \|x_k - v_i\|^2)} \)

Therefore

\[
\mu_{ik} = \left[ \sum_{j=1}^{C} \frac{1 - \exp(-\rho \|x_k - v_j\|^2)}{1 - \exp(-\rho \|x_k - v_i\|^2)} \right]^{\frac{1}{m-1}}
\]

(27)

The relationship of updating the typicality \( t_{ik} \) is obtained by minimizing the following criterion:

\[
\frac{\partial J_{NAFCM}(\mu_{ik}, \eta)}{\partial t_{ik}} = b D_{\Delta,ik}^2 + \eta \log t_{ik} = 0
\]

(28)

\[
t_{ik} = \exp \left( -\frac{b D_{\Delta,ik}^2}{\eta} \right), \forall i, k
\]

(29)

Therefore

\[
t_{ik} = \exp \left( -\frac{b \left(1 - \exp(-\rho \|x_k - v_i\|^2)\right)}{\eta} \right), \forall i, k
\]

(30)

Similarly the equation (10) is rewritten by

\[
\eta_i = K \frac{\sum_{k=1}^{N} \mu_{ik}^m \left(1 - \exp(-\rho \|x_k - v_i\|^2)\right)}{\sum_{k=1}^{N} \left(\mu_{ik}^m \right)}
\]

(31)

In the following, we present the steps of the NAFCM algorithm to determine the centers of clusters \( V = \{v_1, v_2, ..., v_i, ..., v_c\} \), the typicality matrix \( T = [t_{ik}] \) and the fuzzy partition matrix \( U = [\mu_{ik}] \).

**NAFCM algorithm**

Given a set of observations \( X = \{x_1, ..., x_N\} \), NAFCM algorithm is described by the following steps:

**Initialize** \( l = 0 \)

Set the number of clusters \( C, 1 < C < N \)
Set the level of weighting \( m : 2 < m < 4 \)
Run the FCM algorithm and use equation (31) to get \( \eta_i \)
Set the stopping criterion \( \varepsilon, \varepsilon > 0 \)

**Repeat** \( l = l+1 \)

**Step 1**
Update the cluster centers by equation (18).

**Step 2**
Update the membership matrix \( U = [\mu_{ik}] \) by equation (27)

**Step 3**
Update the typicality matrix \( T = [t_{ik}] \) by equation (30)

If \( \|U^l - U^{l-1}\| < \varepsilon \), return to step 1, if not stop.

**C. New Allied Fuzzy C-Means algorithm based on PSO (NAFCM-PSO):**

**PSO**

It is a population-based searching algorithm and is initialized with a population of random solutions, called particles. Each particle flies through the searching space with a velocity that is dynamically adjusted. These dynamical adjustments are based on the historical behaviors of itself and other particles in the population [7][9].

The PSO concept consists of constantly changing velocity (accelerating) each particle in the search space to move and his \( P_{best} \). Acceleration is weighted by random terms, with separate random numbers, which are generated for acceleration toward \( P_{best} \) and respectively . The change of speed (acceleration) and the position of each particle in the search space is iteratively [8]:

\[
v_k^{(l+1)} = K \left[ w v_k^{(l)} + \rho p_{k,best}^{(l)} - x_k^{(l)} \right] + \rho \left[ p_{k,best}^{(l)} - x_k^{(l)} \right]
\]

(32)

\[
x_k^{(l+1)} = x_k^{(l)} + v_k^{(l+1)}
\]

(33)

Where

\( k = 1, ..., N \) : size of particles.
\( D \) : represents the size of the search space.
\( v_k^{(l)} = (v_{k1}, ..., v_{kd}) \) : represents the speed of \( k^{th} \) particle.
\( x_k^{(l)} = (x_{k1}, ..., x_{kd}) \) : represents the best previous position of \( k^{th} \) particle.
\( g \) : Index represents the best particle among all particles in the group.
\( K \) : The constriction factor described by the following relationship:

\[
K = \frac{2}{\sqrt{2} - \sqrt{\phi^2 - 4\phi}}
\]

(34)

Where \( \phi = c_1 + c_2 > 4 \)

\( c_1 \) and \( c_2 \) are two positive constants satisfying the following relationship:

\[
\phi = c_1 + c_2 > 4
\]

(35)

\( \rho_1 \) and \( \rho_2 \) represents two random variables defined as follows:

\[
\begin{align*}
\rho_1 &= r_1 \times c_1 \\
\rho_2 &= r_2 \times c_2
\end{align*}
\]

(36)

\( r_1 \) and \( r_2 \) are two random variables between 0 and 1.

\( w \) : It is the weight of inertia. This factor sets the export capacity of each particle to improve the convergence of the method. \( w \), often, decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weight \( w \) is set according to the following equation:
\[ w = w_{\text{max}} - \frac{w_{\text{max}} - w_{\text{min}}}{\text{iter}_{\text{max}}} \times \text{iter} \]  

(37)

Where \( w_{\text{max}} \) and \( w_{\text{min}} \) are the initial weight and final weight, respectively, \( \text{iter}_{\text{max}} \) is the maximum number of allowable iterations, and \( \text{iter} \) is the current iteration number.

**Fitness function**

The optimal position is measured with said fitness function defines the following optimization problem. This according to fitness following:

\[ f(x_k) = \frac{G}{J_{\text{NAFCM}}(U, T, V)} \]  

(38)

Where

\( G \): It is a positive constant.

\( J_{\text{NAFCM}}(U, T, V) \): represents the objective function of the NAFCM algorithm.

**NAFCM-PSO algorithm**

It is a clustering algorithm that identifies the parameters of Takagi-Sugeno fuzzy model.

Given a set of \( X = \{x_1, x_2, ..., x_N\} \), the NAFCM-PSO algorithm is described by the following steps:

**Initialization l = 0**

Set the number of clusters \( C \): \( 1 < C < N \)

Set the fuzzy degree \( m \): \( 2 < m < 4 \)

Fix the population size \( N \)

Set \( c_1, c_2, r_1 \), and \( r_2 \)

Set the weight of inertia: \( w_{\text{max}}, w_{\text{min}} \)

Set the stopping criterion \( \varepsilon \)

Set the size of the search space: \( D \)

Initialize \( U = [\mu_{1k}] \), \( T = [I_{1k}] \) and cluster centers \( V \) by the FCM algorithm use equation (31) to get \( \eta \)

Initialize the 1\(^{st}\) particle generation.

Initialize the position and velocity of each particle.

Initialize the fitness function \( f(x_k) \)

**Repeat l = l + 1**

**Step 1**

Update the cluster centers by equation (18).

**Step 2**

Update the fuzzy partition matrix \( U = [\mu_{1k}] \) by equation (27).

**Step 3**

Update the typicality matrix \( T = [I_{1k}] \) by equation (30)

**Step 4**

Calculate the new value of fitness for each particle using equation (38).

**Step 5**

Compare the fitness of each particle with \( p_{\text{best}} \), if the value is better than \( p_{\text{best}} \) and then set the \( p_{\text{best}} \) value.

**Step 6**

Compare the fitness value of \( g_{\text{best}} \) with: if the value is better than \( g_{\text{best}} \), \( g_{\text{best}} \) then set equal to this value.

**Step 7**

Update position and velocity of each particle by the equations (32) and (33).

So: get the stability of the fuzzy partition, that is to say

\[ |v_k^{(l+1)} - v_k^{(l)}| < \varepsilon \]

If this condition is satisfied, stop iteration and find the best solution in the last generation. If not, go back to step 1.

**IV. IDENTIFICATION FOR CONSEQUENT PARAMETERS**

The identification of consequent parameters \( \theta_i = [a_i^T, b_i] \) of the Takagi-Sugeno model described by equation (1), we must build the matrix of regression \( X \) and the output vector \( Y \) following certain measures which are defined as follows:

\[ X = \begin{bmatrix} x_{1,1}^r, x_{1,2}^r, \ldots, x_{1,N}^r \end{bmatrix} \]

\[ Y = \begin{bmatrix} y_1, y_2, \ldots, y_N \end{bmatrix} \]

Since the defuzzification method, used in the Takagi-Sugeno model, is linear with the consistent parameters \( a_i \) and \( b_i \), (1), we can use the RWLS algorithm for estimating these parameters.

Either \( \theta_i = [a_i^r, b_i]^T \) represents the vectors of parameters of rule \( i \), and the matrix \( X_e = [X; 1] \) represents an extension of \( X \) . Using the RWLS algorithm using the following recursive procedure:

**Step 1:** initialization of the algorithm

\[ \theta_i(0) = [a_i^T, b_i] - 0 \cdot P(0) \cdot \sigma I \]

Where \( P \) : is the gain matrix value.

**Step 2:** Compute \( G(k) \) at each simple time

\[ G(k) = \frac{1}{\mu_a(k)} + x_e^r(k)P(k-1)x_e(k) \]  

(39)

Where \( x_e^r(k) \) is the observation matrix, and \( 0 < \mu_a(k) < 1 \) is a forgetting factor

**Step 3:** Update the gain matrix

\[ P(k) = \frac{1}{\lambda} \left[ P(k-1) - \frac{P(k-1)x_e^r(k)x_e(k)P(k-1)}{\mu_a(k)} + x_e^r(k)P(k-1)x_e(k) \right] \]  

(40)

**Step 4:** Update the consequent parameter

\[ \dot{\theta}(k) = \dot{\theta}(k-1) + G(k)\varepsilon(k)y(k) \]  

(41)

Where \( \varepsilon(k) = y(k) - x_e^\theta \theta(k-1) \)
V. SIMULATION RESULTS AND VALIDATION MODEL

A. Simulation Results
Consider a nonlinear system described by the following difference equation [10]:

\[
y(k) = \frac{y(k-1) (y(k-2)+2) (y(k-1)+2.5)}{8.5 + y^3(k-1) + y^2(k-2)} + u(k) + e(k)
\]  

(43)

Where 
\( y(k), u(k) \) are the output and the input of the system respectively. 
\( e(k) \) is a noise.

In this part, we have applied the three algorithms, AFCM, NAFCM, NAFCM-PSO and the RWLS method to identify the parameters of fuzzy models which approximate the nonlinear model (43).

The shape of the excitation signal used for identification is illustrated in Figure 1

For another input-output sequence, the figures represent the simulation results for identification by the three algorithms (AFCM, NAFCM and NAFCM-PSO).

B. Model Validation
Once the identification algorithm is applied, a step of validating the Takagi-Sugeno fuzzy model is needed. Several validation tests of the model are presented. Among them, we use the Root Mean Square Error (RMSE) test and the Variance Accounting For (VAF) test.

- Root Mean Square Error (RMSE)

This test (RMSE) calculate the mean squared error between the estimated output and real output.

\[
RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (y(k) - y_{est}(k))^2}
\]  

(44)

Where 
\( y \) is the real output 
\( y_{est} \) is the estimated output

If the model output and real output are combined, the RMSE equal zero.
Variance Accounting For (VAF)

The performance of the fuzzy model is measured by the VAF which calculates the percentage deviation of the variance between the real output and model output. It is defined by:

\[
VAF = 100\% \left[ 1 - \frac{\text{var}(y - y_{est})}{\text{var}(y)} \right]
\]

The test (VAF) tends in 100% when the \( y \) and \( y_{est} \) are combined.

### TABLE I. VALID RESULTS

<table>
<thead>
<tr>
<th>ALGORITHM</th>
<th>AFCM</th>
<th>NAFCM</th>
<th>NAFCM-PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (10^6)</td>
<td>2.156</td>
<td>0.708</td>
<td>0.586</td>
</tr>
<tr>
<td>VAF (%)</td>
<td>99.8984</td>
<td>99.9857</td>
<td>99.9997</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, we proposed another clustering algorithm (NAFCM) for the identification of nonlinear stochastic systems. This Algorithm is an extension of the AFCM algorithm proposed by Wu and Zhou 2006 where one replaced the Euclidean distance by another non-euclidean distance. The proposed algorithm overcomes the problems of sensitivity to noise and aberrant points; however, it is sensitive to initialization and is easily trapped in local optima. The particle swarm optimization method combined with the NAFCM algorithm can solve these problems.

The validation results RMSE and VAF show a better behavior of the NAFCM algorithm compared to AFCM algorithm. This validation results show well the effectiveness of the proposed algorithms (New Allied Fuzzy C-Means algorithm NAFCM and NAFCM-PSO algorithm) however the NAFCM algorithm combined with the PSO algorithm has the best result of identification.

REFERENCES