Robust and Simultaneous Reconstruction of Actuator and Sensor Faults via Sliding Mode Observer

Ali Ben Brahim¹, Slim Dhahri², Fayçal Ben Hmida³ and Anis Sellami⁴

Abstract—This paper proposes a method for robust and simultaneous actuator and sensor faults reconstruction of linear uncertain system based on sliding mode observer (SMO). In comparison with existing work, the observer contains two discontinuous terms to solve the problem of simultaneous faults. The main focus of this paper is to synthesis a SMO of an uncertain linear system based on compensation the actuator and sensor faults through the use of two discontinuous terms. The efficacy of this method will be demonstrated with a VTOL aircraft example taken from the FRE literature.

Keywords: Uncertain system; Linear Matrix Inequalities; Simultaneous reconstruction; Actuator and sensor faults; Sliding mode observer.

I. INTRODUCTION

The sliding mode approach has been the main focus of many research works in the literature. We include, inter alia, for example [1], [2], [3], [4], [5], [6]. The existence of discontinuous nonlinear variable in the dynamic equations of SMOs represents the major difference from the other linear observers.

The fault estimation and reconstruction (FRE) is widely discussed in the literature using observers state [7], in particular sliding mode observers [8], [9], [10], [11], [12], [13]. The main idea is to determine the size, location and the dynamics behavior of the faults. In most time, dynamical systems are subject to parametric variations and external disturbances. It is from this reality, this work has been developed in the uncertain context. The problem of robust fault reconstruction actuators and sensors linear uncertain systems is treated for the first time by Tan and Edwards in 2003 [14]. The concept $H_{\infty}$ and LMI (Linear Matrix Inequality) are used to minimize the transfer between uncertainty and defects reconstructed. However, we note that this work deals separately with the problem of reconstruction of actuator and sensor faults. These same authors have extended this work in 2003 to solve the problem of simultaneous estimation of actuators and sensors faults but for some linear systems [15]. However, in the uncertain linear context this problem is not addressed in the FRE literature.

In this paper, we proposed a method that is able to robust and simultaneous reconstruct actuator and sensor faults for a linear uncertain system. Appropriate filtering of the system output yields fictitious system whose actuator faults are the original actuator and sensor faults.

The main focus of this paper is to synthesis a SMO of an uncertain linear system based on compensation the actuator and sensor faults through the use of two discontinuous terms.

II. UNCERTAIN SYSTEM WITH SIMULTANEOUS ACTUATOR AND SENSOR FAULTS

Consider the following uncertain dynamic system affected by actuator and sensor faults:

$$\dot{x}_p = A_p x_p + B_p u_p + M_p f_a + D_p \xi (x, u, t) \quad (1)$$

$$y_p = C_p x_p + N_p f_s \quad (2)$$

where $x_p \in \mathbb{R}^n$ is the state, $u_p \in \mathbb{R}^m$ is the input, $y_p \in \mathbb{R}^p$ is the measurement output of the system, $f_a \in \mathbb{R}^q$ represent the actuator faults where $f_a \leq \alpha_1$ and $f_s \in \mathbb{R}^p$ represent the sensor faults where $f_s \leq \alpha_2$ with $\alpha_1$ and $\alpha_2$ are a known positive scalars. $A_p \in \mathbb{R}^{n \times n}$, $B_p \in \mathbb{R}^{n \times m}$, $C_p \in \mathbb{R}^{n \times n}$, $M_p \in \mathbb{R}^{n \times q}$, $N_p \in \mathbb{R}^{p \times h}$ and $D_p \in \mathbb{R}^{n \times k}$ are the constant matrices. The signal $\xi : \mathbb{R}^{n \times \mathbb{R}^m} \times \mathbb{R}^+ \rightarrow \mathbb{R}^h$ encapsulates the uncertainty in the system. It is assumed to be unknown but bounded subject to $\|\xi (x, u, t)\| \leq \beta$ where the positive scalar $\beta$ is known. Throughout in this paper we assume that $C_p$ and $N_p$ are both full ranks.

The objective now is to transform the sensor faults to actuator faults whose fictitious faults vector affects only the system state. So we pass through the following steps:

First pass the output $y_p \in \mathbb{R}^p$ through an orthogonal matrix $T_R \in \mathbb{R}^{p \times p}$ such that

$$T_R y_p := \begin{cases} y_1 = C_1 x_p \\ y_2 = C_2 x_p + N_1 f_s \end{cases} \quad (3)$$

where $y_2 \in \mathbb{R}^h$ and $N_1 \in \mathbb{R}^{h \times h}$ is non-singular. The partition of the outputs $y_1$ and $y_2$ represents respectively the faults free sensor signals and the potentially faulty sensors signals.

Now define $z \in \mathbb{R}^h$ as a filtered version of $y_2$ with

$$\dot{z} = -A_f z + A_f C_2 x_p + A_f N_1 f_s \quad (4)$$

where $-A_f \in \mathbb{R}^{h \times h}$ is a stable filter matrix. From the equation above, an augmented state space system of order $n + h$
can be obtained
\[
\begin{pmatrix}
\dot{x}_p \\
\dot{z}
\end{pmatrix} = \begin{pmatrix}
A_p & 0 & x_p \\
C_2 & -A_f & z
\end{pmatrix} + \begin{pmatrix}
B_p \\
0
\end{pmatrix} u + \\
\begin{pmatrix}
M_p \\
\end{pmatrix} f_a + \begin{pmatrix}
D_p \\
0
\end{pmatrix} \xi (x, u, t)
\]
\[
\begin{pmatrix}
y \\
\end{pmatrix} = \begin{pmatrix}
C_1 & 0 & 0 \\
0 & I_h & 0
\end{pmatrix} \begin{pmatrix}
x_p \\
\end{pmatrix}
\]
\[
(5)
\]
Compared with the system (1)-(2), the faults in system are now longer directly fed through to the output and so may all be considered as actuator faults.
We assume in this paper that the following conditions are satisfied:
(A1) \( \text{rang}(CM) = q + h = \tilde{q} \leq p \)
(A2) The invariant zeros of triplet \( (A, M, C) \) are stables, that is \( \text{rang} \begin{bmatrix}
\tilde{A} - M & 0 \\
C & M
\end{bmatrix} = n + \text{rang}(M). \)
**Remarque:** The condition (A1) means that the directions of the actuator and sensor faults are completely independent.

**Lemma 1.** [14]

If the condition (A1) then there exist a change of co-ordinates in which the system triplet \( (A, M, C) \) has the following structure:

\[
T \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} T^{-1} = \begin{bmatrix}
0 & T \\
0 & 0
\end{bmatrix}
\]

\[
T M = \begin{bmatrix}
TM_{a} & T M_{s}
\end{bmatrix} = \begin{bmatrix}
0_{(n+h-p) \times q} & 0_{(n+h-p) \times h}
\end{bmatrix}
\]

\[
(7)
\]
where \( A_{11} \in \mathbb{R}^{(n+h-p) \times (n+h-p)}, T \in \mathbb{R}^{p \times p} \) is orthogonal and

\[
M_{a0} = \begin{bmatrix}
0_{(p-q-h) \times q} \\
M_{s2}
\end{bmatrix}, \quad M_{s0} = \begin{bmatrix}
0_{(p-q-h) \times h}
\end{bmatrix}
\]

\[
(8)
\]
where \( M_{s2} \in \mathbb{R}^{(q+h) \times q} \) and \( M_{s2} \in \mathbb{R}^{(q+h) \times h} \) are both full rank.

\[
A_{11} = \begin{bmatrix}
A_{11}^{0} & A_{12}^{0} \\
0 & A_{22}^{0}
\end{bmatrix}, \quad A_{21} = \begin{bmatrix}
0 & A_{21}^{0}
\end{bmatrix}
\]

\[
(9)
\]
where \( A_{11}^{0} \in \mathbb{R}^{p \times p}, A_{21}^{0} \in \mathbb{R}^{(p-q-h) \times (n+h-p)} \) with \( r \geq 0 \) and the system \( (A_{22}^{0}, A_{21}^{0}) \) are completely observable.

### III. A SLIDING MODE OBSERVER DESIGN

In this section we described a new sliding mode observer that is used for the uncertain system (5)-(6). The SMO which will be used here is of the form

\[
\dot{x} = Ax + Bu - G_{e}y + \left[ \begin{array}{c}
G_{r1} \\
G_{r2}
\end{array} \right] \begin{pmatrix}
v_1 \\
v_2
\end{pmatrix}
\]

\[
(10)
\]
\[
\hat{y} = C \dot{x}
\]

\[
(11)
\]
with

\[
v_1 = \begin{cases}
- \rho_1 \frac{\|P_{e} e_y\|}{\|P_{e} e_y\|} & \text{if } e_y \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
v_2 = \begin{cases}
- \rho_2 \frac{\|P_{e} e_y\|}{\|P_{e} e_y\|} & \text{if } e_y \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
(12)
(13)
\]
where \( e_y := \hat{y} - y \) is the output error estimation and \( P_0 \in \mathbb{R}_{p \times p} \) is the symmetric positive definite (s.p.d.) matrix. \( v_1 \) and \( v_2 \) have the main objective to compensate \( f_a \) and \( f_s \) respectively despite the presence of the uncertainty \( \xi (x, u, t). \)

So the scalar gain functions \( \rho_1 \) and \( \rho_2 \) will also be described formally but it must be represent an upper bound on the magnitude of the actuator and sensor faults respectively plus the uncertainty.
We assume that

\[
\rho_1 \geq \|CM_{a}\| \alpha_1 (t, u) + \gamma_{a0}
\]

\[
\rho_2 \geq \|CM_{s}\| \alpha_2 (t, u) + \gamma_{s0}
\]

\[
(14)
(15)
\]
where \( \gamma_{a0} \) and \( \gamma_{s0} \) are a positive scalars.

Define \( e := \hat{x} - x \) as the state estimation error. It then follows from (5)-(6) and (10)-(11) that

\[
\dot{e} = A_0 e + \begin{pmatrix}
G_{r1} & G_{r2}
\end{pmatrix} \begin{pmatrix}
v_1 \\
v_2
\end{pmatrix} - \begin{pmatrix}
M_{a} & M_{s}
\end{pmatrix} \begin{pmatrix}
f_a \\
f_s
\end{pmatrix} - D \xi (x, u, t)
\]

\[
(16)
\]
where \( A_0 = A - G_{1} C. \)

**Proposition 1.** [14]

If there exists a s.p.d. Lyaponov matrix \( P \), that satisfies \( PA_0 + A_0^{T} P < 0 \) with the structure:

\[
P = \begin{bmatrix}
P_1 & P_1 L \\
(L^0 P_1)^{T} & L^T P_1 L + T^T P_2 T
\end{bmatrix}
\]

\[
(17)
\]
where \( P_1 \in \mathbb{R}^{(n+h-p) \times (n+h-p)}, P_2 \in \mathbb{R}^{p \times p} \) and \( L = \begin{bmatrix}
L^0 & 0
\end{bmatrix} \) with \( L^0 \in \mathbb{R}^{(n+h-p) \times (p-q-h)} \) and if \( P_2 := TP_1 T^T \) then the state estimation error system \( e \) is quadratically stable. Furthermore, sliding occurs in finite time on \( S_{e} := \{ e : e_y = 0 \} \) governed by the system matrix \( A_{11} + LA_{22} \).

Now define scalars

\[
\mu_0 = - \lambda_{\max} \left\{ P A_0 + A_0^{T} P \right\}
\]

\[
\mu_1 = \sqrt{\lambda_{\max} \{ D^T P D \}}
\]

(18)

(19)

**Lemma 2.** [14]

The state estimation error \( e \) in (16) is ultimately bounded with respect to the set

\[
\Omega_{e} = \left\{ e : \|e\| < \frac{2 \mu_1 \beta}{\mu_0} + \varepsilon \right\}
\]

\[
(20)
\]
where \( \varepsilon > 0 \) is an arbitrarily small positive scalar.

**Proof:** see reference[14]

Lemma (2) will be used now to prove the main idea result of this section, that for an appropriate of \( \mu_1 \) and \( \mu_2 \) a sliding
mode motion is induced on $S_g$.

First introduce a co-ordinate change, let $T_L : e \rightarrow e_L$ where

$$T_L = \begin{pmatrix} I_{n+h-p} & L \\ 0 & T \end{pmatrix}$$  \hspace{1cm} (21)

In this new co-ordinate system, the triplet $(A, M, C)$ will be in the form

$$\begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{pmatrix}, \begin{pmatrix} 0 \\ \tilde{M}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \tilde{M}_{12} & M_L \end{pmatrix}, \begin{pmatrix} 0 \\ I_P \end{pmatrix}$$  \hspace{1cm} (22)

where $\tilde{A}_{11} = A_{11} + LA_{21}$ and $\tilde{M}_2 = TM_2$. The nonlinear gain will have the form $\tilde{G}_n = \begin{pmatrix} \tilde{G}_{n1} & \tilde{G}_{n2} \end{pmatrix}$. The linear gain $G_{1L} = T_L \tilde{G}_1$, $G_{2L} = T_L \tilde{G}_2$ and the Lyapunov matrix will be $P_L = (T_L^{-1})^T P (T_L^{-1}) = diag(P_1, P_2)$.

The fact that $P_L$ is block diagonal matrix for $(A_L - G_{1L}C_L)$ implies that $A_{11}$ is stable and hence the sliding motion is stable.

In the new co-ordinates

$$\dot{e}_1 = \tilde{A}_{11} e_1 - \left( \tilde{A}_{12} - \tilde{G}_{11} \right) e_y - \tilde{D}_1 \xi (x, u, t)$$  \hspace{1cm} (23)

$$\dot{e}_y = \tilde{A}_{21} e_1 - \left( \tilde{A}_{22} - \tilde{G}_{12} \right) e_y - \tilde{D}_2 \xi (x, u, t) - \tilde{M}_{22} f_a - \tilde{M}_{21} f_s + v_1 + v_2$$  \hspace{1cm} (24)

where $\tilde{G}_{11}$ and $\tilde{G}_{12}$ represent an appropriate partition of $G_{1L}$.

**Proposition 2.**

If the gain functions $\rho_1$ and $\rho_2$ from (14)-(15) satisfies

$$\rho_1 \geq \frac{2}{\mu_0} \left( \frac{\mu_0}{\mu_1} + \left\| \tilde{D}_2 \right\| \beta + \left\| \tilde{M}_{22} \right\| \alpha_1 (t, u) + \gamma_{a0} \right)$$  \hspace{1cm} (25)

$$\rho_2 \geq \frac{2}{\mu_0} \left( \frac{\mu_0}{\mu_1} + \left\| \tilde{D}_2 \right\| \beta + \left\| \tilde{M}_{21} \right\| \alpha_2 (t, u) + \gamma_{a0} \right)$$  \hspace{1cm} (26)

then the ideal sliding mode motion place on $S_g$ in finite time.

**Proof.**

Consider the Lyapunov function $V_y = e_y^T P_0 e_y$. $P_L$ is a block diagonal Lyapunov matrix for $(A_L - G_{1L}C_L)$.

So that $P_0 \begin{pmatrix} \tilde{A}_{22} - \tilde{G}_{12} \\ \tilde{A}_{21} \end{pmatrix} + \begin{pmatrix} \tilde{A}_{22} - \tilde{G}_{12} \end{pmatrix}^T P_0 < 0$ and hence

$$\dot{V}_y \leq 2 e_y^T P_0 \begin{pmatrix} \tilde{A}_{21} e_1 - \tilde{M}_{21} f_s - \tilde{D}_2 \xi (x, u, t) \\ \tilde{A}_{21} e_1 - \tilde{M}_{22} f_a - \tilde{D}_2 \xi (x, u, t) \end{pmatrix} - 2 \rho_1 \left\| P_0 e_y \right\|$$

$$+ 2 e_y^T P_0 \begin{pmatrix} \tilde{A}_{21} e_1 - \tilde{M}_{21} f_s - \tilde{D}_2 \xi (x, u, t) \\ \tilde{A}_{21} e_1 - \tilde{M}_{22} f_a - \tilde{D}_2 \xi (x, u, t) \end{pmatrix} - 2 \rho_2 \left\| P_0 e_y \right\|$$

$$\leq -2 \left\| P_0 e_y \right\| \left( \left( \rho_1 \left\| \tilde{A}_{21} e_1 \right\| - \left\| \tilde{M}_{21} f_s \right\| - \left\| \tilde{D}_2 \xi (x, u, t) \right\| \right) \\ + \left( \rho_2 \left\| \tilde{A}_{21} e_1 \right\| - \left\| \tilde{M}_{22} f_a \right\| - \left\| \tilde{D}_2 \xi (x, u, t) \right\| \right) \right)$$

From lemma (2), infinite time $e \in \Omega_a$ which implies $\left\| e \right\| < \frac{2 \mu_0}{\mu_1} + \varepsilon$, therefore from the definition of $\rho_1$ and $\rho_2$ in (25)-(26),

$$\dot{V}_y \leq -2 (\gamma_{a0} + \gamma_{a0}) \left\| P_0 e_y \right\| \leq -2 (\gamma_{a0} + \gamma_{a0}) \gamma \sqrt{V_y}$$

where $\gamma := \sqrt{\lambda_{\min} (P_0)}$. This proves that the output estimation error $e_y$ will be reached zero infinite time, and a sliding motion takes place.

**IV. ROBUST AND SIMULTANEOUS RECONSTRUCTION OF ACTUATOR AND SENSOR FAULTS**

In this section the sliding mode observer will be analyzed with regard to its ability to robust and simultaneous reconstruct the actuator and sensor faults despite the presence of the uncertainty $\xi (x, u, t)$.

We assume that a sliding mode observer is designed and that a sliding motion is achieved, then and

$$\dot{e}_1 = \tilde{A}_{11} e_1 - \tilde{D}_1 \xi (x, u, t)$$  \hspace{1cm} (27)

$$v_{1eq} = -\tilde{A}_{21} e_1 + \tilde{M}_{21} f_s + \tilde{D}_2 \xi (x, u, t)$$  \hspace{1cm} (28)

$$v_{2eq} = -\tilde{A}_{21} e_1 + \tilde{M}_{22} f_a + \tilde{D}_2 \xi (x, u, t)$$  \hspace{1cm} (29)

where $v_{1eq}$ and $v_{2eq}$ are the equivalent output error injection terms required to maintain a sliding motion and can be approximated by

$$v_{1eq} = -\rho_1 \frac{P_0 e_y}{\left\| P_0 e_y \right\| + \delta_a} \text{ if } e_y \neq 0$$  \hspace{1cm} (30)

$$v_{2eq} = -\rho_2 \frac{P_0 e_y}{\left\| P_0 e_y \right\| + \delta_s} \text{ if } e_y \neq 0$$  \hspace{1cm} (31)

where $\delta_a$ and $\delta_s$ are small positive scalars, in the ideal objective to minimize the effect of the uncertainty $\xi (x, u, t)$ on the actuator and sensor faults reconstruction respectively.

**A. Robust reconstruction of actuator faults**

The objective now is to reconstruct the actuator faults whilst minimizing the effect of $\xi (x, u, t)$.

Define a would-be reconstruction signal

$$\hat{f}_a = W_a T^T v_{1eq}$$  \hspace{1cm} (32)

where $W_a := \begin{pmatrix} W_{a1} & M^{-1} \end{pmatrix}$ with $W_{a1} \in \mathbb{R}^{(q+h) \times (p-q-h)}$ represents design freedom.

It follows (18)-(19) and (27)-(28)

$$\hat{f}_a = G_a (s) \xi (x, u, t) + f_a$$  \hspace{1cm} (33)

where the transfer functions

$$G_a (s) = W_a A_{21} \{ sI - (A_{11} + L_a A_{21}) \}^{-1} \{ (D_1 + L_a D_2) \} + W_a D_2$$  \hspace{1cm} (34)

Using the Bounded Real Lemma the $L_2$ gain from the exogenous signal $\xi (x, u, t)$ to the $f_a$ will not exceed $\gamma_a$ if the following holds

$$\begin{pmatrix} P_a A_{11} + A_{11}^T P_a & -P_a D_1 & -(W_a T^T A_{21})^T \\ -P_a D_1^T & -\gamma_a I & -(W_a T^T D_2) \\ *( & * & * \end{pmatrix} \begin{pmatrix} * & -\gamma_a I \end{pmatrix} < 0$$  \hspace{1cm} (35)

where $\delta_a$ is a positive scalar. The objective is to find $P_a$, $L_a$ and $W_a$ to minimize $\delta_a$ subject to (35).
Writing

\[ P_a = \begin{pmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{pmatrix} > 0 \]  \hspace{1cm} (36)

where \( P_{11} \in \mathbb{R}^{(n+h-p) \times (n+h-p)} \) and \( P_{12} := \begin{bmatrix} P_{12} \end{bmatrix} \) with \( P_{12} \in \mathbb{R}^{(n+h-p) \times (p-q-h)} \). It follows there is a one-to-one correspondence between the two sets of decision variables \((P_{11}, P_{12}, P_{22})\) and \((P_1, L_a, P_2)\) since

\[ P_{11} = P_1, L_a = P_{11}^{-1} P_{12} \text{ and } P_2 = P_{22} - P_{12}^T P_{11}^{-1} P_{12} \]  \hspace{1cm} (37)

Processing that the linear gain \( G \) be chosen to satisfy

\[
\begin{pmatrix}
PA + A^T P - \gamma a I \\
\gamma a I
\end{pmatrix} < 0
\]  \hspace{1cm} (38)

where

\[
B_d \in \mathbb{R}^{(n+h) \times (p+k)} \\
K \in \mathbb{R}^{p \times (p+k)} \\
H_a \in \mathbb{R}^{(q+h) \times (n+h)} \\
B_d := \begin{bmatrix} 0 & D \end{bmatrix} \\
K := \begin{bmatrix} K_1 & 0 \end{bmatrix} \\
H_a := \begin{bmatrix} 0 & H_2 \end{bmatrix} \\
E_a := \begin{bmatrix} E_1 & E_2 \end{bmatrix}
\]

and \( E_a \in \mathbb{R}^{p \times p} \) is nonsingular user defined parameter and \( H_2 \in \mathbb{R}^{(q+h) \times k} \) depends on \( W_{a1} \).

If the feasible solution to (36) and (38) exists, then the requirements of proposition2 will be fulfilled.

**Proposition3.**

Inequality (38) is feasible if and only if

\[
\begin{pmatrix}
PA + A^T P - \gamma a C^T (KK^T)^{-1} C - \gamma a I \\
\gamma a I
\end{pmatrix} < 0
\]  \hspace{1cm} (39)

where an appropriate choice of \( G \) is given by

\[ G = \gamma a P^{-1} C^T (KK^T)^{-1} \]  \hspace{1cm} (40)

**Proof. See reference [14]**

The objective now is to reconstruct the sensors faults whilst minimizing the effect of \( \xi(x, u, t) \).

A robust reconstruct can be designed of sensors faults, replacing \((P_a, L_a, W_a)\) with respectively

\[
(P_s, L_s, W_s)
\]  \hspace{1cm} (42)

where 42 the main idea is to find \( P_s, L_s \) and \( W_s \) to minimize \( \gamma_s \).

**V. ILLUSTRATIVE EXAMPLE**

The method developed in this paper to obtained a robust and simultaneous reconstruct of actuator and sensor faults will be now demonstrated with an example, which is a VLOT aircraft model taken from reference [14]. The system states, outputs and inputs respectively are

\[
\begin{align*}
x & = \begin{bmatrix} \text{horizontal velocity (kts)} \\
\text{vertical velocity (kts)} \\
\text{pitch rate (deg/s)} \\
\text{pitch angle (deg)} \end{bmatrix} , \\
y & = \begin{bmatrix} \text{horizontal velocity (kts)} \\
\text{vertical velocity (kts)} \\
\text{pitch angle (deg)} \end{bmatrix} , \\
u & = \begin{bmatrix} \text{collective pitch control} \\
\text{longitudinal cyclic pitch control} \end{bmatrix} 
\end{align*}
\]

In the notation of (1)-(2)

\[
A_p = \begin{bmatrix}
-9.9477 & -0.7476 & 0.2632 & 5.0337 \\
52.1659 & 2.7452 & 5.5532 & -24.2221 \\
0 & 0 & 1 & 0 \\
0.4422 & 0.1761 & & \\
3.5446 & -7.5922 & & \\
-5.5200 & 4.4900 & & \\
0 & 0 & & \\
0 & 0 & & 1 \\
0 & 0 & & 1 \\
0 & 0 & & 1
\end{bmatrix} \\
B_p = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix} \\
C_p = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 
\end{bmatrix}
\]

The parametric uncertainty \( \xi(x, u, t) \) is given by

\[
\xi = \begin{bmatrix} 0 & \Delta a_1 & 2 \\
0.1 & \Delta a_2 & 0 
\end{bmatrix} y 
\]

where \( \Delta a_1 = 0.7 \) and \( \Delta a_2 = 0.2 \).

The filter matrix from (4) is chosen \( A_f = 0.47 \). Hence the matrices that make up the system (5)-(6) can be obtained. The invariant zeros of \( (A, M, C) \) are actually the invariant zeros of \( (A_p, M_p, C_p) \).

It was found that \( (A_p, M_p, C_p) \) does not have any invariant zeros (so the assumption2 is satisfied), and also that assumption1 is satisfied. Consequently the new proposed method is applicable.

In designing the sliding mode observer, the positives scalars from (30) and (31) where chosen as \( p_1 = 1, \rho_2 = 20, \delta_s = \delta_s = 0.05 \), the design parameters were chosen as \( D_1 = 10 I_3 \), \( \gamma_{a0} = \gamma_{s0} = 1 \), the optimization scalar
yielded a value of $\gamma_a = 0.1991$ and $\gamma_s = 0.0325$, yielding observer gains of

$$G_{a1} = \begin{pmatrix} -1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 4.9825 & -0.9357 & -0.5991 \\ -0.5297 & -0.0661 & -0.2359 \\ 0 & 0 & 1.0000 \end{pmatrix}$$

$$G_{a2} = \begin{pmatrix} -1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 2.0575 & -0.0029 & 0 \\ -0.0906 & -0.0065 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


The Lyapunov matrix from (17) is given by

$$P_0 = \begin{pmatrix} 17.9898 & -1.3006 & -21.8026 \\ -1.3006 & 2.0751 & 0.9862 \\ -21.8026 & 0.9862 & 43.8609 \end{pmatrix}$$

matrices $W_a$ from (32), $W_a$ from (42), $L_a$ from (37), $L_s$ from (42),

$$W_a = \begin{pmatrix} 12.3647 & -3.5965 & 0.2799 \\ 0.1634 & 0.0073 & -0.5556 \\ 0.2359 & 0.5338 & 0 \\ -0.5991 & 4.8284 & 0 \\ 0.0906 & 0.0065 & 0 \\ 2.0575 & -0.0029 & 0 \end{pmatrix}$$

$$W_s = \begin{pmatrix} M_{a1}^{-1} \\ M_{a1}^{-1} \\ M_{a1}^{-1} \end{pmatrix}$$

$$L_a = \begin{pmatrix} L_{a1} \\ L_{a1} \\ L_{a1} \end{pmatrix}$$

$$L_s = \begin{pmatrix} L_{s1} \\ L_{s1} \\ L_{s1} \end{pmatrix}$$

A. Simulations in the absence of measurement noise

The following figures represent results from simulations in the absence of measurement noise, implying that the actuator and sensor faults occurred simultaneously.

![Fig. 1](image1.png)

(a) real actuator fault and his estimate  (b) real sensor fault and his estimate

Fig. 1: The left subfigure is an actuator fault and its reconstruction. The right subfigure is a sensor fault and its reconstruction.

B. Simulations in the presence of measurement noise

The following figures are from simulations of identical scenarios to the considered above except white noise of standard deviation of $10^{-4}$ has been added to the output signal so that the measured signal which is used by the robust and simultaneous actuator and sensor faults reconstruct observer is corrupted.

![Fig. 2](image2.png)

(a) real actuator fault and his estimate  (b) real sensor fault and his estimate

Fig. 2: The left subfigure is an actuator fault and its reconstruction. The right subfigure is a sensor fault and its reconstruction.

VI. Conclusion

This paper has proposed a method for robust and simultaneous reconstruction of actuator and sensor faults using a sliding mode observer that minimize the effect of system uncertainty on the reconstruction. The idea is bound to generate two signals injection of the output error for the reconstruction of actuator and sensor faults which are guaranteed the stability bounds and robustness of the sliding motion by using the features of lyapunov and matrix inequalities linear (LMI).

REFERENCES


