Directional Virtual Coordinate Systems for Wireless Sensor Networks*

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Abstract— A Directional Virtual Coordinate System (DVCS) is proposed based on a novel transformation that restores the lost directionality information in a Virtual Coordinate System (VCS). VCS is an attractive option to characterize the node locations in Wireless Sensor Networks (WSNs), instead of using geographical coordinates, which are expensive or difficult to obtain. A VCS characterizes each node in a network with the minimum hop distances to a set of anchor nodes. Proposed transformation supplements the VCS, thus preserving all its inherent properties such as geodesic distance information embedded in the coordinates. Virtual directionality introduced alleviates the local minima issue present in original VCS. Properties of this virtual directional domain are discussed. With these directional properties, it is possible, for the first time, to consider deterministic algorithms for routing in the virtual domain, as illustrated with a constrained tree network example. A novel routing scheme, Directional Virtual Coordinate Routing (DVCR) that illustrates the effectiveness of directional virtual coordinate domain is presented. DVCR significantly outperforms existing VCS routing schemes Convex Subspace Routing (CSR) and Logical Coordinate Routing (LCR), while achieving a performance similar to geographical routing scheme Greedy Perimeter Stateless Routing (GPSR), but without the need for node location information.

Index Terms— Routing, Sensor Networks, Virtual Coordinates, Virtual Directions

I. INTRODUCTION

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irtual Coordinate Routing (VCR) and Geographical Routing (GR) are two main classes of address-based routing schemes for WSNs. Geographical routing [1][7][7] relies on physical location information of nodes, and directional information that can be derived from individual node locations. Obtaining location information however requires mechanisms like GPS, which are costly or infeasible in some applications, or localization algorithms, which are complex and error prone as a result of their reliance on measurements such as RSSI or time delay. GR also suffers from poor routability in the presence of concave physical voids. Connectivity based approaches provide an alternative solution to overcome weaknesses associated with location determination and geophysical voids. VCR [2]-[4] uses a Virtual Coordinate System (VCS) that characterizes each node by a coordinate vector of size \(M\), consisting of the shortest hop distance to each of a set of \(M\) anchors, which may be generated using network wide flooding [3]. The number of anchors becomes the networks’ VCS dimensionality.

Routing in virtual domain has two phases. Most of the VCR schemes [2][3][11] use Greedy Forwarding (GF) combined with a back-tracking algorithm. In GF, a packet is simply forwarded to a neighbor that is closer to the destination than the packet holding node. Virtual Coordinates (VCs) of nodes are used for distance evaluation between nodes as well as for node identification (ID). Distance is estimated using either \(L^1\) or \(L^2\) norm based on VCs; such values are often unreliable estimates of the distance as the contributions due to different anchors are not orthogonal. When a closer neighbor cannot be found, i.e., the packet is at a local minima, back-tracking is employed to climb out of it.

Performance of VCR and anchor placement is highly correlated. Anchors may be selected randomly [4] or by selecting nodes with specific properties, e.g., by selecting all the perimeter nodes [10]. When a message reaches a local minima, an expanding ring search is performed in [10] until a closer node is found or TTL (Time-To-Live) expires. In Virtual Coordinate assignment protocol (VCap), the coordinates are defined based on three anchors [2]. At local minima, VCap causes a packet to follow a rule called local detour. In Logical Coordinate based Routing (LCR) [3] backtracking is used when GF fails at a local minima. Aligned virtual coordinate system (AVCS) [9] re-evaluates VCs by averaging its own coordinates with neighboring coordinates to overcome local minima. In Axis-based Virtual Coordinate Assignment Protocol (ABVCap) [11], 5-tuple VC is assigned to each node corresponding to longitude, latitude, ripple, up, and down. All these VCR protocols rely mainly on Greedy forwarding, followed by a backtracking scheme to overcome local minima. Convex Subspace Routing [4], in contrast, selects dynamically changing subsets of anchors to provide convex distance surfaces for routing.

VCS has its inherent advantages and disadvantages. VCS is a connectivity based higher dimensional transformation of the WSN, resulting in some attractive properties such as considerably high routability without need for any geographical information, effectiveness of connectivity information embedded in VCs, and insensitivity to physical voids and to localization errors. Physical domain to virtual domain transformation is many to one as VCs are insensitive

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to directions, which is one main cause of identical coordinates and local minima. If an adequate number of anchors are not appropriately deployed, it may also cause the network to suffer from identical coordinates and local minima [4] resulting in logical/virtual voids. Identification of the optimal number of anchors and proper anchor placement remains a major challenge [4].

Inadequacies associated with VCS are due to loss of directionality information and the lack of information about the physical topology. This paper proposes a novel transformation with which the VCS can regain its lost directionality, and thus acquire some sense of physical location to supplement the connectivity information embedded in original VCS. No additional transmission cost is involved as each node can evaluate the directional values with VCs available locally. Acquiring directionality provides new information hitherto not available in VCS, facilitating a new approach for designing a broad spectrum of WSN algorithms. Technique to identify ‘good’ anchors alleviating the issues in VCS discussed in [4], novel routing schemes and topology preserving maps [5] are among potential applications of Directional Virtual Coordinate Systems (DVCS). As an example, we illustrate a deterministic algorithm for routing in a constrained tree topology, based on new transformed coordinates in directional virtual space. To our knowledge, no deterministic algorithms have been developed before using VCS.

The contributions of this paper are:
1. A novel concept of transforming directionless VCS to directional VCS,
2. Properties of directional VCS and deterministic routing in a constrained tree network, and
3. Directional Virtual Coordinate Routing (DVCR) - Routing in directional VCS.

The proposed routing scheme in directional virtual space - Directional Virtual Coordinate Routing (DVCR) is compared with CSR [4] and LCR [3]. Moreover it is compared with a geographical routing scheme called Greedy Perimeter Stateless Routing (GPSR) [7] which makes greedy forwarding decisions until it fails, for example due to a geographical void, and attempts to recover by routing around the perimeter of the void. DVCR outperforms, CSR and LCR with a noticeable value achieving more or less the similar performance as GPSR.

Section II explains the new transformation of VCS to directional VCS. In Section III, a deterministic routing based on directional coordinates in a simple tree is discussed and in Section IV transformed domain network partition property was illustrated. A novel routing protocol is proposed in Section V. Performance evaluation is in Section VI. Finally, Section VII concludes the contribution of this paper.

II. DIRECTIONLESS VIRTUAL SPACE TO DIRECTIONAL VIRTUAL SPACE TRANSFORMATION

As a virtual coordinate corresponds to the distance to a particular anchor, the physical domain to virtual domain transformation is many to one. The coordinate propagates concentrically around an anchor, thus losing the directional information. Consequences of this mapping include identical coordinates and local minima encountered in routing [4]. A novel transformation of VCs to regain the directionality lost is proposed next. The notations used are summarized in Table 1.

TABLE I

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of network nodes</td>
</tr>
<tr>
<td>$N_i, N_s, N_f \in N$</td>
<td>Node $i$, Destination, Source</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of anchors</td>
</tr>
<tr>
<td>$A_i \subseteq 1:M$</td>
<td>Anchor set (a subset of $N$)</td>
</tr>
<tr>
<td>$h_{N_i N_f}$</td>
<td>Minimum hop distance between node $N_i$ and $N_f$</td>
</tr>
<tr>
<td>$h_{N_i, N_s, \ldots, N_k}$</td>
<td>Node $N_k$’s VC</td>
</tr>
<tr>
<td>$[n_{ij} \ldots n_{ik}]$</td>
<td>Node $N_i$’s transformed VC</td>
</tr>
<tr>
<td>$f = 1: L, L \leq C^N$</td>
<td>$D_{N_i N_f}$ Distance between $N_i$ and $N_f$ in transformed domain</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of neighbors</td>
</tr>
<tr>
<td>$N_{i,\text{Prev}}$</td>
<td>Node that forwarded the packet to current node</td>
</tr>
<tr>
<td>$N_{i,\text{Next}}$</td>
<td>Node that current node will forward the packet to</td>
</tr>
</tbody>
</table>

First consider a 1-D network where one can easily visualize the concept behind the transformation. Table 2 contains the VCS for the 1-D network shown in Fig. 1 with respect to two anchors $A_1$ and $A_2$, which are $h_{A_2 A_1}$ hops apart (8 hops in Fig. 1). Note that $h_{N_i A_1}$ propagates symmetrically from the corresponding anchor, thus losing directionality. Even though $(h_{N_i A_1} - h_{N_i A_2})$ provides the sense of directionality for the region between anchors, as can be seen in Table 2, it remains constant outside the region bounded by anchors, thus failing to provide directional information. Conversely, $(h_{A_2 A_1} + h_{N_i A_2})$ has a constant value in between the anchors, but linearly varies elsewhere. By combining those, a node $N_i$ is characterized using $f(h_{N_i A_1}, h_{N_i A_2})$ that defined as,

$$f(h_{N_i A_1}, h_{N_i A_2}) = \frac{1}{2h_{A_2 A_1}} (h_{N_i A_1} - h_{A_2}) (h_{N_i A_1} + h_{N_i A_2})$$

(1)

$f(h_{N_i A_1}, h_{N_i A_2})$, as shown in Table 2, maps the nodes’ VC $\equiv (h_{N_i A_1}, h_{N_i A_2})$ linearly to the real axis with positive and negative values with center at the midpoint of $A_1$ and $A_2$, providing directional information in the virtual domain. The term $\frac{1}{2h_{A_2 A_1}}$ normalizes the distance to provide a unit difference of the ordinate between two adjacent nodes.

Fig. 1. 1D network with two anchors $A_1$ and $A_2$.

TABLE II

<table>
<thead>
<tr>
<th>NODE ID</th>
<th>$N_i$</th>
<th>$N_s$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
<th>$N_6$</th>
<th>$N_7$</th>
<th>$N_8$</th>
<th>$N_9$</th>
<th>$A_2$</th>
<th>$N_4$</th>
<th>$A_1$</th>
<th>$N_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{A_1}$</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{A_2}$</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{A_1} - h_{A_2}$</td>
<td>-8</td>
<td>-8</td>
<td>-6</td>
<td>-4</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>$h_{A_1} + h_{A_2}$</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>$f(h_{A_1}, h_{A_2})$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.3</td>
<td>-0.2</td>
<td>-0.1</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Prior VC based routing schemes such as LCR [3] encountered local maxima at anchors even for this simple linear array [4], but with this transformation, the local minima problem is overcome completely for this linear topology. Furthermore, one can now view each node as a point in a vector space. Define, \( \vec{u}_{A_1A_2} \) as the unit vector in \( A_1A_2 \) direction, which is named as a virtual direction. Ordinate in Fig. 1 can be written in the form:

\[
\tilde{f}(h_{N_{A_1}}, h_{N_{A_2}}) = f(h_{N_{A_1}}, h_{N_{A_2}})\vec{u}_{A_1A_2}
\] (2)

Now consider a 2D sensor network. Select two coordinates of node \( N_i: h_{N_{A_1}} \) and \( h_{N_{A_k}} \), with respect to anchors, \( A_1 \) and \( A_k \). Then the magnitude of the virtual distance vector component in the \( A_1A_k \) direction is given by

\[
f\left(h_{N_{A_1}}, h_{N_{A_k}}\right) = \frac{1}{2h_{A_1A_k}}\left(h_{N_{A_1}}^2 - h_{N_{A_k}}^2\right)
\] (3)

Since there are \( M \) ordinate available, \( C_M^2 \) different virtual directions (though not orthogonal to each other) may be specified, and can be evaluated locally at each node. Some of the properties in this directional Virtual Space are discussed next. Each transformed domain ordinate can be written in the form:

\[
\tilde{f}\left(h_{N_{A_1}}, h_{N_{A_k}}\right) = f\left(h_{N_{A_1}}, h_{N_{A_k}}\right)\vec{u}_{A_1A_k}
\] (4)

where, \( \tilde{f}\left(h_{N_{A_1}}, h_{N_{A_k}}\right) \) is the vector representation of the transformed ordinate of \( A_1 \) and \( \vec{u}_{A_1A_k} \) is the virtual direction obtained by \( A_1A_k \). We can also define the virtual distance between two nodes \( N_p \) and \( N_q \) in this direction to be,

\[
F_{A_1A_k}(N_p, N_q) = f\left(h_{N_{A_1}}, h_{N_{A_k}}\right) - f\left(h_{N_{A_1}}, h_{N_{A_k}}\right)
\] (5)

Fig. 2 shows the 2D extension of the transformation to a grid network. Transformed coordinates are given by \( f(A_1, A_2), f(A_3, A_4) \), which provides directionality.

![Fig. 2](image)

**Property 1:** Consider a 2-D network with two anchors \( A_1 \) and \( A_2 \), which are \( h_{A_1A_2} \) hops apart, the transformation \( f(h_{N_{A_1}}, h_{N_{A_2}}) = \frac{1}{2h_{A_1A_2}}\left(h_{N_{A_1}}^2 - h_{N_{A_2}}^2\right) \) partitions the network in to two sections with positive and negative ordinates. Moreover, the nodes that are equidistance in terms of hops to \( A_1 \) and \( A_2 \) have \( f(h_{N_{A_1}}, h_{N_{A_2}}) = 0 \).

Proof: Consider the transformation in Eq. (3). The nodes with \( h_{N_{A_1}} > h_{N_{A_2}} \) have positive transformed ordinate while nodes with \( h_{N_{A_1}} < h_{N_{A_2}} \) have negative transformed ordinates. Nodes with \( h_{N_{A_1}} = h_{N_{A_2}} \) have zero ordinate in the transformed domain. QED

### III. Routing in Directional Virtual Domain

In this section we demonstrate how it is possible to exploit properties of DVCS to develop strategies to identify routing paths deterministically, which was not feasible with directionless VCS. For the example presented in this section, that of a simple tree, a DVCS based deterministic routing protocol can be used to guarantee 100% routability. This direction based method can be considered as a foundation for developing relationships to discover routing paths in more complex topologies.

Consider a Constrained Tree (CT) network with branches extending off one main trunk (backbone). Assume that the maximum degree of a node is three, i.e., no two branches occur at same point, and there are no branches off branches (See Fig. 3). CT network topologies fit well in environments such as mine-shafts and pipeline distribution systems. The traditional VCR schemes such as LCR and CSR cannot guarantee 100% routability in these networks. Consider the network shown in Fig. 3, where a packet is to be sent from node \( N_5 \equiv (7,13) \) to \( N_4 \equiv (16,10) \). In this case, the packet will be forwarded to \( N_{4+1} \) in Greedy Forwarding based on VCs, whereas the correct neighbor to forward the packet is \( N_{5-1} \).

![Fig. 3](image)

**Property 2:** In a CT, the gap between any two adjacent nodes in a branch is constant, and is uniquely dependent on the junction node. Specifically, for a branch off node \( N_i = (h_{N_{A_1}}, h_{N_{A_2}}) \), the gap is given by

\[
g = \frac{(h_{N_{A_1}} - h_{N_{A_2}})}{(h_{N_{A_1}} + h_{N_{A_2}})}
\] (6)

Proof: Junction node coordinates, are unique. Consider the junction node \( N_i = (h_{N_{A_1}}, h_{N_{A_2}}) \). A positive integer \( x \) which makes \( (h_{N_{A_1}} + x) - (h_{N_{A_2}} - x) = h_{N_{A_1}} - h_{N_{A_2}} \) does not exist. Hence gap in each branch given by (6) is unique. QED

**Property 3:** In a CT, only the members of the backbone \( N_i = (h_{N_{A_1}}, h_{N_{A_2}}) \) satisfies

\[
h_{N_{A_1}} + h_{N_{A_2}} = h_{A_1A_2}
\] (7)

Proof: This can be proved by the characteristics of VC. QED

**Property 4:** In a CT, junction node \( N_i = (h_{N_{A_1}}, h_{N_{A_2}}) \) can identify the coordinates of members on its branch and how many hops that each member of the branch is away from itself.

Proof: As in property 2, gap in a branch is unique and it is known by the branching node. Assume a node \( N_d = (h_{N_{A_1}}, h_{N_{A_2}}) \) is a member of the branch from junction \( N_i \).
Then virtual ordinates of \(N_d\) satisfies \(h_{N_dA_1} = h_{N_dA_2} + ga\) and \(h_{N_dA_2} = h_{N_dA_1} + ga, g\) can be found as in (6). Thus ‘\(a\)’, the number of hops to the backbone from \(N_d\) can be calculated. Consider another junction node \(N_j = (h_{N_dA_1} + x, h_{N_dA_2} - x)\) with gap in its branch \(g'\). There do not exist a positive integer \(x\) and \(a'\), which satisfy \(h_{N_dA_1} = h_{N_dA_2} + x + g'a'\) and \(h_{N_dA_2} = h_{N_dA_1} - x + g'a'\). Hence \(N_d\) exists only in the branch of \(N_j\). QED

**Theorem 1:** In a tree with branches on two sides off one main trunk, with no branches off branches, with node degree \(\leq 3\), 100% routability can be achieved with two anchors placed one at each extreme of the trunk.

Proof: If the destination is on the same branch as the source, the source can ascertain it by evaluating the gap between itself and its neighbor, and it can forward the packet toward or away from branch node correctly. Otherwise, routing is performed in two steps. Initially the packet is routed to the junction node where the current node holding branch connects to the backbone. Then packet is routed to the destination from the junction node. But current node should find out the coordinates of the junction node. Consider Fig 3. Let the current node be \(N_k\) and destination be \(N_d\). Distance between anchors, \(h_{A_1A_2}\) in (7) is known and (6) is simply the difference between current node and the neighbor. Therefore the VC of the branching node \((h_{N_dA_1}, h_{N_dA_2})\) can be found. Thus ‘\(a\)’, the number of hops to the backbone, can be found. If \(a\) and \((h_{N_dA_1}, h_{N_dA_2})\) are known, a packet can reach the backbone, i.e., node \((h_{N_dA_1}, h_{N_dA_2})\), and then it can be routed to the destination. Any junction node can identify whether \(N_d\) is in its branch or not. If \(N_d\) is a member of the branch, junction node will forward the packet to its neighbor on the branch. If \(N_d\) is not a member of its branch, junction node will forward it the neighbor closest to destination, excluding the neighbor on the branch, to \(N_d\). QED

A_1 and A_2 need not be at extremes, but all the branches should occur in between A_1 and A_2. Moreover, if the number of nodes in-between A_1 and A_2 is odd, and if there is a branch at the middle point, all the nodes in that branch will have zero ordinate (See proof of Property 1). This can be avoided by assuring hop distance between A_1 and A_2 to be even when anchor A_2 is selected. Furthermore, in a tree with branches on two sides (provided branches are off one main trunk), when there is a branch off a branch, it can be treated as a sub-constrained tree network, and hence need to add 'exactly one' additional anchor to ensure deterministic routing. Routing should be done in each constrained sub-tree based on corresponding anchors. This will allow us to get the number of anchors needed for any tree - i.e., any graph without cycles. A simple adjustment can be proposed if there are two branches at the same node. After generating VCS with respect to anchors A_1 and A_2, members of the backbone and branching nodes which have two branches can identify themselves. After that junction nodes with two branches can add one more bit to the coordinate of the nodes in one of the branch to indicate whether it’s the upper or lower branch. This newly added bit can be used to prevent identical coordinates in the upper and lower branches. Theorem 1 holds for the network after this small adjustment. Moreover, as observed, the same approach can be applied in a tree network with degree 3.

**IV. SIMULATION RESULTS: PARTITIONS IN DIRECTIONAL VIRTUAL SPACE**

Effectiveness of the directional domain is evaluated in five examples as shown in Fig. 5, representative of a variety of networks: (a) spiral shaped network with 421 nodes, (b) a grid based network with 100 randomly missing nodes (800 nodes) (c) a 496-node, circular shaped network with three physical voids/holes, (d) a network of 343 nodes mounted on walls of a building, and (e) odd shaped network with 550 nodes. Communication range of a node in all five networks is unity. MATLAB® 2009b was used for the computations.

Network partitioning based on sign of new ordinates:

As stated in property 1, the sign of each ordinate in transformed domain is used to identify different sectors of the network. In the networks shown in Fig. 5, randomly selected three anchors, A_1, A_2, and A_3 were placed. Then using the transformation given in (3), the new coordinates, \([f(h_{lA_1}, h_{lA_2}), f(h_{lA_1}, h_{lA_3}), f(h_{lA_2}, h_{lA_3})]\) were generated by each node locally. Based on the sign of each ordinate in the directional coordinate, i.e. positive/ negative, different sections were colored as shown in Fig. 5. Since 3 anchors’ ordinates are used for transformation, cardinality in transformed domain is \(3 (C_2^3)\). Hence the maximum possible sign combinations in the network is \(2^3\). As in Fig. 5, not all the sign combinations exist but existing combinations clearly partition the network.

**V. DIRECTIONAL VIRTUAL COORDINATE ROUTING - DVCR**

In this section we present a novel routing scheme based on transformed coordinates. In a network with \(M\) randomly selected anchors, a node can evaluate its transformed coordinates of cardinality \(P = C_2^M\) locally. Let the transformed current node coordinate be \(N_i = [n_{i1}, ..., n_{ij}, ..., n_{ip}]\) and that of the destination be \(N_d = [n_{dl}, ..., n_{dj}, ..., n_{dp}]\). \(L^2\) distance between \(N_i\) and \(N_d\) using transformed coordinates is,

\[
D_{N_iN_d} = \sqrt{\sum_j (n_{ij} - n_{dj})^2}; \quad j = 1: L \leq C_2^M
\]

A packet is forwarded to a neighbor using Greedy Forwarding (GF). To overcome the local minima, the minima node performs an approximate hop distance estimation from itself and also from neighbors as explained next, based on
The transformed domain has three ordinates generated by 
\[(A_1, A_2), (A_1, A_3), \text{ and } (A_2, A_3)\] pairs.

Assumption is that \(L^I\) in transformed domain (see (5)) is a good representation of the hop distance. Hence there exists a neighbor which has lower hop distance (estimated) to destination. For this estimation, two directional ordinates are sufficient. To overcome the local minima, the minima node performs an approximate hop distance estimation from itself and also from neighbors as explained next, based on (9),(10) and (11). Assumption is that \(L^I\) in transformed domain (see (5)) is a good representation of the hop distance. Hence there exists a neighbor which has lower hop distance (estimated) to destination. For this estimation, two directional ordinates are sufficient. Define the ordinate difference set \(\Delta_{A_p A_q}\) between current node \(N_i\) and all the neighbors \(N_k \in K\) as,

\[
\Delta_{A_p A_q} = \left| F_{A_p A_q}(N_i, N_k) \right| ; \ N_k \in K \tag{9}
\]

Since there are \(|K|\) number of neighbors, size of \(\Delta_{A_p A_q}\) is the same as that of \(|K|\). Consider \(\Delta_{A_1 A_2}\) with respect to \(f(A_1, A_2)\). Let \(\alpha_{12}\) and \(\beta_{12}\), be max \((\Delta_{A_1 A_2})\) and min \((\Delta_{A_1 A_2})\) respectively. Then the approximate hop distance between \(N_i\) and \(N_d\) is represented with respect to \(f(A_1, A_2)\),

\[
\alpha_{12} n + \beta_{12} m = \left| F_{A_1 A_2}(N_i, N_d) \right| \tag{10}
\]

Similarly \(\alpha_{34}\) and \(\beta_{34}\) is obtained following the same method with respect to another ordinate \(f(A_3, A_4)\). Another representation of the approximate hop distance between \(N_i\) and \(N_d\) with respect to \(f(A_3, A_4)\),

\[
\alpha_{34} n + \beta_{34} m = \left| F_{A_3 A_4}(N_i, N_d) \right| \tag{11}
\]

By solving (10) and (11) the approximate hop count from current node to destination, \(n + m\), can be estimated. This can be repeated for neighbors set \(K\) to get the hop distances from neighbors to destination. Packet will be greedily forwarded to the neighbor selected by this hop count approximation. Algorithm of the routing protocol can be summarized as in Fig. 6.

![Fig. 5. Partitions of (a) Spiral shaped network with 421 nodes, (b) A grid based network with 100 randomly missing nodes (c) A 496-node circular shaped network with three physical voids/holes, (d) A network of 343 nodes mounted on walls of a building (e) Odd shaped network with 550 nodes, based on the sign of the ordinates in transformed domain created by three randomly selected anchors \(A_1, A_2, A_3\). Transformed domain has three ordinates generated by \((A_1, A_2), (A_1, A_3), \text{ and } (A_2, A_3)\) pairs.](image)

**VI. PERFORMANCE OF DIRECTIONAL VIRTUAL COORDINATE ROUTING**

The performance of proposed Directional Virtual Coordinate Routing (DVCR) scheme is evaluated next for the five networks introduced in Fig. 5. Performance of DVCR is compared with two virtual coordinate-based routing schemes - Logical Coordinate Routing (LCR) [3]and Convex Subspace Routing (CSR) [4], and a geographic routing scheme - Greedy Perimeter Stateless Routing (GPSR) [7]. In LCR implementation, we assumed that the entire path traversed is available at each node so that backtracking can be perfectly performed avoiding any loops; i.e., the implemented case is the best case of LCR, and is not achievable in practice due to the cost involved in transmitting the required information. Time-To-Live (TTL) of the packet is set to 100 hops.

Average routability, average path length that packets traversed, and average energy consumption per successfully delivered packet are used as the performance metrics. Average routability evaluation considers all source-destination pairs; i.e., each node generated a set of \((N-1)\) messages, with one message for each of the remaining node as destination.

**Average routability** is given by

\[
R_{AVG,\%} = \frac{\text{Total # of packet that reached the destination}}{\text{Total number of packet generated}} \times 100\% \tag{12}
\]

**Average path length** is given by

\[
H_{AVG} = \frac{\text{Cumulative number of hops that each packet traversed}}{\text{Total number of packet generated}} \tag{13}
\]

Note that the average path length calculation includes the path lengths for unrouted messages as well. In order to have a,
fair estimation of the energy consumption, the average energy consumption per successfully routed packet is defined as,

\[ E_{AVG} = \frac{E_a \times H_{AVG} \times \text{Packet size}}{R_{AVG}} \]  

(14)

where \( E_a \) is average energy per byte. For all the routing schemes a fixed packet length of 12 bytes was assumed where 4 bytes each for destination ID, current node ID and VC/Physical coordinates. For the random anchor placement, the performance was averaged over five random anchor configurations.

Performance comparison in terms of \( R_{AVG} \%, H_{AVG} \) and \( E_{AVG} \) are as shown in Fig. 7. With random anchor placement, the proposed schemeDVCR outperforms CSR and LCR with \( H_{AVG} \) to shortest distance path length (\( H_s \)) ratio close to unity. Also it out performs GPSR in spiral and grid with missing nodes and achieves almost the same performance in rest of the networks. Even though DVCR achieves a higher \( R_{AVG} \% \), \( E_{AVG} \) (see Fig 7 (c)) is less than that of GPSR while very close to \( E_{AVG} \) of CSR and LCR. It is important to note that GPSR relies on accurate location information, achievable via expensive hardware such as GPS, or localization schemes subject to significant complexity and estimation errors. The importance of directionality information was illustrated by the performance of DVCR and anchor selection mechanism. More importantly, the required number of anchors is 5, which is a significantly lower number compared to the anchors used in other literature to achieve \( R_{AVG} \) over 95%. This simply indicates that DVCS alleviates the local minima problem. Moreover, proposed scheme does not require any costly anchor placement though proper anchor placement increases the routability of DVCR as observed [6]. The most fascinating fact is that no additional communication is required for the DVCS calculation, and it is a transformation performed locally at each node.

VII. CONCLUSIONS

A simple and novel transformation is proposed for virtual coordinates that for the first time allows VCS to recover directionality lost during the coordinate generation, thereby significantly increasing the effectiveness of virtual coordinate systems in routing. The issues such as identical coordinates and local minima, caused mainly due to the loss of directionality in virtual coordinate system generation, are mitigated in the directional domain. Directional space contains the inherent connectivity information, as well as a sense of directions with respect to the node arrangement. The proposed Directional Virtual Coordinate Routing (DVCR) outperforms Convex Subspace Routing (CSR) and Logical Coordinate Routing (LCR) with 38.9% and 44.6% average increment in average routability over five network types respectively, with an average path length to shortest path length ratio of 1.35 with only five randomly selected anchors, corresponding to less than 1.5% of nodes. Directionality provides new information hitherto not available in VCS, facilitating a new domain for designing a broad spectrum of WSN algorithms. Technique to identify ‘good’ anchors and boundary detection are among potential applications of Directional Virtual Coordinate Systems (DVCS).

REFERENCES