Impact of Region-Based Faults on the Connectivity of Wireless Networks in Log-normal Shadow Fading Model

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Abstract—The traditional studies on fault-tolerance in networks assume that the faults are random in nature, i.e., the probability of a node failing is independent of its location in the deployment area. However, this assumption is no longer valid if the faults are spatially correlated. In this paper we focus on the study of the impact of region-based faults on wireless networks. Most of the studies on connectivity of wireless networks assume a unit disk graph model, i.e., links exist between two nodes if they are within a circular transmission range of one another. However, the unit disk graph model does not capture wireless communication environment accurately. The log-normal shadow fading model for communication was introduced to overcome the limitations of the unit disk graph model. In this paper we investigate connectivity issues of wireless networks in a log-normal shadow fading environment where the faults are spatially correlated. If \( d_{\text{min}}(G) \) denotes the minimum node degree of the network, we provide the analytical expression and method for computing \( P(d_{\text{min}}(G) \geq 1) \) in a region-based fault scenario, where \( P(d_{\text{min}}(G) \geq 1) \) denotes the probability of the minimum node degree being at least 1. Through extensive simulation, we find \( P(\kappa(G) \geq 1) \), where \( \kappa(G) \) represents the connectivity of the graph \( G \) formed by the distribution of nodes on a 2D plane and examine the relationship between \( P(d_{\text{min}}(G) \geq 1) \) and \( P(\kappa(G) \geq 1) \).

I. INTRODUCTION

The connectivity of a graph is defined as the fewest number of nodes/links whose deletion disconnects the graph. Connectivity is traditionally considered as the primary metric for evaluation of the fault-tolerance capability of both wired and wireless networks. Most of the traditional studies on fault-tolerance in networks [1]–[7] assume that the faults are random in nature, i.e., the probability of a node or link failing is independent of its location in the deployment area. However, the assumption of random node failure is not valid in many scenarios. In fact in some networks, e.g., network in military environment or sensor network, faults may be spatially-correlated or confined in a region. Connectivity analysis in such localized fault scenario is considerably different than random fault or fault-free scenario. In this paper we study the impact of region-based faults on the connectivity of wireless networks in log-normal shadow fading model.

Most of the studies [1]–[7] in geometric wireless networks use unit disk graph model as communication model with the assumption that transmission range of the nodes in the network is uniform. But the unit disk graph model does not capture wireless communication environment accurately. This model assumes that two nodes located at points \( p \) and \( q \) on a 2D plane forms a link if distance between them does not exceed a threshold value \( r_t \) (transmission range).

However, measurement studies of wireless radio signal show that signal strength not only depends on the distance between the transmitter and the receiver, but also on various other factors like communication environments. The log-normal shadow fading model for communication discussed in [8] overcomes the limitations of the unit disk graph model. In this model [8], signal strength varies log-normally around the mean transmission power. This phenomenon of variation of signal strength is called shadowing effect. These variations are unavoidable in wireless communication and are caused by obstruction and irregularities in surrounding environment. The implication of shadowing effect is that all nodes within the transmission range of a transmitter are not guaranteed to have a communication link with the transmitter. On the other hand some nodes outside the transmission range may form a communication link with transmitter (see Fig 1). Log-normal Shadow Fading Model of radio propagation [8] captures this variation. Connectivity study [9], [10] of wireless networks considering shadow fading model as network communication model is a more appropriate approach in this regard.

Also most of the studies on connectivity of the wireless networks either considers fault-free scenario [1]–[6], [11] or assuming random faults [7]. In a wireless network not only the nodes can fail, the failure may not be random. In fact, there may be considerable amount of correlation between the failed nodes, particularly spatial. The spatially correlated faults is specially relevant in military networks, where an enemy bomb may destroy a large number of nodes confined in a particular area (region) or an enemy jammer jams a part of the network disabling the nodes in that part only.
Spatially correlated failures may be encountered in sensor networks as well where nodes of a region may get destroyed due to fire or increase in temperature. A wireless network formed by the nodes and a fault region is shown in Fig. 2. Recently, there has been considerable interest in study of spatially-correlated or region-based faults on networks, both wired and wireless [12]–[15]. The authors in [12] introduced the notion of region based faults and region based connectivity and showed how this new metric (region-based connectivity) can be utilized to design networks with the same level of robustness as the metric connectivity, but with significantly lesser amount of networking resources (e.g., transmission power of the nodes). The idea of region-based faults was further extended in [16], where faults were confined to be in multiple regions instead of just one. The results presented in this paper are based on the region-based fault models considered in [12], [16].

We provide a taxonomy of networks, faults and communication models and then identify the class where we focus our attention. To the best of our knowledge, this is the first study that presents connectivity related results in wireless networks in log-normal shadow fading model and with localized fault scenario. In our taxonomy, we have four parameters - network description, network state, fault model and communication model. By network description, we imply a topological description of the network, or a geometric description. By network state, we imply a fault-free or a faulty network. By fault model we imply a random fault or a spatially correlated fault and finally, by communication model we imply a unit disk graph or a log-normal shadow fading model. The taxonomy is presented in Table I. A summary of related work in this domain is presented in Table II.

Penrose in [1] proved that in wireless network with uniformly distributed nodes and with uniform transmission range of all the nodes, if node density is very high, then probability of a random geometric network becoming $k$-connected is equal to the probability of each node in the graph having minimum degree of $k$. In [6] authors verified that statement through simulation and analytical studies in circular deployment area. The implication of the above result is that in highly dense random geometric network with uniform transmission range for all nodes, the network will become connected as soon as the transmission range is large enough to achieve a network with no isolated nodes. So in this scenario probability of having no isolated nodes (i.e. $P(d_{min}(G) \geq 1)$ where $d_{min}(G)$ denotes the minimum node degree of the network) is a good estimation of probability of connectivity (i.e. $P(\kappa(G) \geq 1)$) where $\kappa(G)$ represents the connectivity of the graph $G$.

But all these results are valid in fault-free network scenario and with unit-disk communication model. In this paper we investigate whether probability of having no isolated nodes (i.e. $P(d_{min}(G) \geq 1)$) is still a good estimate of connectivity of wireless network with region-based fault scenario and log-normal shadow fading communication model.

The contributions of the paper are as follows:

1) We provide an analytical expression for computing $P(d_{min}(G) \geq 1)$ where node distribution is uniform and faults are region-based,

2) Through extensive simulation we find $P(\kappa(G) \geq 1)$ and compare it with $P(d_{min}(G) \geq 1)$ and

3) Study the impact of region based faults on the connectivity of the wireless network under unit disk graph and log-normal shadow fading model.

The rest of the paper is organized as follows: in section II we describe the node distribution, wireless channel model and region-based fault model used in this paper, in section III we give the analytical expression for computing $P(d_{min}(G) \geq 1)$, in section IV we describe the simulation setup and discuss the results and conclude the paper in section V.

II. SYSTEM MODELS

In this section we discuss about the node distribution model, wireless channel model and fault model considered in this paper.

A. Node distribution

We assume that nodes are distributed randomly and uniformly on an infinitely large two dimensional plane with a constant node density $\rho$. We consider a circular subarea $A$ of radius $r_d$ containing $n$ nodes from this infinite plane such that $\rho = n/A$. In limiting case for large $n$, the uniform distribution of $n$ nodes in an area $A$ is well approximated by Poisson point process.

B. Signal propagation model

In wireless radio communication, received signal power decreases logarithimically as the distance between transmitter
and receiver increases. This phenomenon is called path loss. If transmitter $u$ transmits a signal at power $P_t(u)$ and receiver $v$ receives the signal at power $P_r(v)$, then

$$\frac{P_r(v)}{P_t(u)} \propto \frac{1}{d^\alpha}$$

where $d$ is the separation distance between transmitter and receiver and $\alpha$ is the path loss exponent. $\alpha$ depends on the propagation environment and terrain structure and can vary from 2 in free space to 6 in dense urban environment [8].

A wireless link is established between $u$ and $v$ if the received power $P_r(v)$ at $v$ is greater than a threshold power $P_{th}(v)$. The most commonly used signal propagation model in wireless ad-hoc networks literature assumes that maximum transmission range of a transmitter is equal to the distance $R_0$ for which $P_r(v) = P_{th}(v)$ and the received signal power is uniform at any receiver $v$ at a distance $r$, $0 \leq r \leq R_0$ [6]. So the probability of having a link between two nodes $u$ and $v$ at a distance $r$ is a normalized function of distance $r/R_0$ and is given by $p(r) = 1$ if $r \leq 1$ and $p(r) = 0$ otherwise. The graph formed by this model is called unit-disk graph.

Wireless network graph construction based on the above model may be highly inaccurate as it does not consider the variation of the signal strength due to presence of different obstructions in its path. Measurement studies of wireless signal power show that the mean power of signal at different distances from the transmitter varies log-normally (with standard deviation $\sigma$) around the area mean power [8]. The value of $\sigma$ varies from 0 to as large as $12 \, \text{dB}$ [8]. This model is known as log-normal shadowing model and is very close to real radio propagation model. The link probability between two nodes at distance $r$ (as shown in [9] [10]) is given by:

$$p(r) = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{10\alpha}{\sqrt{2}\sigma} \log_{10} \frac{r}{R_0} \right) dB.$$  

where the normalization factor $R_0$ is the maximum distance at which a link can be formed in absence of shadowing effect, i.e. at $\sigma = 0$ and erf(.) denotes the error function. The implication of this link probability is that some nodes at a normalized distance $\hat{r} < 1$ from the transmitter may not have a link while some nodes at a normalized distance $\hat{r} > 1$ may form a link with transmitter. Here we assume that all transmitters transmit at equal power, i.e., $R_0$ is the same for all transmitters and all links formed in the network are undirected.

### C. Fault Model

In this paper we consider that faults are localized and confined in a single region. We consider fault region to be circular of radius of $r_f$. It is assumed that all the nodes in this fault region are faulty and do not participate in the network connectivity. Since we test the connectivity of the nodes only within a circular subarea $A$ of radius $r_d$, any region fault center occurring within a distance of $(r_d + r_f + R)$ from the center of $A$, will either remove some of the nodes in area $A$ or from the neighborhood of the nodes in area $A$. We call this Potential Fault Location area (PFL). Fault centers occurring outside this area will have no affect on the nodes of $A$ or on their neighbors.

### III. PROBABILITY OF HAVING NO ISOLATED NODES IN THE GRAPH

Let $X$ and $Y$ be the random variables denoting the location of a node in the area $A$, and the location of the center of a Fault region (FR), respectively. In the node distribution model we considered nodes are uniformly distributed over $A$ with area $A = \pi r_d^2$. For these uniformly distributed nodes over finite area $A$, the probability density function (pdf) is given by:

$$f_X(x) = \begin{cases} \frac{1}{\pi r_d^2} & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}.$$

The node density of the area $A$ is $\rho = n/A$. The distribution of $Y$ is uniform over PFL with area $A_f = \pi(r_d + r_f + R)^2$.
and probability density function (pdf):
\[
h_Y(y) = \begin{cases} 
\frac{1}{\pi}, & \text{if } y \in PFL, \\
0, & \text{otherwise} 
\end{cases}
\]

In presence of a random fault region (FR), a node within a circle of \(\pi r_f^2\) from the fault center will be considered non-operational or dead. Any node outside FR will be considered operational or live. Only live nodes will be considered to be in the network system. Now we define another random variable \(Z\) which denotes location of a live node in the system after fault. Note that random variable \(Z\) can only take a subset of values that random variable \(X\) can take. So the probability density function of \(Z\) over \(A\) is a sub-probability density function [21] and consequently the probability of \(Z\) over \(A\) will not sum up to 1. Given a fault at location \(y\), region of overlap of FR with \(A\) is denoted by \(A'(y)\). The conditional sub-probability density function of \(Z\) given \(y\) is given by:
\[
g_{Z|Y}(z | y) = \begin{cases} 
\frac{1}{\pi}, & \text{if } z \in A \setminus A'(y) \\
0, & \text{otherwise} 
\end{cases}
\]

Then sub-probability density function of \(Z\) is given by
\[
g_Z(z) = \int_{A_f} g_{Z|Y}(z | y) dy = \int_{A_f} g_{Z|Y}(z | y) h(y) dy 
= \frac{1}{\pi} \int_{A_f} \int_{\|y\| \leq rd + rf + R} \|z - y\| > rf 
= \frac{(A_f - \pi r_f^2)}{A_f} = \bar{p}, \text{if } z \in A 
= 0, \text{otherwise}
\]

where \(\|\cdot\|\) denotes the Euclidean distance of a location from the center of \(A\).

Here we are considering a circular subarea \(A\) out of infinite plane. The nodes in area \(A\) are said to be connected if there exists at least one path between each pair of nodes in \(A\). It should be noted that we are not considering border affect of this finite area \(A\). So expected degree of any node in the border region of \(A\) is same as any node in the central region. The nodes outside the region \(A\) can act as relay node and help in connecting two nodes inside \(A\). But due to any region fault in the area \(A_f\) the expected degree of the nodes in \(A\) will decrease more than the fault free scenario. As stated earlier, \(P(\text{no isolated node}) \geq P(\text{connectivity})\) for any graph. The goal of this paper is to investigate how tight is this upper bound of \(P(\text{connectivity})\) in region-based fault scenario with log-normal shadow model.

### A. Probability of having no isolated nodes in the graph

Let us consider a node at location \(x\). The probability of node at \(x\) having a link with another node at a distance \(r\) is given in equation(1). Degree \(D\) of a node is the number of neighbors of that node. Let the expected degree of the node at \(x\) is \(E(D | x) = \mu(x)\). For Poisson point process the probability of a node having degree \(d\) is
\[
P(D = d | x) \approx \frac{\mu(x)^d}{d!} e^{-\mu(x)}
\]

Then the probability that the given node is isolated is
\[
P(\text{node iso}|x) = P(D = 0 | x) \approx e^{-\mu(x)}
\]

Thus the probability of having any isolated node is given by
\[
P(\text{node iso}) = \int_A P(\text{node iso}|x) g_Z(x) dx
\]

Probability that none of the \(n\) nodes in subarea \(A\) are isolated is
\[
P(\text{no node iso}) = (1 - P(\text{node iso}))^n
\]

Again applying Poisson approximation,
\[
P(\text{no node iso}) \approx \exp \left( - n \int_A e^{-\mu(x)} g_Z(x) dx \right)
\]

In section III-B, we show the analytical expression for calculating of the value of expected degree at any possible node location \(x \in A\).

### B. Expected degree of a node in \(A\) due to any random region fault

In absence of a fault the expected value of \(D\) of a node \(X\) located at \(x\) can be computed by integrating \(pp(r)\) over the entire system plane and is given as [9]:
\[
E_1 = \rho \int_0^{2\pi} \int_0^{\infty} p(r) r dr d\theta
\]

In presence of a fault with center \(Y\) located at \(y\), the expected degree of node \(X\) will decrease. Let \(d(x,y)\) be the distance between \(X\) and fault center \(Y\). Then the loss of neighbors by node \(X\) at a distance \(d(x,y) \geq rf\) from fault center \(Y\) can be approximately given as a function of distance \(d(x,y)\)
\[
E_2(d(x,y)) = \rho \int_0^{2\pi} \int_{d(x,y) - rf}^{d(x,y) + rf} p(r) r dr d\theta
\]

We get the above expression by considering that the effect of fault will be symmetrical around the node in the annular area of width \(2rf\) at a distance between \(d(x,y) - rf\) and \(d(x,y) + rf\) from the node \(X\) (see Fig(3)). So the net loss of neighbors by a single fault at distance \(d(x,y)\) will be proportional to the value of the net loss of neighbors in this whole annular area. The multiplicative factor \(\frac{rf}{d(x,y)}\) is the ratio of area of a single fault to the area of the whole annular region. Then expected degree of a node given the node location and fault location can be stated as
\[
E(D|x,y) = \begin{cases} 
E_1 - E_2(d(x,y)), & \text{if } d(x,y) \geq rf \\
0, & \text{if } d(x,y) < rf
\end{cases}
\]

Expected degree of a node for any location of fault center in PFL can be given as
\[
E(D|x) = \int_{A_f} E(D|X,Y) h(y) dy
\]
Using (14) in expression (10) we get the probability of no isolated node in the graph.

X and Y being random variables and independent of each other we can employ Monte-Carlo integration [22] to numerically evaluate \( P(\text{no node iso}) \) in equation (10). We generate \( N \) random points for variable \( X \) uniformly in area \( A \) and for each \( X \) we generate \( N \) random fault center for variable \( Y \) uniformly within the fault region \( A_f \). Then according to Monte-Carlo integration method if \( N \) is very large :

\[
E(D|x_i) = \frac{1}{N} \sum_{i=1}^{N} E(D|x_i, y_i)
\]

and approximate value of \( P(\text{no node iso}) \) is given by:

\[
P(\text{no node iso}) = \exp \left( -n \pi r_f^2 \frac{N}{2} \sum_{i} \exp(-E(D|x_i)) \right)
\]

where \( x_i, y_i \) are \( N \) points \( i = 1, \ldots, N \) such that \( x_i \in A \) and \( y_i \in A_f \). We start the evaluation with \( N = 1000 \) and increase \( N \) thereafter until equation (10) converges.

IV. SIMULATION RESULTS AND DISCUSSION

We perform extensive simulations in order to study the relation between the probability of the graph being connected and the probability of having no isolated node in presence of a region based faults. We also study the impact of \( r_f \) (fault radius) on the probability of the graph being connected for different values of node density and different link formation models. We consider a circular simulation area \( B \) containing the circular observation subarea \( A \) in the middle. We test the probability of the graph being connected and the probability of no isolated nodes, only for the nodes inside area \( A \). Let radius of area \( A \) is \( r_d \). In order to minimize border effect of area \( A \) we consider radius of \( B \) twice as the radius of \( A \).

First, \( n_B = \rho B \) nodes are uniformly and randomly distributed on the deployment area \( B \) to ensure the density of area \( A \) remains \( \rho \). Then a circular fault area of radius \( r_f \) is generated with its center placed randomly over the circular area within distance \( (r_d + r_f + R) \) from the center of the deployment area \( A \). All the nodes (say \( n' \)), which lie in fault region are removed and graph \( G(V', E) \) is formed with the remaining \( (n_B - n') \) nodes. The edges are formed between nodes following the link probability given in equation (1). Then we check whether area \( A \) has any isolated nodes or there exists a path between any two pairs of nodes in \( A \). We used the java library Jgrapht to construct the graphs by this process. In all these experiments area \( A \) is taken as \( 2.5 \times 10^3 \) m². We study the impact of two parameters on the probability of the graph being connected: (i) the density of nodes \( n \) in \( A \) and (ii) the fault region radius \( r_f \) keeping other factors \( \alpha, \sigma \) and \( R_0 \) (see equation (1)) as constant. For a particular \( r_f \) and \( \rho \), the same experiment is repeated for \( k_1 \) number of different node locations and \( k_2 \) number of times for different fault locations, and finally, the percentage of connected topologies and the percentage of topologies with no isolated nodes are computed. If \( k_1 k_2 \) is large enough, this experiment gives us fairly good estimate of probability of the graph being connected \( P(\text{conn}) \) and probability of no isolated nodes \( P(\text{no iso node}) \) in area \( A \). In our experiments, the value of \( r_f \) is varied from 0 to 80 at steps of 20 and the value of \( \rho \) is varied from 0.0001 to 0.0008 at steps of 0.00004 with \( k_1 k_2 \) taken as 10,000. In all the experiments we consider value of \( \alpha = 3, \sigma = 4, R_0 = 43.34 \text{ m} \) and \( R = 120 \text{ m} \).

In the first set of experiments, we compared the probability of no isolated nodes in the graph with probability of the graph being connected due to a region based fault (in a random location) in the graph for different values of \( \rho \). We varied \( \rho \) from 0.0001 to 0.0008 for \( r_f = 0 \) and 80. Probability of the graph being connected decreases when fault radius is 80 \( (r_f = 80) \) as opposed to \( r_f = 0 \). Results are shown in Fig. 4. It may be noted that for the same \( r_f \) probability of the graph being connected is always less than probability of no isolated nodes in the graph for low value of \( \rho \), but when \( \rho \) is relatively high both these probabilities merges and eventually becomes 1. This means that even with region based faults, if nodes are distributed uniformly, probability of having no isolated nodes in the graph is a good estimate of probability of connectivity of the graph.

In the second set of experiments, we studied the impact of increase of fault radius on the probability of the graph being connected. The value of \( r_f \) is varied from 0 to 80 for \( \rho = 0.00074 \) and \( \rho = 0.0005 \). At node density \( \rho = 0.00074 \) the the graph is connected with probability 1 in fault free scenario (i.e. \( r_f = 0 \)). But with increase in fault radius the probability of connectivity decreases. Results for change of graph connectivity with fault radius is shown in Fig. 5. An interesting observation is that decrease in probability of connectivity with the increase in fault radius \( r_f \) is insignificant when node density is high.

In third set of experiments, we studied the effect of increase in fault radius on probability of the graph being connected \( P(\text{conn}) \) in two different communication link models: i) Unit Disk graph (UDG) model and ii) Log-normal Shadow fading model. In order to make these two models comparable, we kept the expected degree of a node equal in no fault scenario for both these models. The transmission range \( r_g \) of a node in UDG model is required to be set to 6m to achieve the same. The value of \( r_f \) is varied from 0 to 80 keeping the node-density.
Simulation results show that even in case of region-fault at high node density $P(d_{\text{min}}(G) \geq 1)$, is a good estimate of $P(\kappa(G) \geq 1)$, where $d_{\text{min}}(G)$ represents the minimum node degree and $\kappa(G)$ represents the connectivity of the graph $G$. Further study on this work can be done by relaxing the strict requirement of all nodes to be connected and focusing on the impact of region-based faults on the size and number of connected components in a wireless network disconnected due to fault.

REFERENCES