A 2D Numerical Model For Simulation of Bend-Flow And Pollutant Diffusions With Effects of Streamline Curvature

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Abstract— The purpose of this paper is to present a 2D depth-averaged model that includes the effects of streamline curvature under orthogonal curvilinear coordinates for simulating flows and pollutant diffusions in channel bends. The proposed model uses an orthogonal curvilinear coordinate system efficiently and accurately to simulate the flow field with irregular boundaries. As for the numerical solution procedure, The SIMPLEC solution procedure has been used for the transformed governing equations in the transformed domain. Practical application of the model is illustrated by an example, and a fair agreement between the values measured and computed demonstrates the model’s capabilities.

Keywords-orthogonal curvilinear coordinate; numerical simulation flow; effects of streamline curvature

I. INTRODUCTION

Flow characteristics in channel bends are much more complicated than those in straight reaches [1-4]. The occurrence of the secondary flow is one of the dominant features of flows in bends. Secondary flow results from the imbalance between the transverse water surface gradient force and centrifugal force over the depth due to the vertical variation of the primary flow velocity, which brings about much more difficulties in simulating bend-flows. Many studies have been conducted on flows in bends [2-4]. The 3D numerical models have been developed to simulate the complicated spiral flow motion in the bend. However, the 2D depth-averaged models are often adopted in practice by hydraulic engineers because of their easy implementation and application. In order to combine the simplicity (i.e., easily adopted into other programs or models), generality (suitable for different geometries), physical rationale, and efficiency (less computing time), the present study applies the curvature correction method by Launder et al. and Sharma [4] in the two-equation $k-\varepsilon$ turbulence model under orthogonal curvilinear coordinates. Thus, a 2D depth-averaged model for prediction of flows and pollutant diffusions with effects of streamline curvature under orthogonal curvilinear coordinates has been established and will be widely used in the prediction of complex flows in hydraulic engineering.

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II. MATHEMATICAL MODEL

A. Governing equations in transformed plane

The unsteady 2D depth-averaged flow and pollutant transportation governing equations can be written in orthogonal curvilinear coordinates in the following general form as [2-4]

$$
\frac{\partial \left( H \phi \right)}{\partial t} + \frac{1}{h_i h_j} \frac{\partial}{\partial \xi} \left( H U \phi_i \right) + \frac{1}{h_i h_j} \frac{\partial}{\partial \eta} \left( H V \phi_\eta \right) = \frac{1}{h_i h_j} \frac{\partial}{\partial \xi} \left( \gamma_\phi H h_i \phi_i \right) + \frac{1}{h_i h_j} \frac{\partial}{\partial \eta} \left( \gamma_\phi H h_\eta \phi_\eta \right) + S_\phi
$$

(1)

where $U$ and $V$ are the velocity components in the $\xi$ and $\eta$ directions, respectively; $H$ water depth; $\phi$ represents any dependent variable of interest; $S_\phi$ is the source term in the $\xi-\eta$ coordinate system; $\gamma_\phi$ is the effective viscosity. If the conditions $\phi = 1, S_\phi = 0$ and $\gamma_\phi = 0$ are satisfied, Eq. (1) represents the continuity equation. $S_\phi$ and $\gamma_\phi$ for each of the transport quantities ($U$, $V$, $C$, $k$, $\varepsilon$ ) are as follows, respectively

$\phi = U$ ( $\xi$ momentum equation)

$$
S_\nu = g_\nu \frac{gU \sqrt{U^2 + V^2}}{C_f} \frac{gH \partial h_i}{h_i \partial \xi} \frac{HVU \partial h_\eta}{h_\eta \partial \eta} + \frac{H \nu \partial h_i}{h_i \partial \xi} \partial \left( \phi \right) + \frac{1}{h_i h_\eta} \frac{\partial}{\partial \xi} \left( H \gamma_\nu \frac{\partial V}{h_i \partial \xi} \frac{\partial h_i}{h_i \partial \xi} + \frac{2V \partial h_i}{h_\eta \partial \eta} \right) + \frac{1}{h_i h_\eta} \frac{\partial}{\partial \eta} \left( H \gamma_\nu \frac{\partial V}{h_i \partial \xi} \frac{\partial h_\eta}{h_\eta \partial \eta} + \frac{\partial U}{h_i \partial \xi} \partial \phi \right) \partial \xi \partial \eta (2a)
$$

$$
+ \frac{2H \gamma_\nu \frac{\partial V}{h_i \partial \xi} \frac{\partial U}{h_i \partial \xi} \partial \phi \partial h_i \partial \eta} + \frac{2H \gamma_\nu \frac{\partial V}{h_i \partial \xi} \frac{\partial U}{h_i \partial \xi} \partial \phi \partial h_i \partial \eta} + \frac{2H \gamma_\nu \frac{\partial V}{h_i \partial \xi} \frac{\partial U}{h_i \partial \xi} \partial \phi \partial h_i \partial \eta}
$$
\[ \gamma_U = \gamma + \gamma_i \]  
\[ \varphi = V(\eta \text{ momentum equation}) \]
\[ S_i = g_s \frac{-gV\sqrt{U^2 + V^2}}{C_j} - gH \frac{\partial h}{\partial \eta} - HUV \frac{\partial h_y}{\partial \xi} \]
\[ + \frac{HU^2}{h_y h_z} \frac{\partial h_z}{\partial \eta} + \frac{1}{h_y h_z} \frac{\partial}{\partial \eta} \left[ H \gamma \left( \frac{2V}{h_y} \frac{\partial h_y}{\partial \eta} + \frac{2V}{h_z} \frac{\partial h_z}{\partial \xi} \right) \right] \]
\[ + \frac{1}{h_y h_z} \frac{\partial}{\partial \xi} \left[ H \gamma \left( \frac{2U}{h_y} \frac{\partial h_y}{\partial \eta} + \frac{2U}{h_z} \frac{\partial h_z}{\partial \xi} \right) \right] \]
\[ + \frac{2H}{h_y h_z} \left( \frac{\partial U}{\partial \xi} + \frac{\partial U}{\partial \eta} \right) \left( \frac{\partial h_z}{\partial \xi} + \frac{\partial h_z}{\partial \eta} \right) \]
\[ \gamma_i = \gamma + \gamma_{\text{t}} \]  
\[ \varphi = C \left( C \text{ transportation equation} \right) \]
\[ \varphi = k \left( k \text{ transportation equation} \right) \]
\[ S_i = H \left( P_i + P_{\text{ir}} - \varepsilon \right) , \gamma_i = \gamma + \frac{\gamma_{\text{t}}}{\sigma_{\gamma}} \]  
\[ \varphi = \varepsilon \left( \varepsilon \text{ transportation equation} \right) \]
\[ S_r = H \left( C_{\nu} P_i \frac{\varepsilon}{k} - C_{\nu} \frac{\varepsilon^2}{k} + P_{\text{ir}} \right) , \gamma_r = \gamma + \frac{\gamma_{\text{t}}}{\sigma_{\gamma}} \]

where \( C_{\nu} \) is the modelling constant in the turbulent viscosity formulation, and has the value of 0.09, as shown in Eq.(9).

\[ P_{\text{ir}} = \frac{U^3}{H} \frac{1}{\sqrt{C_f}} , \quad P_{\text{ir}} = 1.8 C_{\nu} U^4 \sqrt{C_f} / H^2 C_f^{3/4} \]

As stated above, this is the standard two-equation \( k - \varepsilon \) model in an orthogonal curvilinear coordinate system. The \( k - \varepsilon \) model can not predict complex turbulent flows. A few suggestions for modifying the \( k - \varepsilon \) model have been published, which aimed at the \( \varepsilon \) -equation. Launer and Sharma\[4\] selected a special form of the gradient Richardson number, involving a typical turbulence frequency to characterize the influence of the curvature:

\[ R_s = \frac{k^2 \hat{U} \rho(U \hat{j})}{r^2} \]

where \( \hat{U} \) is the velocity in the streamline direction, \( r \) is the radius of curvature of the streamline.

For the modified destruction term in the \( \varepsilon \) equation, they proposed

\[ -C_{\nu} \frac{1}{k} (1 - C_s R_s) (e^2 / k) \]

and assigned a value of 0.20 to the constant \( C_s \). This modification gave good predications for data on a curved surface\[4\]. This basic idea has been adopted in this paper.

### B. Discretization scheme

A staggered grid system is used where the control volumes for \( U \) and \( V \) are Centred on the faces of the control volumes for the scalar variables, \( h \) and \( k \) and \( \varepsilon \), the pressure nodes are located at the centre of the continuity control volume, which is known to overcome the wiggles or the wiggles or the checkboard patterns of the pressure\[1,3,4\]. The SIMPLEC solution procedure has been used for the transformed governing equations in the transformed domain. In the numerical solution of unsteady flow, the riverbed may be exposed to the water surface. In order to deal with the changeable computational region, the technique of moving boundary (or the method of condensation) is used in the computation\[6\].

### C. Boundary conditions

a) Inlet plane: \( U, V, k \) and \( \varepsilon \) are specified.

b) Exit plane: \( U = 0, V = 0, k = 0 \) and \( \varepsilon = 0 \) are satisfied.

c) Body surface: Wall-functions are used for turbulent kinetic energy \( k \) and its rate of dissipation \( \varepsilon \); and no slipping condition is used for velocity.
Chang (1971) performed a series of experiments in rectangular meandering channels measuring with both flow and pollutant concentration \(^5\). One of his channels with a single meander, which consists of two reversing 90° channel bends, is used to test the reliability of the proposed model to predict the features of a reversing flow. The channel had a smooth bed and uniform 90° bends in alternating directions interconnected by a straight reach. The channel was 2.34 m wide, the radius of the channel centerline was \(R_c = 8.53\) m, the interconnected straight part was 4.27 m, and the straight inlet and outlet reaches were 2.13 m each. Fig. 1 shows the plan view of this channel. The water depth was 0.115 m and the velocity was 0.366 m/s. The measurements were carried out along the second bend of the channel.

The curvilinear coordinate grid for the computed region is shown in Fig. 2. The region concerned is divided into 100×10 elements. In the computation, the time step \(\Delta t\) is about 3s; the Courant number is less than 5; the Coriolis parameter \(f\) is neglected, owing to the experimental flow region being computed is small; the roughness \(n\) is about 0.012. The computation converges at 400 time steps.

The results shown in Fig. 3 compare the experimental and numerically predicted depth averaged longitudinal velocity distributions across the flume at different cross sections. It can be observed from Fig. 3 that the proposed models simulate the longitudinal velocity distributions across the flume at different locations very well.

The results shown in Fig. 4 compare the experimental and numerically predicted depth averaged pollutant concentration distributions across the flume at different cross sections. It can be observed from Fig. 4 that the proposed models simulate the pollutant concentration distributions across the flume at different locations very well.

**Fig. 1** Sketch of meandering channel in laboratory (unit: m)

**Fig. 2** Computational grid of model

**Fig. 3** Comparisons of velocity for sections 1 and 2

**Fig. 4** Comparisons of pollutant concentration for sections 1 and 2
Advantages of this proposed mathematical model are that the effects of streamline curvature to properties of turbulence flow are included and the orthogonal curvilinear coordinate grids are used to deal with the complicated computational region boundaries in the numerical simulation of complex turbulence flow. The computed examples show that this proposed model has good stability, convergence and accuracy; and this mathematical model can be used to accurately predict tidal flows and will find more applications in hydraulic engineering.

REFERENCES


