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APPLICATION OF MULTICHANNEL 2-D LINEAR PREDICTION TO COLOR IMAGE CODING

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ABSTRACT

Linear predictive coding is an efficient method for image data compression [1]. This paper describes multichannel 2-D linear prediction and its application to predictive coding of color images. Two algorithms are compared initially. In the first the whole frame of the image is divided into subframes and the predictor coefficients are computed separately for each. In the second, predictor coefficients are obtained for the whole frame of the image. Both of these methods have the disadvantage that the linear prediction coefficients matrices must be computed in real time, and transmitted to the receiver as side information. This significantly increases complexity of the coding system. As an alternative we consider using a fixed set of prediction matrices, i.e., one that does not depend on the specific image being coded. In this way both receiver and transmitter has the linear prediction matrices and no side information has to be transmitted. Such prediction matrices can be generated by various averaging methods discussed in the paper. We compare the results of this coding to that resulting from the previous two methods.

1. INTRODUCTION

Image coding is a fairly new subject which has been practiced over the last 15 years. Many efforts have been made toward the implementation of digital image coding. The main purpose of coding therefore, is to decrease the transmitted bit rate, but to maintain a certain acceptable level of fidelity.

In this paper multichannel 2-D AR models [2,3] are applied to the coding of color images. We are concerned here with predictive coding, i.e., linear prediction followed by quantization of the prediction error.

2. PREDICTIVE CODING

Predictive coding (also known as differential pulse code modulation (DPCM)) involves 2-D linear prediction applied to the image and transmission of the quantized error residuals. The prediction of the sample to be encoded in predictive coding is generated from the previously coded samples [1]. For our case we will be dealing with the three (red, green, and blue) components of a color image so the prediction process is multichannel 2-D linear prediction. The vector-valued error $e(n_1,n_2)$ that is the difference between the predicted value and the actual value of the color element $x(n_1,n_2)$ is quantized into one of a number of given discrete levels. These levels are assigned to codewords which may be fixed or variable depending on the type of coding used. The encoded information is then transmitted through the channel to the receiver. Fig. 1 shows the general predictive coding system. Fig. 1(a) shows the transmitter part of the system while Fig. 1(b) shows the receiver part.

a. Linear Prediction of Color Images

The most important part of the predictive coding system is the linear predictor. Linear prediction for multichannel 2-D random processes was developed and studied in Refs. [2,3]. Color images as we mentioned before, can be considered as 3-channel 2-D random processes. If $x(n_1,n_2)$ are a set of color picture elements, a linear prediction for the $(n_1,n_2)$th color element using previous elements can be written as:

$$
\hat{x}(n_1,n_2) = - \sum_{(i,j) \in R} A_{i,j} x(n_1-i,n_2-j)
$$

where $R$ is the region of support for the prediction filter and $\hat{x}$ represents the prediction based
on the true image values. The filter coefficients \( A_{ni} \) can be obtained by minimizing the mean-square prediction error and solving a set of Normal equations. Then the prediction error of the color image
\[
e(n_1, n_2) = x(n_1, n_2) - \hat{x}(n_1, n_2)
\] (2)
is quantized and transmitted. In the above analysis we neglect the effect of quantization in the predictive coding system at the transmitter.

b. Image Quantization

The signal values to be transmitted (i.e. the prediction error values) are quantized. Two types of quantization can be used namely scalar quantization and vector quantization. Our concern here is with scalar quantization.

Signal-to-noise ratio (SNR) is used to measure the overall performance of a predictive coder.
\[
SNR = 10 \log \left( \frac{\text{Covar}(x)}{\text{Covar}(x - \hat{x})} \right)^{1/2}
\] (3)
where Covar [ . ] represents the covariance matrix for the quantity.

c. Side Information

Great system complexity results from the need to compute the linear prediction matrices and transmit them as side information.

In this paper we considered a possible way to overcome the disadvantage of the real time computation and transmission of side information. The idea is to select a fixed set of predictor parameters and apply it directly to the data. In this case no real time computations are required, and there is no need to transmit the side information. The criteria for selecting the fixed predictor parameters will be explained in detail in the next section.

3. EXPERIMENTAL RESULTS

The original color pictures used in this paper are 128x128 pixels in size. A variety of pictures were used in this experiment (only two of them are shown in Fig. 2).

The linear predictive models developed in [2,3] were applied to the coding of these color images. Two different procedures were used initially. In the first procedure the total frame of the image is divided into four subframes with 64x64 pixels each. The predictor coefficients and the error covariance matrix are obtained separately for each subframe by solving a set of Normal equations. The different linear predictive filters derived are then applied to the different subframes. The residual, error covariance matrix, and the set of filter coefficients for each subframe are coded and transmitted.

In the second procedure, the whole frame of the image is taken, and the Normal equations are solved to get only one set of predictor matrices. This single error covariance matrix and the set of filter coefficients for the whole frame are coded and transmitted along with the residuals as in the first procedure.

Note that in both of these procedures the predictor matrices must be calculated in real time. In the first case the real time calculation is done for each subframe and in the second case it is done for the whole frame. These real time computations and the need to send results as side information, greatly increase the complexity of the coding system.

As an alternative we considered two new procedures which overcome the disadvantages of the previous ones. In the first of these new procedures a fixed set of predictor matrices is used. We refer to this in the experiments as the third procedure. The fixed set of predictor matrices is chosen on the basis of the following criteria. The filter parameters for multichannel 2-D linear prediction are determined for each of the different images separately. We used 8 different pictures not including the jelly beans of Fig. 2b. A new fixed set of parameters is obtained by averaging the parameters corresponding to the different images. Then this new set is applied in our process.

In the second new procedure a fixed autocorrelation matrix is generated by averaging a group of autocorrelation matrices estimated from the same data, i.e. all images except the jelly beans. The predictor parameters are developed by solving a set of Normal equations involving the fixed average autocorrelation matrix. As in the previous case the residual is quantized and sent to the receiver and there is no need to send any side information. We refer to this in the experiments as the fourth procedure.

Different experiments are done to show the performance of the third and forth (new) procedures by comparing their results with the results of the first and second (original) procedures. Although our experiments here used
scalar quantization, the ideas can also be applied to the case of vector quantization.

In this experiment we used a uniform two-level quantizer with step size $\Delta$ defined by

$$\Delta = \gamma \sigma^2$$  \hspace{1cm} (4)

where $\gamma$ is a factor controlling the dynamic range of the quantizer (this also has an effect on the granular noise and the sharpness of the edges) and $\sigma^2$ represents the prediction error variance. The behavior of the quantizer is varied by varying the step size $\Delta$. A second order linear predictive filter and a 2-level quantizer with step size $\gamma = 0.7071$ were used in this experiment. The value of $\gamma$ was taken from the Max table for the Laplacian distribution.

Fig. 3 shows the results of three procedures where (a) is the result of using the first procedure (dividing total frame into four subframes), (b) is the result of using the second procedure (using the whole frame), (c) shows the simulation results using the third procedure (applying the averaged set of filter coefficients). The result of the fourth procedure is approximately the same as the third one.

When the reconstructed coded pictures are compared with the original picture shown in Fig. 2a good results are obtained. Note that the alternative procedures (three and four) produce good coded results which are quite similar to the results of the first and second procedures.

A quantized error picture is shown in Fig. 4. This is generated by quantizing the difference between the original picture and the reconstructed one. Most of the essential information is retained in the quantized error picture.

Table 1 shows the SNR values obtained by applying the four different procedures, for the red, green, and blue components of the color picture (diagonal elements of the SNR matrix). The SNR of the second procedure is given as the average of the values obtained for the four different subframes. It is clear from the table that the SNR values for the various procedures are close to each other. The highest SNR values are obtained for the first procedure but there is only about a 1 dB improvement.

To complete this experiment and to check the performance of the new procedures on a picture that was not in the set used to derive average parameters, the jelly bean picture (Fig. 2b) was tested. Fig. 5 shows the quality of the picture obtained by applying the third procedure. The quality of the reconstructed picture for this procedure is again similar to that for the first and second procedures (not shown). Table 2 shows the SNR results. The highest SNR appeared for the picture coded by applying the first procedure. The second, third, and fourth procedures yielded SNR results that are very close.

### Table 1 SNR for Coded Images

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Red</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Procedure</td>
<td>22.90</td>
<td>19.98</td>
<td>17.82</td>
</tr>
<tr>
<td>Second Procedure</td>
<td>22.30</td>
<td>19.93</td>
<td>17.14</td>
</tr>
<tr>
<td>Third Procedure</td>
<td>21.92</td>
<td>19.73</td>
<td>16.45</td>
</tr>
<tr>
<td>Fourth Procedure</td>
<td>21.90</td>
<td>19.75</td>
<td>16.44</td>
</tr>
</tbody>
</table>

### Table 2 SNR for Bean Coded Picture

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Red</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Procedure</td>
<td>22.87</td>
<td>21.98</td>
<td>20.00</td>
</tr>
<tr>
<td>Second Procedure</td>
<td>22.48</td>
<td>21.90</td>
<td>19.86</td>
</tr>
<tr>
<td>Third Procedure</td>
<td>21.35</td>
<td>21.76</td>
<td>19.05</td>
</tr>
<tr>
<td>Fourth Procedure</td>
<td>21.35</td>
<td>21.76</td>
<td>19.05</td>
</tr>
</tbody>
</table>

### 4. CONCLUSIONS

The foregoing results showed that using a fixed set of prediction parameters yielded good image reconstructions, which are close to the results of using real-time computed filter parameters. Also the variation of the SNR from one procedure to the other is small. The new procedures can be applied to any image outside the training set. In addition a 2-level quantizer is sufficient for these pictures.

### 5. REFERENCES


Fig. 1 Block diagram of predictive coding.

(a) Transmitter

(b) Receiver

Fig. 2 Original pictures.

Fig. 3 Coded lady’s face images.

Fig. 4 Quantized error image.

Fig. 5 Coded jelly beam image.