Network Coverage in Multicarrier Broadcast System

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Abstract—In this paper, we discuss the network coverage of the generic multicarrier (or OFDM) based broadcast system by exploiting the frequency diversity over subcarriers. We first define the performance metric and discuss the relationship among broadcast rate, coverage area and the outage probability. To calculate the actual broadcast coverage area, we provide analytical results for both the low SNR and high SNR cases. In addition, we extend our discussion to the multicarrier multicell broadcast network. Simulation results show that multicell cooperation(MCC) has the best performance.

Index Terms—Broadcast, OFDM, MCC, SFN, Coverage.

I. INTRODUCTION

Nowadays, with the fast development of wireless communications, the demand for multimedia data keeps increasing at an unprecedented speed. In particular, conventional TV broadcasting is experiencing a growing diffusion of new services. In many countries, the existing TV networks are converting from analog broadcasting to digital broadcasting with HDTV programs. On the other hand, cellular operators are also starting to offer multimedia services such as mobile TV and video on Demand (VoD). These new challenges require advanced transmission techniques like Orthogonal Frequency Division Multiplexing (OFDM) and Multi-cell Cooperation (MCC) to enhance the network performance.

Compared with single carrier broadcasting, multicarrier transmission (i.e., OFDM) supports broadband data transmission without complicated time-domain equalization and has been adopted in many digital video broadcasting standards such as DVB and MediaFLO. In multicarrier transmission, channel encoding can be implemented either independently on each subcarrier or jointly over all subcarriers. In general, multicarrier independent coding is easier to implement but increases the density of the transmission. The broadcast coverage with multicarrier independent coding has been studied in [1], where a collaborative broadcast and unicast hybrid network is presented. As to our best knowledge, the broadcast coverage with joint coding has not been studied. Comparing with independent coding, joint coding can minimize the average distortion when transmitting with random states (e.g slow-fading channels) [2]. It allows for a much more simple strategy to reach the optimal rate. The use of joint coding in the context of digital broadcasting of HDTV is shown to be an efficient alternative to the single-resolution independent coding technique [3]. It has been widely used in the multi-resolution TV broadcasting and many standards such as DVB-H and ATSC[4]. Due to the advantage of joint-coding in broadcasting, it is important to study its performance in terms of network coverage.

For broadcasting with multiple transmitters (base stations), the current networks mainly support two transmission modes: the traditional multi-frequency network (MFN) and the new single frequency network (SFN). In SFN, several transmitters simultaneously send the same signal over the same frequency channel, increasing the spatial diversity. Comparing to MFN, SFN is efficient in utilization of the radio spectrum, allowing more radio and TV programs. Also, SFNs offer advantages in terms of coverage and decrease the receiving outage in comparison to MFN [5]. To further improve the performance, multi-cell cooperation (MCC) prescribes the joint encoding of the transmitted signals from multiple base stations (BS), where BSs are inter-connected via the high capacity backbone network [6]. Comparing to SFN, MCC has larger broadcasting coverage with the same transmitting power [7]. Previous studies on SFN and MCC are mainly in single carrier network [8][9][10]. To the best of our knowledge, the broadcast coverage in multicarrier multicell network with joint coding has not been studied.

In this paper, we study the maximum broadcasting coverage in the multicarrier multicell network with joint-coding and MCC. We further discuss the result in [7] in joint-coding MCC and provide the analytical boundaries of the broadcast coverage. The paper is organized as follows. In Section II, the performance metric and system model are defined. Then in Section III, we analytically evaluate the performances of broadcast with OFDM transmission in single cell and multicells in both high SNR and low SNR cases. The simulation results are presented in Section IV. Finally, a conclusion is drawn in Section VI.

II. SYSTEM MODEL

The term broadcast has been used for both TV/radio broadcasting and cellular downlink system in wireless communications. To avoid ambiguity, we define broadcast network as the delivery of common information from the base station or tower to silent users in multicast applications. For the convenience of discussion, we refer to the two-way cellular network as a unicast network. Comparing to unicasting, there is no uplink in broadcast and the channel state information (CSI) is not available at the transmitter.
A. Performance metric

Channel capacity is the fundamental performance metric for communication systems. In information theory, there are two types of channel capacity definitions of a broadcast channel with an uninformed transmitter: the ergodic capacity (also known as the Shannon capacity) and the outage capacity. The ergodic capacity defines the maximum data rate that can be achieved with negligible error probability through all the fading states. It requires complicated receivers and leads to significant decoding delay, which is not suitable for broadcast applications. On the other hand, the outage capacity defines the maximum data rate that can be transmitted with a certain outage probability that the received data can not be decoded with negligible probability of error. By allowing some outage, the broadcast receiver can decode the message in every fading state so the delay requirement is met. Therefore, we choose the more practical outage capacity as the figure of merit to evaluate the broadcast system.

Generally, the broadcast receivers decode the same received signals with different outage probability based on different fading statistics since they are distributed over a large area. Within a given area \( A \), we define \( q(A) \) as the outage probability associated with the worst receiver in area \( A \). All the other receivers in the area have the outage probability lower than \( q(A) \). We denote the broadcast rate as \( r_0 \) and the maximum outage probability allowed as \( q_0 \). In order to optimize the broadcast performance, there are three alternative ways:

1) With the given broadcast rate \( r_0 \) and the maximum outage probability allowed \( q_0 \), maximize coverage \( A \) such that \( q(A) \leq q_0 \).
2) With the given coverage \( A \) and the maximum outage probability allowed \( q_0 \), maximize the broadcast information rate \( r_0 \) such that \( q(A) \leq q_0 \).
3) With the given broadcast rate \( r_0 \) and the coverage area \( A \), minimize the worst user’s outage probability \( q(A) \).

The above three ways are interchangeable. With any two of the three parameters given, we can optimize the third. In a practical broadcast network, \( r_0 \) and \( q_0 \) are usually prefixed and the goal is to maximize the coverage area \( A \) with \( q(A) \leq q_0 \) to ensure the quality of service (QoS).

To calculate the broadcast coverage, we assume the channel is Rayleigh flat fading and the envelope of the complex channel gain \( |h| \) for each spatial channel has the following distribution:

\[
 f(|h|) = \frac{|h|}{\sigma^2} \exp\left(-\frac{|h|^2}{\sigma^2}\right)
\]  
(1)

where \( E[|h|^2] = 2\sigma^2 \) is determined by path loss. For most of the calculation of the free space transmission path loss, the Hata model is widely used for distances of 1-100km. The formula for empirical path loss in urban areas using the Hata model is:

\[
P_L(d) dB = 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_t) - a(h_r) + (44.9 - 6.55 \log_{10}(h_t)) \log_{10}(d)
\]
(2)

where \( f_c \) is the carrier frequency, \( h_t \) and \( h_r \) are the height of the transmitter and receiver antenna, respectively, and \( d \) is the distance between the transmitter and receiver. For big cities at frequencies \( f_c > 300 \) MHz, the correction factor \( a(h_r) \) is given by:

\[
a(h_r) = 3.2(\log_{10}(11.75h_r))^2 - 4.97 \text{dB}
\]
(3)

From (2), the average received power from a cell on a channel at distance \( d \) is:

\[
2\sigma^2 = P10^{P_L(d)/10}
\]
(4)

where \( P \) is the total transmitted power.

According to Equation (2), the average path loss is a function of the distance between the base station and the receiver. With longer distance, the signal experiences more path loss and the receiver gets higher outage probability. For this reason, for a given coverage area \( A \), we can always assume the worst user \( w \) is on the edge of \( A \) and define \( q^w \) as the outage probability associated with this edge user \( w \). In this way, instead of considering all the users within area \( A \), we only need to focus on the performance of user \( w \).

B. Broadcast coverage in OFDM systems

In an OFDM broadcast network, the total system bandwidth \( B \) is divided into \( N \) frequency flat parallel orthogonal subcarriers. In the single cell case, according to the performance metric defined in Section II.A, the optimization problem is formulated as:

\[
\begin{align*}
\text{max} & \quad A, \\
\text{s.t.} & \quad q^w \leq q_0,
\end{align*}
\]
(5)

\[
q^w = \text{Prob}[h_w^w > \sum_{n=1}^{N} B_N \log_2\left(1 + \frac{P_n|h_w^w|^2}{N_0B_N}\right) < r_0],
\]
(7)

and

\[
\sum_{n=1}^{N} P_n = P.
\]
(8)

Equation (7) is the formula for calculating the outage probability with joint coding. \( N_0 \) is the power density of the white noise. \( B_N = \frac{B}{N} \) is the subcarrier bandwidth and \( r_0 \) is the predetermined broadcast rate. The \( h_w^w \) is the channel gain between the transmitter and the worst user \( w \) on subcarrier \( n \), which is a Rayleigh random variable. \( P_n \) is the transmitting power allocated to subcarrier \( n \) and is our optimization variable. In equation (7), \( \{h_w^w, h_2^w, \ldots, h_N^w\} \) are the only random variables, the outage probability \( q^w \) depends on the power allocation \( P_n \) and the distributions of \( h_n^w, n = 1, 2, \ldots, N \). Our objective is to maximize the broadcast coverage \( A \) with constraints (6) to (8).

In the multicell case, we focus on SFN and MCC. In SFN with \( N_t \) BSs, because all the BSs transmit the same signal, the effective channel gain for \( w \) is given by \( \sum_{m=1}^{N_t} \sqrt{P_n} h_n^w(m) \). Therefore, \( q^w \) in SFN is given by:

\[
q^w = \text{Prob}[h_w^w > \sum_{n=1}^{N} B_N \log_2\left(1 + \frac{\sum_{m=1}^{N_t} \sqrt{P_n} h_n^w(m)^2}{N_0B_N}\right) < r_0].
\]
(9)
Compared to SFN, MCC is expected to offer a larger broadcast coverage area with the same transmitting power since the BSs within the network collaboratively transmit signals, forming a distributed multiple input single output (MISO) system. If we denote the input signal as vector \(\mathbf{x} = [x(1), x(2), ..., x(N_t)]^T\), the outage probability \(q^w\) is given by [10]:

\[
q^w = \text{Prob} [\mathbf{h}^w : B \log_2 \det(\mathbf{I} + \frac{\mathbf{h}^w\mathbf{h}^w^H}{N_0B}) < r_0],
\]

where \(\mathbf{h}^w = [h^w(1), h^w(2), ..., h^w(N_t)]\) is the channel gain from each BS to the receiver \(w\) and \(\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H]\) is the input covariance matrix. As we have discussed, in the broadcast network, the base stations do not have the channel information of the receiver. We assume the channel follows the zero-mean spatially white (ZMSW) model. According to [11], the optimal power allocation scheme is equally allocating power to each base station, which indicates \(\mathbf{Q} = \frac{P}{N_t}\). Then Equation (10) becomes:

\[
q^w = \text{Prob} [\mathbf{h}^w : B \log_2 \det(\mathbf{I} + \frac{|\mathbf{h}^w|^2}{N_0B}) < r_0].
\]

It is worth noting that (10) is also applicable to the MIMO case, where each receiver has multiple antennas. In this case, we only need to change the vector \(\mathbf{h}^w\) to the channel gain matrix \(\mathbf{H}\).

III. BROADCAST ANALYSIS

From the previous discussion, we have already known that with a given outage probability threshold \(r_0\), the broadcast coverage area \(A\) is solely determined by the distribution of the random variable \(\Sigma\), which is defined as:

\[
\Sigma = \sum_{n=1}^{N} B_n \log_2 (1 + \frac{P_n |h_n|^2}{N_0 B_n}),
\]

s.t. \(\sum_{n=1}^{N} P_n = P\). In the equation above, channel gain \(|h_n|\) is a Rayleigh random variable for a general user so that \(|h_n|^2\) is chi-squared distributed with the following probability density function (PDF):

\[
f(x) = \frac{1}{\sigma^2} \exp(-\frac{x}{\sigma^2})
\]

where \(E[|h|^2] = \sigma^2\) is determined by Equations (3) and (4).

Notice that \(\frac{P_n|h_n|^2}{N_0 B_n}\) is the SNR on the subchannel \(n\) and can be denoted as \(SNR_n\). Equation (12) can be written as:

\[
\Sigma = \sum_{n=1}^{N} B_n \log_2 (1 + SNR_n).
\]

In order to calculate the broadcast coverage, the PDF of \(\Sigma\) needs to be known. Unfortunately, we cannot obtain the closed form formula due to the log operation and n-dimensional convolution in (14). The following analysis will induce the closed form expression for \(\Sigma\)'s distribution in low SNR case. In high SNR case, we are able to derive closed bounds for the PDF of \(\Sigma\).

A. \(\Sigma\) in the Low SNR Case

When the SNR on each subcarrier is very small, we have the following approximation of (12):

\[
\Sigma = \frac{1}{\ln 2} \sum_{n=1}^{N} B_n SNR_n = \frac{1}{\ln 2} \sum_{n=1}^{N} P_n |h_n|^2.
\]

Denote \(\sigma_n^2 = \frac{\sigma_n^2 P_n}{\ln 2 N_0 N_t} |h_n|^2\), then \(|h_n|^2\) is still a chi-squared distribution. The PDF of \(\Sigma\) can be obtained by doing the convolution:

\[
f_\Sigma(y) = f_{h_1^2}(y) \times f_{h_2^2}(y) \times \cdots \times f_{h_N^2}(y).
\]

The calculation of (16) is nontrivial. Due to the space limit, we give the following results without detailed derivations. We obtain the PDF of \(\Sigma\) as:

\[
f_\Sigma(y) = \prod_{i=1}^{N} \frac{\sigma_i^{-2N-4}}{\ln 2 - \sigma_i^2} \exp(-\frac{y}{\sigma_i^2}).
\]

It is worth noting that in (17), for any \(i \neq j, \sigma_i^2 \neq \sigma_j^2\). In the special case where \(\sigma_i^2 = \sigma_j^2 = \cdots = \sigma_N^2 = \sigma^2\), (17) becomes:

\[
f_\Sigma(y) = \frac{y^{N-1}}{(N-1)!\sigma^{2N}} \exp(-\frac{y}{\sigma^2}).
\]

In the case of MCC, using the similar approach and letting \(\sigma_n^2 = \frac{\sigma_n^2 P_n}{\ln 2 N_0 N_t}\) and \(|h_n|^2 = \frac{P_n}{\ln 2 N_0 N_t} |h_n|^2\), we derive the PDF of \(\Sigma\) as:

\[
f_{\Sigma,MCC}(y) = \prod_{i=1}^{N} \frac{\sigma_i^{-2N-4}}{\ln 2 - \sigma_i^2} \exp(-\frac{y}{\sigma_i^2}).
\]

When the number of subcarriers \(N\) is large, (17)-(19) are still very complicated to calculate. In order to compute \(\Sigma\)'s PDF for a large \(N\), we use the Large Number Theorem to approximate \(\Sigma\) as Gaussian random variable, then we only need to calculate the mean and variance of \(\Sigma\), which are given as:

\[
E[\Sigma] = \frac{1}{\ln 2} \sum_{n=1}^{N} P_n \sigma_n^2
\]

and

\[
E[\Sigma^2] = \frac{1}{(\ln 2)^2 N_0^2} \left[ \sum_{n=1}^{N} P_n^2 \sigma_n^4 + \sum_{n \neq m} P_n P_m \sigma_n^2 \sigma_m^2 \right].
\]
B. Σ in the High SNR case

If the SNR is very large, Equation (12) can be written as:

\[ \Sigma = \sum_{n=1}^{N} \left[ \ln(SNR_n) \times \frac{B_n}{\ln 2} \right]. \tag{22} \]

Since \( SNR_n \) is deterministic, we only need to focus on \( \sum_{n=1}^{N} \ln(|h_n|^2) \) to determine the distribution of \( \Sigma \). Based on (13), the PDF of \( \ln(|h|^2) \) is:

\[ f_n(y) = \frac{1}{\sigma_n^2} \exp\left(-\frac{y}{\sigma_n^2}\right). \tag{23} \]

Similar to (16), the PDF of \( \Sigma \) can be obtained:

\[ f_\Sigma(y) = \prod_{n=1}^{N} \left[ \exp\left(-\frac{y}{\sigma_n^2}\right) \times \exp\left(-\frac{y}{\sigma_n^2}\right) \times \cdots \times \exp\left(-\frac{y}{\sigma_n^2}\right) \right]. \tag{24} \]

Unfortunately, when calculating the convolution, we need to compute the integral \( \int \exp(Ae^y - Be^{-y})dy \), which has no closed form. Next, we derive both the upper bound and the lower bound for the function \( f_\Sigma(y) \). Without the loss of generality, we assume \( \sigma_1^2 \leq \sigma_2^2 \leq \cdots \leq \sigma_N^2 \). The convolution of the first two terms is calculated as:

\[ \exp\left(y - \frac{\sigma_1^2}{\sigma_2^2}\right) \times \exp\left(y - \frac{\sigma_1^2}{\sigma_3^2}\right) = e^y \int_1^{\infty} \frac{1}{k} \exp\left[-\left(\frac{\sigma_1^2}{\sigma_2^2} + \frac{1}{\sigma_1^2}\right) k \right] \, dk. \tag{25} \]

Let \( k = e^z \), and re-write the Equation (25) and define \( g(y) \) as:

\[ g(y) = e^y \int_1^{\infty} \frac{1}{k} \exp\left[-\left(\frac{\sigma_1^2}{\sigma_2^2} + \frac{1}{\sigma_1^2}\right) k \right] \, dk. \tag{26} \]

Because \( k \geq 1 \), we have the following inequality:

\[ e^y \int_1^{\infty} \frac{1}{k} \exp\left[-\left(\frac{\sigma_1^2}{\sigma_2^2} + \frac{1}{\sigma_1^2}\right) k \right] \, dk \leq g(y) \]

\[ \leq e^y \int_1^{\infty} \frac{1}{k} \exp\left[-\left(\frac{\sigma_1^2}{\sigma_2^2} + \frac{1}{\sigma_1^2}\right) k \right] \, dk. \tag{27} \]

For the left side of the Equation (27):

\[ e^y \int_1^{\infty} \frac{1}{k} \exp\left[-\left(\frac{\sigma_1^2}{\sigma_2^2} + \frac{1}{\sigma_1^2}\right) k \right] \, dk = \exp\left(y - \frac{\sigma_1^2}{\sigma_2^2} \right) \Gamma\left(0, \frac{1}{\sigma_2^2}\right) \tag{28} \]

where \( \Gamma(s, x) = \int_x^{\infty} t^{s-1} \exp(-t) \, dt \) is called the upper incomplete gamma function. Because \( 0 \leq y \) and \( e^{-y} \leq 1 \leq \frac{\sigma_1^2}{\sigma_2^2} \), we have:

\[ \Gamma\left(0, \frac{1}{\sigma_1^2}\right) - \Gamma\left(0, \frac{1}{\sigma_2^2}\right) \leq \Gamma\left(0, \frac{1}{\sigma_1^2}\right) - \Gamma\left(0, \frac{\sigma_1^2}{\sigma_2^2}\right) \tag{29} \]

Repeat the above procedures for (25) to (29) for \( \sigma_2^2, \sigma_3^2 \) and so on, we can get the lower bound as:

\[ \exp\left(y - \frac{\sigma_1^2}{\sigma_N^2}\right) \prod_{n=1}^{N-1} \left[ \Gamma\left(0, \frac{1}{\sigma_n^2}\right) - \Gamma\left(0, \frac{1}{\sigma_{n+1}^2}\right) \right]. \tag{30} \]

Similarly, the upper bound of the Equation (27) is calculated as:

\[ \exp\left(y - \frac{\sigma_1^2}{\sigma_N^2}\right) \times \Gamma\left(0, \frac{1}{\sigma_N^2}\right) - \Gamma\left(0, \frac{\sigma_1^2}{\sigma_N^2}\right) \times \prod_{n=2}^{N-1} \left[ \Gamma\left(0, \frac{1}{\sigma_n^2}\right) - \Gamma\left(0, \frac{1}{\sigma_{n+1}^2}\right) \right]. \tag{31} \]

For \( \Gamma(0, x) \), we can compute it with the following continued fraction:

\[ \Gamma(0, x) = \frac{e^{-x}}{x + \frac{1}{1 + \frac{1}{x + \frac{1}{1 + \frac{1}{x + \cdots}}}}} \tag{32} \]

IV. SIMULATION RESULTS

In this section, the numerical results are provided to evaluate the performance of the broadcasting system under different configurations. Without loss of generality, we choose the broadcast \( r_0 = 515 \) Kbps and the total power to be \( P = 1 \) watt such that the radius \( d_0 = 1 \) km is the benchmark distance with outage probability \( q_0(d_0) = 5\% \). Fig.1 shows the distribution of \( \Sigma \) in the low SNR region with \( N = 32 \). Compared to the single carrier case, the performance of multicarrier with joint coding is much better. Also, we can see the PDF curve of \( \Sigma \) is close to the Gaussian approximation, which is reasonable according to the Central Limit. Theorem and Large Number Theorem. The simulation results match our analysis in Section III very well. Fig.2 shows the coverage of the multicarrier and single carrier transmission with different outage probabilities \( q_0 = 1\%, 5\% \) and 10\%. We can see multicarrier with joint coding provides significant channel gain compared to the single carrier case.

![Fig. 1. PDF of Σ with N = 32](image)

![Fig. 2. Broadcast coverage area with N = 32](image)

In Fig.3 and 4, we show the broadcast coverage area for two and three cell cases. We can see the coverage area with MCC multicarrier (MCC-MC) is the largest in both cases. This
is due to the full exploitation of the transmission diversity in both frequency and space. Table I and II give the numerical values of the coverage area with 3 cells.

Fig. 5 shows the relationship of the separation distances to the total broadcast coverage in multi-cell case. We can see that the coverage area will initially increases with the BSs separation distances. When the separation distance increases to a threshold which is about twice the radius of the coverage of single cell, multiple BSs are no longer cooperating and the coverage area becomes the sum of the coverage of each individual cell.

V. CONCLUSION

In this paper, we investigated the network coverage in a OFDM broadcast system with multicarrier joint coding. The outage capacity and the coverage area were chosen to be the figure of merit. We analyzed how to calculate the broadcast coverage. In particular, a closed form of the $\Sigma$ distribution in low SNR region is derived. In the high SNR region, we provided the lower and upper bounds for the PDF of $\Sigma$. Simulation results showed that the broadcast coverage area can be significantly increased by adopting the OFDM transmission with joint subcarrier encoding and the MCC.

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