Acceleration Sensitivity of Small-Gap Capacitive Micromechanical Resonator Oscillators

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Abstract—The vector components of acceleration sensitivity $\Gamma$ for a closed-loop oscillator referenced to a wine-glass disk array-composite resonator employing tiny (~92nm) electrode-to-resonator capacitive transducer gaps were measured along axes perpendicular and parallel to the substrate to be $\Gamma_{\text{vertical}}$~13.6ppb/g and $\Gamma_{\text{lateral}}$~4.92ppb/g, respectively, which are on par with commercial quartz-based oscillator products. Interestingly, the measured acceleration sensitivity greatly exceeds the prediction of theory. In particular, models for frequency shifts due to variations in electrical stiffness and mechanical stress predict acceleration sensitivities orders of magnitude lower than measured here. Consideration of other microphonic contributors reveals that the measurements of this work were probably limited by the bond wires and package stresses of the board-level realization of the oscillator, so are very likely not representative of the performance actually achievable by a fully-integrated micromechanical resonator oscillator, where MEMS and transistors share a single chip. Still, the measured microphonic performance on par with mid-grade quartz oscillators at least provides some reassurance that the tiny electrode-to-resonator gaps used in high frequency capacitively transduced micromechanical resonators will not compromise the stabilities of oscillators referenced to them in conventional applications that currently accept mid-grade quartz resonators.

Keywords—MEMS, microphonics, acceleration sensitivity, vibration, capacitive transducer, small gap, electrical stiffness.

I. INTRODUCTION

Micromechanical resonators constructed via MEMS technology have recently been spotlighted as potential next generation mechanical signal processors for use in oscillators, filters, mixers, and even amplifiers. Beyond offering substantial reductions in size, power consumption, and manufacturing cost, their potential for single-chip integration with CMOS integrated circuits is expected to yield a degree of system-level miniaturization and resultant parasitic loss reductions that encourage a paradigm shift in wireless communication system architecture [1]. Indeed, capacitively transduced micromechanical resonators, such as shown in Fig. 1, that use voltage-induced electric fields across electrode-to-resonator gaps to excite and sense resonance have now been demonstrated with long-term stability less than $\pm 2$ ppm over 10,000 hours of operation [2]; excellent temperature stability of better than $\pm 0.05$ ppm over -20°C–80°C using less than 18 mW of oven power [3]; quality factor $Q$ greater than 10,000 at GHz frequencies [4], and $fQ$ products exceeding $5.1 \times 10^{13}$ [5]. Moreover, the recent demonstration of phase-noise performance that meets commercial global systems for mobile communication (GSM) specifications suggests that micromechanical resonators should soon be usable in higher-end applications beyond consumer electronics, where micromechanical resonator-based oscillators are already commercialized and now encroaching on markets traditionally dominated by quartz [6].

While the use of capacitive transduction provides many benefits, including higher $Q$ and a larger available set of materials, early capacitive transducer realizations suffered from a relatively weaker electromechanical coupling factor compared to other transduction methods. The resulting high motional

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impedance then made difficult matching to low (e.g., 50Ω) impedance systems. Among methods for improving electromechanical coupling in capacitive transducers, reducing the electrode-to-resonator gap spacing is mathematically the most effective [7]. Doing so, however, is not a simple prospect when the gaps are lateral ones, such as in Fig. 1. For example, although e-beam lithography and subsequent etching to form lateral gaps has been suggested as one possible way to reduce gaps over those obtainable via conventional lithography [8], its low throughput and high cost pose significant barriers to volume production. As a result, sidewall sacrificial spacer methods, such as first used in [9], have dominated among methods for achieving lateral gaps down to 50nm, but below this suffer from etchant and etch by-product diffusion limitations that compromise yield. The latest remedy to this uses atomic layer deposition (ALD) to deposit high-k dielectric material into a sacrificial spacer-defined gap, partially (but not fully) filling it to achieve a much smaller effective gap spacing [7]. This approach recently achieved repeatable gap spacings as small as 37nm that enabled the first micromechanical resonators with simultaneous high Q (>70,000) and low motional impedance (Rx<130Ω) at 61 MHz [10].

But gap reduction also raises concerns regarding susceptibility to acceleration, i.e., to microphonics. Indeed, many oscillators find use in dynamic platforms, for which resilience against vibration is important. Acceleration from environmental vibration is well known to degrade the short-term stability of oscillators, especially ones referenced to mechanical resonators that respond to acceleration by shifts in resonance frequency. Needless to say, acceleration sensitivity has been studied extensively in quartz-crystal oscillators [11].

The acceleration-induced resonance frequency shift Δf for a mechanical resonator is often expressed as,

\[ \Delta f = \frac{\Gamma \cdot \tilde{a}}{f} \quad \text{with} \quad \Gamma = \Gamma_\text{x} \hat{x} + \Gamma_\text{y} \hat{y} + \Gamma_\text{z} \hat{z} \]  

where f is the resonance frequency, \( \tilde{a} \) is the acceleration in vector form, and \( \Gamma \) is the acceleration sensitivity of the resonator gauging the amount of the frequency shift for a given acceleration.

If an oscillator with a mechanical reference resonator experiences a vibration at frequency \( f_\text{v} \), its output signal phase is modulated by the instantaneous frequency dependency of the resonator governed by (1), creating sideband peaks in the output power spectrum as shown in the schematic in Fig. 2. Since only power at the carrier frequency constitutes the output of an oscillator, and power at all other frequencies is considered noise, these side peaks can be interpreted as noise at a frequency offset of \( f_\text{v} \) from the carrier, and their relative height can be expressed as

\[
L(f_\text{v}) = 20 \log \left( \frac{(\Gamma \cdot \tilde{a}) \cdot f}{2f_\text{v}} \right)
\]

It should be noted that, in the presence of random vibration, this noise would appear as \( 1/f^2 \) noise, and would therefore dominate the oscillator phase noise performance. Indeed, resonator acceleration sensitivity often governs oscillator performance in dynamic platforms.

Recently, the acceleration sensitivities of oscillators referenced to capacitively transduced micromechanical resonators with gaps of 1μm and 150nm were measured and reported [12, 13]. This work focuses on smaller-gapped capacitive micromechanical resonator oscillators, with an aim to elucidate the fundamental mechanisms governing the acceleration sensitivity of micromechanical resonators that employ electrode-to-resonator gaps with spacings less than 100nm. Two main mechanisms are considered: 1) changes in electrical stiffness caused by acceleration-induced changes in electrode-to-resonator gap spacing; and 2) the effect of acceleration-induced stress on the resonant structure. Using ANSYS finite element simulation, silicon surface micromachined resonators are modeled and analyzed, and these models are compared with measurement data. Other potential mechanisms associated with the testing apparatus that can dominate the acceleration sensitivity of tested resonator oscillators are discussed, as well.

II. Potential Contributors to Acceleration Sensitivity

A. Acceleration-Induced Electrical Stiffness Changes

Spring softening due to electrical stiffness is a well known non-linear dynamical phenomenon associated with capacitive transduction. In capacitive transduction, as described in Fig. 3, the electrical drive force generated by a constant bias voltage \( V_p \) applied across the electrode-to-resonator gap increases as the resonator approaches the electrode, i.e., as the capacitive gap decreases, and decreases as the resonator moves farther from the electrode. Since the change in force is proportional to displacement, it can be modeled as an effective electrical stiffness that can be expressed as

\[
k_e = \frac{\partial F_e}{\partial x} = \frac{\varepsilon A V_p^2}{d^2}
\]

where \( \varepsilon \) and A are the permittivity and overlap area, respec-

Fig. 3: Lumped model of a capacitively transduced micromechanical resonator. The electrostatic force depends on the gap size between the resonator and the electrode; as the resonator displacement \( x \) increases, the gap size decreases resulting in an increase in the electrostatic force. This displacement dependency of the electrostatic force behaves similarly to a mechanical restoring force, but in the opposite direction.
tively, of the electrode-to-resonator gap. Since it acts against the restoring force of the resonator’s mechanical stiffness, \( k_e \) subtracts from the overall stiffness of the resonator structure, yielding a resonance frequency given by

\[
f_r = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{k_e - k_w}{m}} = f_o \sqrt{1 - \frac{k_e}{k_w}} \tag{4}
\]

where \( k \) is the overall stiffness, \( m \) is the resonator mass, \( k_w \) is the mechanical stiffness, and \( f_o \) is the original natural frequency of the resonator (with \( V_p = 0V \)).

Using (3) and (4), the fractional frequency change of the resonance frequency brought about by acceleration-induced changes in gap spacing and overlap area can be expressed as

\[
\frac{\Delta f}{f} = \frac{1}{2} \frac{V_p^2}{d^2 k_w} \Delta d \quad \text{for gap spacing change}
\]

\[
\frac{\Delta f}{f} = -\frac{1}{2} \frac{V_p^2}{d^2 k_n} \Delta A \quad \text{for overlap area change}
\]

where \( \Delta d \) is change in gap spacing and \( \Delta A \) is change in overlap area. Note that as the electrode-to-resonator gap \( d \) shrinks, (5) predicts much larger shifts in frequency shifts due to acceleration-induced shifts in both cases.

B. Acceleration-Induced Stress Changes

As a mechanical resonator experiences acceleration, a stress gradient is induced in the resonant structure, which results in a shift in resonance frequency. Reference [12] investigated this stress effect for the case of beam-type MEMS, for which the principal mechanism for acceleration sensitivity is axial stress due to the D’Alembert force from a floating coupled mass. This magnitude of this effect depends on the magnitude of the induced force, making the resonator mass an important factor. In other words, as mass increases, acceleration induces more force, which then generates a larger shift in the resonance frequency. In addition, the stress distribution, i.e., locations of highest stress, also influence the amount of frequency shift. Finite element simulation is an effective means for modeling such acceleration induced-stress distributions on a resonant structure and for linking them to the resulting resonance frequency change.

C. Vibration Susceptibility of Non-Resonant Components

Many microwave electronic components are known to be sensitive to vibration, from coaxial cables that typically exhibit acceleration sensitivities from \( 10^{-3} \) to \( 10^{0}\text{rad/g} \) [14]; to bandpass filters; mechanical phase shifters; and printed circuit boards [15]. Indeed, investigations on the role of such components on oscillator acceleration sensitivity [14, 16] reveal that they can often dominate. Thus, the resolution of any board-level test set-up gauging the acceleration sensitivity of the micromechanical resonator oscillators of interest, here, is ultimately limited by contributions from non-resonant board-level components.

III. MODELING

To investigate the acceleration sensitivity of small-gap capacitively transduced micromechanical resonators, this work employs the polysilicon wine-glass mode disk array-composite resonator depicted in Fig. 4. Here, five disk resonators are coupled by half-wavelength beams to form a composite resonator structure that vibrates at a designed mode frequency. Because it uses several resonators, this composite resonator handles more power and exhibits lower impedance than a single stand-alone resonator, both of which are beneficial to oscillator design and performance. Each disk has a radius of \( R=32\text{um} \) and thickness of \( h=3\text{um} \), and is suspended by two support beams attached opposite one another at wine-glass mode nodal points, as indicated in Fig. 4.

When \( n \) resonators are assembled into an array-composite, the composite structure takes on values of stiffness and mass that are \( n \) times the value at the same location on a single stand-alone resonator. In other words, the composite structure is \( n \) times stiffer than any one of its constituent resonators. However, the mass of the composite is also larger, so forces generated by accelerations will likely also be approximately \( n \) times larger, suggesting that the acceleration response of the composite should be similar to that of a single resonator, although not exactly the same. In order to simplify the analysis so as to more easily delineate important dependencies, the analyses to follow are done on a single resonator, with the understanding that they will only be approximate for the array-composite actually tested.

Using models from [17], the mechanical stiffness at the anti-nodes of a stand-alone constituent disk resonator in Fig. 4 is calculated to be \( k_w=6.61\times10^9\text{ N/m} \). The capacitive transducer area for the disk is \( A=5.04\times10^{-10}\text{m}^2 \). The resonance frequency is \( f=61\text{MHz} \), and the sacrificial sidewall spacer-defined [18] electrode-to-resonator gap is \( d=92\text{nm} \). At \( V_p=12\text{V} \), using (3), the electrical stiffness is calculated to be \( k_e=885\text{N/m} \). These values are used to obtain some of the numbers in the analyses to follow.

A. Acceleration-Induced Electrical Stiffness Changes

1) Vertical Acceleration

When a disk resonator experiences a vertical (or \( z \)-directed) acceleration, the resonator displaces in that direction, yielding a change in transducer area \( A \), as shown in Fig. 5. The resulting change in electrical stiffness, \( k_e \), can be expressed as
The fractional transducer area change, \( \Delta A/A \), can be approximated as the resonator displacement, \( \delta \), with respect to the resonator thickness, \( h \), after which equation (6-1) can be expressed as,

\[
\Delta k_e = k_e \frac{\delta}{h} \tag{6-2}
\]

Using ANSYS finite element modeling, the resonator displacement per unit acceleration in the \( z \)-direction is simulated to be \( \delta = 8.71 \times 10^5 \text{nm/g} \). Using this in (1), (5), and (6-2), the \( z \)-direction acceleration sensitivity for a capacitively transduced micromechanical disk resonator becomes

\[
\Gamma_{z, \text{elec.stiff}} = 1.81 \times 10^{-11} / \text{g} \tag{7}
\]

2) Lateral Acceleration

When a disk experiences a lateral (i.e., in plane) acceleration, its transducer gap, \( d \), changes as shown in Fig. 6a. Unlike the case of vertical acceleration, since the resonator is anchored diagonally, its displacement varies depending on the orientation of the acceleration. As shown in the ANSYS finite element simulation of Fig. 6b, the resonator displacement is maximized when the acceleration is perpendicular to the anchors and bends the supporting beams flexurally. In contrast, when the acceleration is parallel to the anchors, it must act against substantially stiffer restoring forces that minimize displacements along this direction, thus, minimizing the acceleration sensitivity. Fig. 7 presents plots of \( x \) and \( y \) direction resonator displacements (\( \delta_x \) and \( \delta_y \)) with respect to the orientation of the acceleration. The directional dependence of resonator displacements in units of nano-meter per g can be expressed as

\[
\Gamma_{x, \text{elec.stiff}} = 1.80 \times 10^{-16} / \text{g} \tag{8}
\]

For lateral accelerations, the overall change in electrical stiffness becomes more complicated when electrodes are placed symmetrically around the resonator, as is the case for the wine-glass disk under consideration. Fig. 8a shows an example case when the disk displaces in the \( y \)-direction. Here, the electrode-to-resonator gap decreases by \( \delta \) on the upper side, but increases by \( \delta \) on the lower side. As a result, \( k_e \) increases at the upper side, but decreases at the lower side. The overall resonator stiffness contribution from electrical stiffness can thus be expressed as the difference between these two:

\[
\Delta k_e = \Delta k_{e,x} - \Delta k_{e,y} = 6 \frac{A}{d} \frac{e/\varepsilon}{2} \left( \frac{\delta}{d} \right)^2 = \frac{3}{2} k_e \left( \frac{\delta}{d} \right)^2 \tag{9}
\]

Combining (8) and (9), expressions for lateral acceleration sensitivity can be expressed as

\[
\Gamma_{x, \text{elec.stiff}} = 1.27 \times 10^{-16} / \text{g} \tag{10}
\]

\[
\Gamma_{y, \text{elec.stiff}} = -1.27 \times 10^{-16} / \text{g}
\]
and plotted as shown in Fig. 8b. Note that the predicted lateral direction acceleration sensitivities of (9) are substantially smaller than the vertical direction sensitivities modeled in (7) by several orders of magnitude. Clearly, cancellation of electrical stiffness contributions by symmetrical electrode placement is an effective means for nulling electrical-stiffness-based acceleration sensitivity in the lateral direction.

**B. Acceleration-Induced Stress Changes**

The change in resonance frequency due to acceleration-induced stress changes was explored by means of ANSYS finite element simulation, as shown in Fig. 9. Again, lateral acceleration sensitivity exhibits directionality with respect to the acceleration orientation, i.e. when the acceleration is perpendicular to the anchors maximum stress/strain is induced yielding the largest frequency shift, and when the acceleration is parallel to the anchors minimum stress/strain is induced, thus producing the least frequency shift. From the ANSYS-simulated plot in Fig. 9b, expressions for stress-based acceleration sensitivity for the resonance frequency of a wine-glass disk resonator can be summarized as

\[
\Gamma_{\text{perpendicular to anchors, } \text{elec. stiff}} = -1.80 \times 10^{-16} / g \\
\Gamma_{\text{parallel to anchors, } \text{elec. stiff}} = -6.48 \times 10^{-19} / g
\]

and plotted as shown in Fig. 8b. Note that the predicted lateral direction acceleration sensitivities of (9) are substantially smaller than the vertical direction sensitivities modeled in (7) by several orders of magnitude. Clearly, cancellation of electrical stiffness contributions by symmetrical electrode placement is an effective means for nulling electrical-stiffness-based acceleration sensitivity in the lateral direction.

\[
\Gamma_{x, \text{stress}} = 1.73 \times 10^{-14} / g \\
\Gamma_{y, \text{stress}} = -1.71 \times 10^{-14} / g \\
\Gamma_{z, \text{stress}} = -2.02 \times 10^{-15} / g
\]

or

\[
\Gamma_{\text{perpendicular to anchors, stress}} = -4.94 \times 10^{-14} / g \\
\Gamma_{\text{parallel to anchors, stress}} = -2.64 \times 10^{-16} / g
\]

(11)

It should be noted that the acceleration sensitivity values in (11) are several orders of magnitude smaller than those for tuning fork beam-type resonators reported in [12], which were \(\Gamma_{\text{tuning fork}} = 10^{-15} \sim 10^{-9} / g\). This is because 1) the anchors for the wine-glass mode resonators in this study are strategically placed at vibration nodes, so that induced stresses are mostly local with very little impact on the rest of the resonant structure, as can be seen in Fig. 9a; and 2) high-frequency disk resonators have smaller masses, so induce much less force than low frequency tuning forks under identical accelerations.

**IV. MEASUREMENT**

Fig. 10 illustrates the experimental setup used to measure acceleration sensitivity of oscillators referenced to wine-glass disk array-composite resonators, such as shown in Fig. 4. The rudimentary sustaining oscillator used in these measurements consisted of an SA5211 transimpedance amplifier and an AD8322 clamping amplifier, and connected with the resonator die on the printed circuit board (PCB) shown in Fig. 10b. Freescale accelerometers (MMA1213 and MMA3201) housed on another PCB were used to accurately measure acceleration. A Bruel & Kjaer Type 4809 shaker, to which both PCB’s were attached, provided accelerations while an Agilent 8711A spectrum analyzer measured the output power of the oscillator. Fig. 11 presents a measured plot of the oscillator output power spectrum when the oscillator experiences a lateral vibration of 15g (rms) at 200Hz. From plots like Fig. 11, the heights of sideband peaks were determined and the acceleration sensitivity, \(\Gamma\), extracted using (3). A statistically relevant set of data was obtained by exciting the oscillator in both lateral and vertical directions with various acceleration amplitudes at 200kHz and 300kHz.

Fig. 12 summarizes the measured acceleration sensitivity values, which are seen to vary very little with respect to the
the disk array-composite resonator-based oscillators were vibration frequencies or the acceleration amplitudes. The oscillator circuit might dominate among frequency shifting mechanisms. Theoretical models predict much lower acceleration sensitivities than measured here, which suggests that components other than the resonator in the oscillator circuit might dominate among frequency shifting mechanisms.

From the data, the micromechanical resonator oscillators of this work performed on par with mid-range commercial quartz-based oscillators. However, theoretical models predict much lower acceleration sensitivities than measured here. Although on par with commercial quartz-based oscillator products, the measured values are still far higher than the 0.0181 ppb/g predicted by theory, when assuming electrical stiffness and mechanical stresses as major mechanisms for acceleration sensitivity. Non-resonant board-level components, such as reconfiguring wires and bond wires, are suspected as limiting sources of acceleration sensitivity in this work. Work to fully integrate micromechanical resonator oscillators on single-chips, so as to remove the board and its acceleration sensitive components, is presently underway with the aim of measuring the true acceleration insensitivity achievable via MEMS technology.

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